APPLICATION OF GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY (GARCH) MODEL TO DETERMINE THE VALUE AT RISK ON THE ANALYSIS OF RISK INVESTMENT

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ABSTRACT: *The aims of this study were to find the best Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, to predict volatility, and to determine the Value at Risk on the Data Composite Stock Price Index (IHSG) over the period from January 2011 to February 2016. In time series data, sometimes we faced a problem that the variance is not a constant or heteroscedasticity. One of a model that can be used to deal with this condition is GARCH model. The GARCH model can be used to forecast volatility. Based on the results of calculation of Value at Risk, GARCH model can be used to estimate the risk of investment. Based on the analysis results, it was found that the best model is the ARMA (2,2)-GARCH (1,1) and the value of the Value at Risk on the level of confidence 95% for one period ahead. By square root of the forecast of the variance, we found that the forecast of the volatility is as big as 0.011943077. This means that if an investor allocates fund Rp.100,000,000.00 to invest, there is a 5% chance of occurrence of losses in excess of Rp1.966.458,00 during the next 24 hours.*

Keywords: heteroscedasticity, Volatility, Value at Risk, GARCH model

1. INTRODUCTION

The time series data is a set of observations, each one being recorded at a specific time [1]. The data are either has a pattern or did not have a pattern. The time series that has a repeating pattern called seasonal time series data. For example, data movement share price of a company. In the case of non-seasonal time series, the method of Box Jenkins modeling can be used by determining some criteria to find the ARMA and ARIMA models [2, 3, 4, 5]. Model *Seasonal-*ARIMA (SARIMA) sometimes gives the estimation values far from the expected values [6]. While Autoregressive Moving Average (ARMA) model has assumption that the variance is constant. It is commonly found that the time series data has not constant variance. For example, the share price has return price which is not constant across time, and this is called heteroscedasticity. Heteroscedasticity is a disturbance for regression model. If it is known that the time series data have non constant variances over time, and then we force to use ARMA model. This implies that the forecast values will have a wide confidence interval. Value at Risk is a measurement of the worse risk of investment at certain level of confidence at the normal market condition. To calculate the Value at Risk, we need to forecast the volatilities. Volatility, or the standard deviation of returns, is the main measure of risk in most financial analysis. It is sufficient as a risk measure only when financial returns are normally distributed [7]. However, an assumption of normality for returns is violated for most if not all financial returns. For that reason the use of volatility as a risk measure can lead to misleading conclusions [7, 8].Time series model with assumption that the variances are not constant (heteroscedasticity), we can use volatility modeling. According to Bollerslev [9], time series data which have property heteroscedasticity can be modeled by using Generalized Autoregressive Conditional Heteroscedastic (GARCH) model. In this study, we will use GARCH model to calculate the *Value at Risk* for daily share price data at closing time of Composite Stock Price Index (IHSG Jakarta).

2. TIME SERIES MODELING

Prior we conduct an analysis of time series data, the assumption of stationarity should be fulfill by the data. Stationarity is one of the fundamental concepts in time series analysis [10, 11]. A stationary time series is one whose statistical properties such as mean, variance, autocorrelation are all constant over time.

Augmented Dickey-Fuller (ADF) Test

Some time series data tend to be nonstationary, for example a price series data, the nonstationary due to the fact that there is no fixed level for price. Such a nonstationary series is called unit-root nonstationary time series [8], . a **unit root** is a feature of some stochastic processes that can cause problems in time series modeling [3]. Let the regression equation is as follows:

$$x_t = \delta + \sum_{i=1}^{p} \phi_i x_{t-i} + \varepsilon_t$$

One of the unit root tests is Augmented Dickey Fuller (ADF) test. The ADF test is conducted through the calculation of the value of τ (tau) statistic by the following formula:

$$\tau = \frac{\hat{\phi}_k}{SE(\hat{\phi}_k)}$$

The hypothesis is as follows: $H_0 : \phi = 0$ (x_t is nonstationary) vs $H_a : \phi \neq 0$ (x_t is stationary).
If τ statistic $< \tau$ table, then H_0 is not rejected and the data are nonstationary [1].

Autoregressive Of Order p, AR(p), Model

The general form of the-p order Autoregressive model is

$$x_t = \delta + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + \varepsilon_t$$

where ε_t is *white noise*. The equation above can be written as:

$$\Phi(B)x_t = \delta + \varepsilon_t$$

where $\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$.

