MALAYSIAN TRAINEE TEACHERS' MATHEMATICAL KNOWLEDGE OF FRACTIONS

Mazlini Adnan[1], Marzita Puteh[2], Effandi Zakaria[3]

[1, 2]Department of Mathematics, Faculty of Science and Mathematics, Universiti Pendidikan Sultan Idris, 35900 Tg. Malim, Perak.

[2]Department of Innovation in Teaching and Learning Faculty of Education, Universiti Kebangsaan Malaysia, 43650 Bangi, Selangor.

[*]For correspondence; Tel. + (60) 196546250, E-mail: mazlini@fsmt.upsi.edu.my

ABSTRACT*: The study of trainee teachers' knowledge of fractions is important because it is one of the topics that is difficult to learn and teach. Unfortunately, studies of teacher trainees' fraction knowledge are limited. The main purpose of this study was to evaluate the Malaysian trainee teachers' mathematical knowledge of fractions. This study included all four operations to provide a more comprehensive understanding of this knowledge. One hundred and eleven (111) trainee teachers enrolled in a 4-year teacher education program from two public universities in Malaysia participated in the Fraction Knowledge Test (FKT). Results showed that trainee teachers displayed better knowledge of fraction on procedure than on concept. Findings indicated that trainee teachers need more stimuli to construct their conceptual knowledge. Implications for teacher education are also discussed.*

Keywords: Mathematical Knowledge, Conceptual knowledge, fractions, trainee teachers.

1. INTRODUCTION

By the year 2020, Malaysia will be known as a developed nation. As a developed nation, each individual must possess high skills in various fields. Mathematics is one of the basic subjects that is crucial in producing individuals with high skills. Therefore, mathematics is a subject that has to be mastered and liked by the students. How to make the students master and like mathematics? Therefore, the role of teachers, especially mathematics teachers, is significant in shaping the students' interest. Teachers come second after parents in sharing a very close relationship with the students.

According to Teacher Preparation Research: Current Knowledge, Gaps and Recommendation [1], an effective teacher should have good knowledge about the subjects he/she teaches. However, evidence shows that teachers' conceptual knowledge is still very poor [1]. Therefore, many average teachers are still not fully prepared to teach mathematics [2,3]. There is much evidence in the literature, which shows that teachers consider mathematics as a fixed and repeated knowledge that is learnt effectively by memorizing, algorithms, and recurring procedures [4]. On the basis of the literature, most of the mathematics teachers are still using traditional methods of teaching, namely, teaching via memorization, procedures or facts, and the memorization of the steps of obtaining the solution [5,2] .

The topic of fractions is fundamental to a student's mathematical development. However, fractions are considered to be difficult by students and teachers [6]. The operation of division as a subtopic in fractions is the most difficult and mechanical to solve. The problem of understanding the fractions and decimals concept is not faced only by the form 1 and 2 students. In fact, according to [7], the fractions concept is least understood by most of the students. Students' weakness in understanding the fractions and decimal concept cannot be solved in a short time because they are the most difficult concepts to learn [8]. The question is why does this scenario occur? Have the teachers delivered the conceptual-based explanations well? Is the teachers' knowledge deep and sufficient enough to be able to provide conceptual-based reasons to students regarding the laws, rules, and tips together with algorithms, of fractions and decimals?

2. LITERATURE REVIEW

2.1 Mathematical Knowledge for Teaching

Almost three decades ago Shulman's [9] idea on teachers knowledge needed for teaching inspired most researchers with more theories, models and measures in mathematics education [10]. The concept of knowledge for teaching mathematics has spin on the question, what mathematical reasoning, justifications, perception, understanding, and skills are required for a person to teach mathematics well at primary level. Many have worked towards developing theoretical models and measures to address these questions.

Ball and her colleagues leading the way in this area of Pedagogical Content Knowledge and Subject Matter Knowledge [11]. Primary school teachers need to know, understand and which enables them to teach mathematics effectively is not associated with having an advance level or degree in the subject [12]. This is not to suggest that formal academic qualification is not important to teach mathematics but teachers must have deep understanding on curriculum taught. Literature indicates something else necessary for teachers. Effective teachers of mathematics have other knowledge area to consider which is hard to pin down and measure, meanwhile formal qualification is easy to quantify and record.