For AR (p) stationary we have

$$E(X_t) = \mu = \frac{\delta}{1 - \phi_1 - \phi_2 - \cdots - \phi_p}$$

and

$$\gamma_y(k) = cov\ (x_t, x_{t-k})$$
$$= cov(\ \delta + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p}$$
$$+ \varepsilon_t, x_{t-k})$$
$$= \sum_{i=1}^{p} \phi_i\ cov(x_{t-i}, x_{t-k}) + cov(\varepsilon_t, x_{t-k})$$
$$= \sum_{i=1}^{p} \phi_i\ \gamma_y(k-i) + \begin{cases} \sigma^2 & if\ k = 0 \\ 0 & if\ k > 0 \end{cases}$$

Then we have

$$\gamma_y(0) = \sum_{i=1}^{p} \phi_i\ \gamma_y(i) + \sigma^2$$

$$\Rightarrow \gamma_y(0)\left[1 - \sum_{i=1}^{p} \phi_i \rho_y(i)\right] = \sigma^2$$

The result from division of equation above by $\gamma_y(0)$ for k > 0 can be used to find the value of ACF for the AR (p) process which is satisfy the *Yule-Walker* equation

$$\rho_y(k) = \sum_{i=1}^{p} \phi_i\ \rho_y(k-i) \quad k = 1, 2, \ldots$$

Moving Average (MA) Model
Moving average model of order q is denoted by MA (q) and defined as:

$$x_t = \mu + \varepsilon_t - \theta_1\ \varepsilon_{t-1} - \theta_2\ \varepsilon_{t-2} - \theta_3\ \varepsilon_{t-3} - \ldots - \theta_q\ \varepsilon_{t-q}\ ;$$
$$\varepsilon_t \sim N\ (0, \sigma^2)$$

where x_t is the value at the-t time index, ε_t is the error at the-t time, θ_i is the regression coefficient i= 1,2,3, …,q.

The equation above can be written by backshift operator (B), as follows:

$$x_t = \mu + (1 - \theta_1\ B - \theta_2\ B^2 - \ldots - \theta_q\ B^q)\ \varepsilon_t$$
$$= \mu + (1 - \sum_{i=1}^{q} \theta_i B^i)\ \varepsilon_t = \mu + \Theta(B)\ \varepsilon_t$$

where $\Theta(B) = 1 - \sum_{i=1}^{q} \theta_i B^i$

Since ε_t is white noise, the expected value of MA (q) is
$$E\ (x_t) = E\ (\mu + \varepsilon_t - \theta_1\ \varepsilon_{t-1} - \theta_2\ \varepsilon_{t-2} - \theta_3\ \varepsilon_{t-3} - \ldots - \theta_q\ \varepsilon_{t-q}) = \mu$$
$$Var\ (x_t) = \gamma_y(0) = Var\ (\mu + \varepsilon_t - \theta_1\ \varepsilon_{t-1} - \theta_2\ \varepsilon_{t-2} - \theta_3\ \varepsilon_{t-3} - \ldots - \theta_q\ \varepsilon_{t-q}) = \sigma^2\ (1 + \theta_1^2 + \theta_2^2 + \ldots + \theta_q^2).$$

By the similar method we can find the autocovariance at lag k
$$\gamma_y(k) = Cov\ (x_t, x_{t+k})$$
$$= E\ [(\mu + \varepsilon_t - \theta_1\ \varepsilon_{t-1} - \ldots - \theta_q\ \varepsilon_{t-q})\ (\mu + \varepsilon_{t+k} - \theta_1\ \varepsilon_{t+k-1} - \ldots - \theta_q\ \varepsilon_{t+k-q})].$$

$$= \begin{cases} \sigma^2\left(-\theta_k + \theta_1\theta_{k+1} + \cdots + \theta_{q-k}\theta_q\right) & k = 1, 2, \ldots, q \\ 0 & k > q \end{cases}$$

and the autocorrelation at lag k is
$$\rho_y(k) = \frac{\gamma_y(k)}{\gamma_y(0)}$$
$$= \begin{cases} \dfrac{(-\theta_k + \theta_1\theta_{k+1} + \cdots + \theta_{q-k}\theta_q)}{1 + \theta_1^2 + \cdots + \theta_q^2} & , k = 1, 2, 3, \ldots q \\ 0 & , k > q \end{cases}$$

From the above calculation, we can find the ACF value which can be used to identify MA model and cut off order after lag q [10]. One of the criteria for selection the best model is based on the minimum AIC value (*Akaike's Information Criterion*) and SBC (*Schwarz Bayesian Criteria*), the formula to find AIC and SBC are: AIC = T ln (*MSE*) + 2k, and

$$\text{SBC = T ln } (MSE\ ) + k\ ln\ (T)$$

where $MSE = \frac{1}{T-k}\ (SSE)$, $SSE = \sum_{t=0}^{T}(x_t - \hat{x}_t)^2$, k is the number of parameters, and T is the number of observations. The minimum values of AIC and SBC for the model gives an indication that the said model is the best model [6].

Autoregressive Conditional Heteroscedasticity(ARCH) Model
Conditional variance of the residual ε_t which is σ_t^2, can be written as

$$\sigma_t^2 = \omega + \lambda \varepsilon_{t-1}^2 + \lambda_2 \varepsilon_{t-2}^2 + \cdots + \lambda_q \varepsilon_{t-q}^2$$

where the variance residual depend on the-q squares of residual, and is called Autoregressive Conditional Heteroscedasticity (ARCH). The ARCH model can be written as.

$$x_t = \delta + \sum_{i=1}^{p} \phi_i x_{t-i} - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t$$
$$\varepsilon_t \sim N(0, \sigma^2\ )$$
$$\sigma_t^2 = \omega + \lambda_1 \varepsilon_{t-1}^2 + \lambda_2 \varepsilon_{t-2}^2 + \cdots + \lambda_q \varepsilon_{t-q}^2$$

x_t is the equation of conditional mean [12].

Generalized ARCH (GARCH) Model
GARCH model was developed by Bollerslev[9]. GARCH model allows the conditional variance depend on the conditional variance of the previous lag. So that, the equation of conditional variance become

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \lambda_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

If the present values of the conditional variance was parameterized and depend on the-q lag from the squares residual and the-p lag of the conditional variance, then it can be written as GARCH (p,q). So, GARCH model if its time varying conditional variance is heteroscedastic with both autoregression and moving average [13]. GARCH model can be written as.

$$x_t = \delta + \sum_{i=1}^{p} \phi_i x_{t-i} - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t$$
$$\varepsilon_t \sim N(0, \sigma_t^2\ )$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \lambda_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

x_t is the equation of conditional mean (Bollerslev, 1986).

Return
Return is the relative change in the price of a financial asset over a given time interval, often expressed as a percentage [7]. Return series is easier to handle than price series because return has more attractive statistical properties [8]. There are some definitions of return. The one period of simple return is defined as follows:

$$R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where P_t is the price of an asset at time index t. [7, 8].

Value at Risk (VaR)
Risk is the magnitude of the deviation between the expected return and the return achieved (actual return). The greater the deviation means the greater the risk. If expressed as the extent to which the results obtained can deviate from the expected results, then we can used the measurement of distribution. Statistical tools that can be used as a measure of dispersion are the variance or

standard deviation. The greater the value is, the greater the deviation or the greater the risk are [14, 15]. Value at Risk is a concept used in the measurement of risk, risk management is defined as the estimated maximum losses that will be obtained during a specific time period under a normal market conditions and at a given confidence level [8]. Value at risk (VaR) is widely used in financial risk measurement and management as an estimate of the worst loss that can occur for a given time period and at a given confidence level [7].

3. RESULTS AND DISCUSSION

The data which were analyzed in this study is daily closing share price return IHSG. There are some steps of the analysis data. The first step is we check the stationary data. To check the stationary, we plot the data and look the pattern of the graph. The plot of the data daily closing share price return can be seen in Figure 1. The plot of the log return (ln return) daily closing share price can be seen in Figure 2.
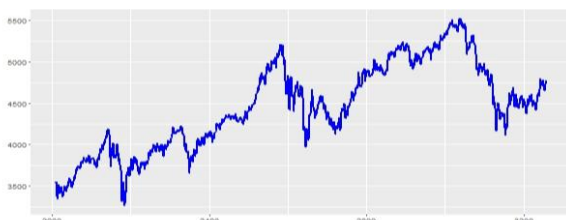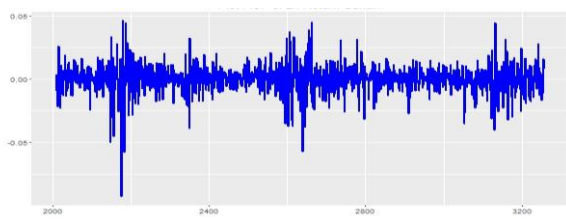

Fig.1. Plot Return share price

Fig.2. Plot log return share price

The second step is we check the stationarity data return and log return by using the Augmented Dickey Fuller (ADF) test with the null hypothesis and alternative hypothesis are as follows:

$H_0 : \phi = 0$ (there is a unit root or data are nonstationary).
$H_1 : \phi < 0$ (there is no a unit root or data are stationary).
Reject H_0 if p-value < 0.05.

Table 1. Results of ADF test for *return* and ln return data

Data	Lag order	ADF test	p-value
Return	0	-1.8918	0.0588
Ln return	0	-33.7349	0.0100

Table 1 shows that the p-value=0.05882 > 0.05 and the statistic value of ADF $|\tau|$= 1,8918. Thus, we conclude that there is not enough evidence to reject H_0. This means that the closing daily stock price data return IHSG over the periods from January 2011 to February 2016 have a unit root. This implies that the data are nonstationary. Table 2, shows that the p-value = 0.01 < 0.05 and statistical values of the Dickey Fuller $|\tau|$ = 33.7349 > critical value = 1.95 (Table Dickey Fuller at 5% significance level). Thus, we conclude that the Ho is rejected. This means that the data *log return*s of the closing daily stock price data return over

the periods from January 2011 to February 2016 are stationary.

Estimation of ARIMA Model

Since the data have been fulfill the stationarity, then the next step is going to identify and estimate the ARIMA model. Table 2 below shows the results of the estimation along with the best model suggested by using software R 3.2.3.

Table 2. The results of the best model for ARIMA

ARIMA(2,0,2) with non-zero mean : -7668.619	
ARIMA(0,0,0) with non-zero mean	: -7647.471
ARIMA(1,0,0) with non-zero mean	: -7648.789
ARIMA(0,0,1) with non-zero mean	: -7649.22
ARIMA(0,0,0) with zero mean	: -7649.108
ARIMA(1,0,2) with non-zero mean	: -7646.664
ARIMA(2,0,1) with non-zero mean	: -7645.257
ARIMA(1,0,1) with non-zero mean	: -7646.788
ARIMA(2,0,2) with zero mean	: -7670.157
ARIMA(1,0,2) with zero mean	: -7648.358
ARIMA(2.0.1) with zero mean	: -7645.768

Based on the results in Table 2, the best model is ARMA (2.2) with the constant is equal to zero. Furthermore, the smallest AIC value is -7670,538. Besides, the selection of the best model can also be based on the smallest values of SBC and AICC [4]. Tables 3 and 4 below give the values of SBC and AICC.

Table 3. The value of SBC for ARMA (2,2)

ARIMA(p,d,q)	SBC
ARIMA(2,0,2) with non-zero mean	**-7637.795**
ARIMA(0,0,0) with non-zero mean	-7637.196
ARIMA(1,0,0) with non-zero mean	-7633.377
ARIMA(0,0,1) with non-zero mean	-7633.809
ARIMA(0,0,0) with zero mean	-7643.971
ARIMA(1,0,1) with non-zero mean	-7626.239

Table 4. The value of AICC on ARMA (2,2) model

ARIMA(p,d,q)	AICC
ARIMA(2,0,2) with non-zero mean	-7668.551
ARIMA(0,0,0) with non-zero mean	-7647.461
ARIMA(1,0,0) with non-zero mean	-7648.77
ARIMA(0,0,1) with non-zero mean	-7649.201
ARIMA(0,0,0) with zero mean	-7649.105
ARIMA(1,0,2) with non-zero mean	-7646.616
ARIMA(2,0,1) with non-zero mean	-7645.209
ARIMA(1,0,1) with non-zero mean	-7646.756
ARIMA(2,0,2) with zero mean	**-7670.109**
ARIMA(1,0,2) with zero mean	-7648.326
ARIMA(2,0,1) with zero mean	-7645.736
ARIMA(1,0,1) with zero mean	-7648.464

The model is said to be the best if the values of AIC, AICC and SBC are minimum. Based on the analysis by

using R, we found that the minimum values for AIC, AICC and SBC are -7,670.538, -7,670.109 and -7637,795 respectively. Based on the criteria of the minimum values of AIC, AICC and SBC, the best model is ARMA(2,2). For the estimation values of the mean model are given in the Table 5 below.

Table 5. The estimation parameter ARMA (2,2) model

Parameter	Estimate	t value	*p-value*
AR (1)	1,2909	9,9453	0,0000
AR (2)	-0,6999	-6,8017	0,0000
MA (1)	-1,2428	-8,4775	0,0000
MA (2)	0,5982	5,0017	0,0000

The mean model can be written as follows:
$$x_t = 1,2909x_{t-1} - 0.6999\ x_{t-2} - 1,2428\varepsilon_{t-1} + 0.5982\varepsilon_{t-2} + \varepsilon_t$$
where x_t : data ln return at the-t time, ε_t : residual at t, and k = 0,1,2, … , n.

Evaluation of ARIMA Model
At the stage of evaluation ARIMA model, we will see whether the residuals of ARIMA are *white noise* and normally distributed.
To check whether or not the residuals are white noise, it will be tested by using the Ljung-Box test. The following are the results of Ljung-Box test.
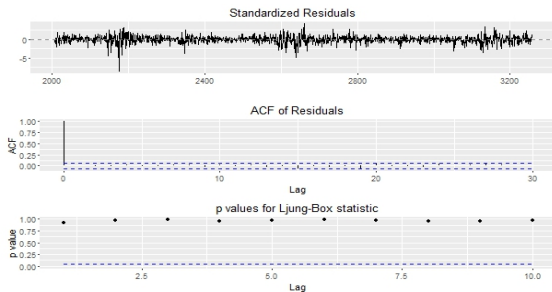

Fig. 3. The results of Ljung-Box test for Residual ARMA (2, 2) model

Figure 3. shows that the p-value for Ljung-Box statistic is above 0.05, this mean that the residual data are not correlated and due to the result we can conclude that the residuals of the ARMA (2,2) model are white noise.
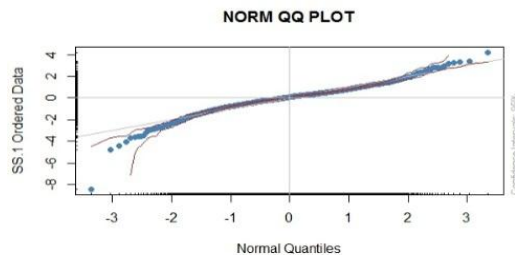
Normality Test

Fig.4. Normal Quantil Quantil Plot with confidence interval 95% for ARMA (2, 2)

Figure 4 shows that there are some residuals are not in the area of 95% confidence interval and the distribution of the data does not follow the straight line. Thus, the residuals of the model ARMA (2, 2) are not normally distributed.

Besides, using Normal QQ Plot, normality test can also be checked by using the Kolmogorov-Smirnov test. Table 6 gives the results of the normality test by using Kolmogorov-Smirnov test.

Table 6. The results of Kolmogorov Smirnov test for model ARMA (2, 2)

```
Title:
  One-sample Kolmogorov-Smirnov test

Test Results:
  STATISTIC:
    D: 0.483
  P VALUE:
    Alternative Two-Sided: < 2.2e-16
    Alternative     Less: < 2.2e-16
    Alternative  Greater: < 2.2e-16
```

Hypothesis
H_0 : data normally distributed
H_a : data not normally distributed.
The test statistic :
$D = \sup | S(x) - F_0(x) | = 0,483$
$D_{0,05\ ;\ 1259} = 0,03804$
With $\alpha = 5\%$, since $D > D_{0,05\ ;\ 1259}$, then Ho is rejected. Thus, the residuals are not normally distributed.

Lagrange Multiplier (LM) Test
The time series data are often have a problem with autocorrelation and heteroscedasticity. A test that can be used to detect the presence of heteroscedasticity or the existence of ARCH effect is Lagrange Multiplier test. The null and the alternative hypothesis are as follows.
H_0: $\lambda_0 = \lambda_1 = \cdots = \lambda_p = 0$ (there is no ARCH effects)
H_a : Some $\lambda_i \neq 0$ i= 1,2,3, … ,p (there is ARCH effects)
Level of significance $\alpha = 0.05$.
Decision: Reject Ho if p-value < 0.05.

Table 7. Test ARCH Lagrange Multiplier for ARMA (2, 2)

```
ARCH LM-test; Null hypothesis:
 no ARCH effects
data:  t
Chi-squared = 171.29,
 df = 8, p-value < 2.2e-16
```

Table 7 shows that the *p-value* $< \alpha$ significant level for ARMA(2,2) model. Thus, we can conclude that the ARMA (2,2) model has ARCH properties, so it is suggested to use the ARCH or GARCH model.

GARCH Model
The model selection model based on the estimation of means and variances function simultaneously with the GARCH model is ARMA(2,2)–GARCH (1,1). The results of the estimation can be found in Table 8 below.
From Table 8, GARCH (1,1) model can be written as follows:
$$x_t = 0.000552 + 1.36448x_{t-1} - 0.709029x_{t-2} - 1.354150\varepsilon_{t-1} + 0.638741\varepsilon_{t-2} + \varepsilon_t$$
and
$$\sigma_t^2 = 0.000003 + 0.111435\varepsilon_{t-1}^2 + 0.864804\sigma^2{}_{t-1}$$

Table 8. Parameters estimation

Variable	Coefficient	Standard Error	p-value
δ	0.000522	0.000212	0.013782
$\emptyset_1$	1.364400	0.206032	0.000000
$\emptyset_2$	-0.709029	0.165621	0.000019
θ_1	-1.354150	0.224720	0.000000
θ_2	0.638741	0.195625	0.001094
ω	0.000003	0.000002	0.135713
α_1	0.111435	0.020379	0.000000
β_1	0.864804	0.023751	0.000000

Parameter Estimation of GARCH (1,1) Model

To estimate the parameters $\omega, \lambda_1, \beta_1$ in GARCH(1,1) model, we can also use the method of Maximum Likelihood Estimation (MLE) as follows: The process ε_t can be modeled as follows $\varepsilon_t = \epsilon_t \sigma_t$, $\epsilon_t \sim N(0,1)$ where ϵ_t is i.i.d. Where $\sigma_t^2 = \varepsilon_t | \varphi$ is conditional variance of error $E(\varepsilon_t | \varphi) = 0$ and φ is vector parameter of $f(\varepsilon_t)$ $\varphi = [\omega, \lambda_1, \beta_1]$. General form of GARCH model is

$$\sigma_t^2 = \omega + \lambda_i \varepsilon_{t-i}^2 + \beta_j \sigma_{t-i}^2 , \varepsilon_t | \varphi \sim iid\ N(0, \sigma_t^2)$$

where $\omega > 0, \beta_j \geq 0, j = 1,2,\ldots,p\ dan\ \lambda_i \geq 0, i = 1,2,\ldots,q$. Likelihood function:

$$L(\varepsilon_t | \varphi) = \prod_{t=1}^n f(\varepsilon_t | \varphi)$$

$$= \prod_{t=1}^n \frac{1}{\sqrt{2\pi\sigma_t^2}} e^{-\frac{1}{2\sigma_t^2}(\varepsilon_t - 0)^2}$$

$$\ln L(\varepsilon_t | \varphi) = \sum_{t=1}^n ln\left\{ \frac{1}{\sqrt{2\pi\sigma_t^2}} e^{-\frac{\varepsilon_t^2}{2\sigma_t^2}} \right\}$$

$$= \sum_{t=1}^n \left\{ \ln \frac{1}{\sqrt{2\pi\sigma_t^2}} + \ln e^{-\frac{\varepsilon_t^2}{2\sigma_t^2}} \right\}$$

$$= \sum_{t=1}^n \left\{ \ln(2\pi)^{-\frac{1}{2}} + \ln(\sigma_t^2)^{-\frac{1}{2}} - \frac{\varepsilon_t^2}{2\sigma_t^2} \right\}$$

$$= \ln(2\pi)^{-\frac{n}{2}} + \ln(\sigma_t^2)^{-\frac{n}{2}} - \frac{\sum_{t=1}^n \varepsilon_t^2}{2\sigma_t^2}$$

$$= -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma_t^2) - \frac{\sum_{t=1}^n \varepsilon_t^2}{2\sigma_t^2}$$

Since $\sigma_t^2 = \omega + \lambda_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$, then the equation above become

$$\ln L(\varepsilon_t | \varphi) =$$
$$-\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\omega + \lambda_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2) - \frac{\sum_{t=1}^n \varepsilon_t^2}{2(\omega + \lambda_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)}$$

The derivative with respect to vector parameter φ :

$$\frac{\partial L}{\partial \varphi} = \frac{\partial L}{\partial \sigma_t^2} \frac{\partial \sigma_t^2}{\partial \varphi} = \left[-\frac{1}{2\sigma_t^2} + \frac{\varepsilon_t^2}{2(\sigma_t^2)^2} \right] \frac{\partial \sigma_t^2}{\partial \varphi}$$

$$= \left[\sum_{t=1}^n -\frac{1}{2(\omega + \lambda_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)} \right.$$
$$\left. +\sum_{t=1}^n \frac{\varepsilon_t^2}{2(\omega + \lambda_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)^2} \right] \frac{\partial \sigma_t^2}{\partial \varphi}$$

The first derivative with respect to ω

$$g_1 = \frac{\partial L}{\partial \omega} = \frac{\partial L}{\partial \sigma_t^2} \frac{\partial \sigma_t^2}{\partial \omega}$$

$$= \frac{1}{2\sigma_t^2} \left[\frac{\varepsilon_t^2}{\sigma_t^2} - 1 \right]$$

$$= \frac{1}{2} \left(\sum_{t=1}^n \frac{\varepsilon_t^2}{(\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2)^2} \right.$$

$$-\sum_{t=1}^n \frac{1}{\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2})$$

The first derivative with respect to λ_1

$$g_2 = \frac{\partial L}{\partial \lambda_1} = \frac{\partial L}{\partial \sigma_t^2} \frac{\partial \sigma_t^2}{\partial \lambda_1}$$

$$= \frac{1}{2} \left(\sum_{t=1}^n \frac{\varepsilon_{-1+t}^2 \varepsilon_t^2}{(\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2)^2} \right.$$

$$-\sum_{t=1}^n \frac{\varepsilon_{-1+t}^2}{\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2})$$

The first derivative with respect to β_1

$$g_3 = \frac{\partial L}{\partial \beta_1} = \frac{\partial L}{\partial \sigma_t^2} \frac{\partial \sigma_t^2}{\partial \beta_1}$$

$$= \frac{1}{2} \left(\sum_{t=1}^n \frac{\varepsilon_t^2 \sigma_{-1+t}^2}{(\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2)^2} \right.$$

$$-\sum_{t=1}^n \frac{\sigma_{-1+t}^2}{\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2})$$

From the equations above, the equations still contain some parameters, namely ω, λ_1 and β_1. Next, we need to apply a method of iteration to find $\hat{\omega}, \hat{\lambda}_1$ and $\hat{\beta}_1$.

Newton Raphson Method

If at the first derivative there are still exist some parameters, then we continued by using Newton Raphson Method by iteration:

$$\begin{bmatrix} \hat{\lambda}_{t+1} \\ \hat{\beta}_{t+1} \end{bmatrix} = \begin{bmatrix} \hat{\lambda}_t \\ \hat{\beta}_t \end{bmatrix} H^{-1} G, \quad \text{and} \quad G = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}$$

$$G = \begin{bmatrix} \frac{1}{2}(\sum_{t=1}^n \frac{\varepsilon_t^2}{(\omega+\lambda_1\varepsilon_{-1+t}^2+\beta_1\sigma_{-1+t}^2)^2} - \sum_{t=1}^n \frac{1}{\omega+\lambda_1\varepsilon_{-1+t}^2+\beta_1\sigma_{-1+t}^2}) \\ \frac{1}{2}(\sum_{t=1}^n \frac{\varepsilon_{-1+t}^2\varepsilon_t^2}{(\omega+\lambda_1\varepsilon_{-1+t}^2+\beta_1\sigma_{-1+t}^2)^2} - \sum_{t=1}^n \frac{\varepsilon_{-1+t}^2}{\omega+\lambda_1\varepsilon_{-1+t}^2+\beta_1\sigma_{-1+t}^2}) \\ \frac{1}{2}(\sum_{t=1}^n \frac{\varepsilon_t^2\sigma_{-1+t}^2}{(\omega+\lambda_1\varepsilon_{-1+t}^2+\beta_1\sigma_{-1+t}^2)^2} - \sum_{t=1}^n \frac{\sigma_{-1+t}^2}{\omega+\lambda_1\varepsilon_{-1+t}^2+\beta_1\sigma_{-1+t}^2}) \end{bmatrix}$$

The second derivative (g1) with respect to ω

$$H_1 = \frac{\partial g_1}{\partial \omega} = \frac{\partial g_1}{\partial \sigma_t^2} \frac{\partial \sigma_t^2}{\partial \omega}$$

$$= \frac{1}{2} \left(-\sum_{t=1}^n \frac{2\varepsilon_t^2}{(\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2)^3} \right.$$

$$\left. +\sum_{t=1}^n \frac{1}{(\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2)^2} \right)$$

The second derivative (g1) with respect to λ_1

$$H_2 = \frac{\partial g_1}{\partial \lambda_1} = \frac{\partial g_1}{\partial \sigma_t^2} \frac{\partial \sigma_t^2}{\partial \lambda_1}$$

$$= \frac{1}{2} \left(-\sum_{t=1}^n \frac{2\varepsilon_{-1+t}^2 \varepsilon_t^2}{(\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2)^3} \right)$$

$$+\sum_{t=1}^n \frac{\varepsilon_{-1+t}^2}{(\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2)^2}$$

The second derivative (g1) with respect to β_1

$$H_3 = \frac{\partial g_1}{\partial \beta_1} = \frac{\partial g_1}{\partial \sigma_t^2} \frac{\partial \sigma_t^2}{\partial \beta_1}$$

$$= \frac{1}{2} \left(-\sum_{t=1}^n \frac{2\varepsilon_t^2 \sigma_{-1+t}^2}{(\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2)^3} \right.$$

$$\left. +\sum_{t=1}^n \frac{\sigma_{-1+t}^2}{(\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2)^2} \right)$$

The second derivative (g2) with respect to ω

$$H_4 = \frac{\partial g_2}{\partial \omega} = \frac{\partial g_2}{\partial \sigma_t^2} \frac{\partial \sigma_t^2}{\partial \omega}$$

$$= \frac{1}{2} \left(-\sum_{t=1}^n \frac{2\varepsilon_{-1+t}^2 \varepsilon_t^2}{(\omega + \lambda_1\varepsilon_{-1+t}^2 + \beta_1\sigma_{-1+t}^2)^3} \right)$$

$$+\sum_{t=1}^{n}\frac{\varepsilon^2_{-1+t}}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^2}$$

The second derivative (g2) with respect to λ_1

$$H_5 \quad =\frac{\partial g2}{\partial\lambda_1}=\frac{\partial g2}{\partial\sigma_t^2}\frac{\partial\sigma_t^2}{\partial\lambda_1}$$

$$=\frac{1}{2}(-\sum_{t=1}^{n}\frac{2\varepsilon^4_{-1+t}\varepsilon_t^2}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^3}$$

$$+\sum_{t=1}^{n}\frac{\varepsilon^4_{-1+t}}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^2})$$

The second derivative (g2) with respect to β_1

$$H_6 \quad =\frac{\partial g2}{\partial\beta_1}=\frac{\partial g2}{\partial\sigma_t^2}\frac{\partial\sigma_t^2}{\partial\beta_1}$$

$$=\frac{1}{2}(-\sum_{t=1}^{n}\frac{2\varepsilon^2_{-1+t}\varepsilon_t^2\sigma^2_{-1+t}}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^3}$$

$$+\sum_{t=1}^{n}\frac{\varepsilon^2_{-1+t}\sigma^2_{-1+t}}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^2})$$

The second derivative (g3) with respect to ω

$$H_7 \quad =\frac{\partial g3}{\partial\omega}=\frac{\partial g3}{\partial\sigma_t^2}\frac{\partial\sigma_t^2}{\partial\omega}$$

$$=\frac{1}{2}(-\sum_{t=1}^{n}\frac{2\varepsilon_t^2\sigma^2_{-1+t}}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^3}$$

$$+\sum_{t=1}^{n}\frac{\sigma^2_{-1+t}}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^2})$$

The second derivative (g3) with respect to λ_1

$$H_8 \quad =\frac{\partial g3}{\partial\lambda_1}=\frac{\partial g3}{\partial\sigma_t^2}\frac{\partial\sigma_t^2}{\partial\lambda_1}$$

$$=\frac{1}{2}(-\sum_{t=1}^{n}\frac{2\varepsilon^2_{-1+t}\varepsilon_t^2\sigma^2_{-1+t}}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^3}$$

$$+\sum_{t=1}^{n}\frac{\varepsilon^2_{-1+t}\sigma^2_{-1+t}}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^2})$$

The second derivative (g3) with respect to β_1

$$H_9 \quad =\frac{\partial g3}{\partial\beta_1}=\frac{\partial g3}{\partial\sigma_t^2}\frac{\partial\sigma_t^2}{\partial\beta_1}$$

$$=\frac{1}{2}(-\sum_{t=1}^{n}\frac{2\varepsilon_t^2\sigma^4_{-1+t}}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^3}$$

$$+\sum_{t=1}^{n}\frac{\sigma^4_{-1+t}}{(\omega+\lambda_1\varepsilon^2_{-1+t}+\beta_1\sigma^2_{-1+t})^2})$$

In the form of Hessian matrix, it can be written as:

$$H=\begin{bmatrix}H_1 & H_2 & H_3\\H_4 & H_5 & H_6\\H_7 & H_8 & H_9\end{bmatrix}$$

The iteration will be stopped if $\begin{bmatrix}\hat{\lambda}_{t+1}-\hat{\lambda}_t\\\hat{\beta}_{t+1}-\hat{\beta}_t\end{bmatrix}<\varepsilon$, where ε is defined as 0,0001. By using software R, the estimation parameters ω,λ_1, and β_1 by using Newton Raphson method we have $\hat{\omega}$ =0,000003000, $\hat{\lambda}_1$ =0,111435001 and $\hat{\beta}_1$=0,864804000.

4. CONCLUSION
Forecasting For Volatility
Forecasting the variance of log returns from the time t by using GARCH (1, 1) model one step ahead is

$$\sigma_{t+1}^2=\sigma_t^2(1)=\omega+\lambda_1\varepsilon_t^2+\beta_1\sigma_t^2.$$

Forecasting for volatility log return one period ahead can be obtained by square root of the results of the forecast of the variance. The value of the forecasting for variance one step ahead is 0.0001426371 and the forecast for the volatility is $\sqrt{0.0001426371}$ = 0.011943077.

The Calculation of VaR
Based on the result of normality test for the residuals, it was found that the residuals are not normality distributed. Thus, Z_α that is going to be used for the calculation of VaR must be corrected by using the Cornish Fisher expansion (Z') [16]. Table 9 shows the results of the calculations by using the Cornish Fisher expansion with four difference levels of significance (α's).

Table 9. The results of Cornish Fisher Expansion

No	α	Z_α	Skewness Value (S)	Z'
1	90%	1,282	0,0001426371	1,281985
2	95%	1,645	0,0001426371	1,644959
3	99%	2,326	0,0001426371	2,325895
4	99,5%	2,576	0,0001426371	2,57586

Furthermore, we calculate the amount of VaR if we assume that we have invested the funds as much as Rp.100,000,000.00 in the shares of IHSG, then the results for the four difference levels of significance are given in Table 10. Furthermore, Table 10 also calculates the estimate for forecasting volatilities of GARCH at four difference levels of significance (α's)..

Table 10. The results of VaR Model ARMA(2,2) GARCH (1,1)

α	Z_α'	Forecasting Volatilitas	VaR	VaR in Rp
90%	1.281985	0.011943077	0.0153108461	1.531.084
95%	1.644959	0.011943077	0.0196645871	1.966.458
99%	2.325895	0.011943077	0.0277783430	2.777.834
99,5%	2.57586	0.011943077	0.0307636941	3.076.369

By using a 95% confidence level means that there is a 5% chance of a loss for more than Rp 1,966,458.00 during the next 24 hours if we invested funds as much as Rp 100,000,000.00.

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