Ball and her colleagues have championed instructional model for mathematics [11]. They have divided the teaching of mathematical knowledge into two major components, Pedagogical Content Knowledge (PCK) and Subject Matter Knowledge (SMK). PCK is divided into Knowledge of Content and Student (KCS); Knowledge of content and teaching (KCT); Knowledge Content and Curriculum (KCC), SMK is divided into three subdomains; Common Content Knowledge (CCK); Specialized Content K(SCK); Horizon content Knowledge (HCK). This study focuses more on the Mathematical knowledge for teaching (MKT) for Fraction. One of the main contributions of Mathematical knowledge for teaching (MKT), is the identification of knowledge in terms that are purely mathematical and specific to the profession, SCK [11]. There has been largely well received by the research community in that it specifies the teachers' knowledge.

2.2 Specialised Content Knowledge (SCK)

Teacher needs SCK knowledge since it plays important role in teaching and learning of mathematics [11]. Generally, it is accepted that specialized content knowledge includes a range of factual, theoretical and practical knowledge as well as competencies and skills in a particular discipline or profession. SCK is a mathematical knowledge that is unique to teaching and is not used in professions outside or other than teaching [11].

2.3 Mathematics Pedagogical Content Knowledge (MPCK)

In general, pedagogical content knowledge affects how teachers think about their subject matter knowledge. A skilful and very knowledgeable teacher has the potential to make the learning of mathematics more meaningful to the students [13]. Teaching or instructions without deep understanding is meaningless. Effective instruction should involve PCK. PCK covers conceptual and procedural knowledge, and the stages of understanding that they are likely to pass through in moving from a state of having little understanding to the mastery of it. As such, teachers should have knowledge on how to teach, their students effectively. The National Council of Teachers of Mathematics pointed out that effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies [14].

2.4 MPCK and Fractions

Learning and mastering fractions is still a major issue for students in both primary and middle schools [15]. Fractions is considered as one of the problematic topics to teach and to understand especially at elementary or primary level. Difficulty with fractions among teachers is well documented in many countries, and many authors consider fractions to be the most difficult area of mathematics covered in primary schools [16]. Studies have shown that teachers and students, have difficulties on fraction concept and division in fractions [17,18]. Findings also shows that confusion arise among teachers, to determine which answer is right or wrong provided by their students for the given questions [19].

$$7\frac{2}{5} - 7 \text{ x } \frac{2}{5}$$

a) 0 b) $\frac{2}{5}$ c) 4 $\frac{3}{5}$ d)) $\frac{23}{5}$

The above question with multiple responses is a challenge for the teachers' knowledge about the concepts. The conversion of mixed number into fractions are learned as multiplication of the whole number and the denominator followed by addition of the numerator. This procedural understanding develops a belief of the existence of a multiplication sign between 7 and $\frac{2}{5}$. The existence is also supported by the rules from algebra as it is often said that if there is no sign between two letters (or a letter and a number) then there is a multiplication sign. So *xy* indicates *x* x *y*. Similarly, 7 $\frac{2}{5}$ indicates 7 ×) $\frac{2}{5}$. These lead teachers towards wrong answer i. e., option (a) 0. But interestingly when these teachers are interviewed and asked about the reasons for the rest of the answers, they started thinking about students reasoning. This unpacking of what students thought, gave them insight about the structure of fraction representation itself. Thus, we can determine the level of teacher's conceptual understanding and relational connectedness in teaching of fractions. PCK is assumed to help teachers to sequence their instructions in a workable way. Conceptual and relational understanding will help teachers to understand their students thinking or reasoning. In order to become a better teachers of mathematics they need to deepen their understanding of the mathematics. They are also expected to apply PCK whenever they teach subject matter for the various levels of abilities of their students.

2.5 Mathematical Knowledge Studies in the Malaysian Context

In the context of Malaysia, there are few studies on mathematical knowledge of teachers [20]. As educators, we are concerned about the effectiveness of the teaching of teachers. Others claim that we are "deskilling" or "deprofessionalizing" teachers by "testing" them. This is counterproductive. In conclusion, there is a powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction. Mathematics teacher educators assume a significant responsibility for enhancing the content and pedagogical knowledge of prospective teachers as well as influencing their belief regarding mathematics teaching and learning and their awareness of their students' mathematical dispositions. In our context, we need to assess the teacher education system in Malaysia for better and quality instructions in the future.

3. RESEARCH METHODOLOGY

The research involves 111 respondents who are the trainee teachers in their third year. They are made up of two IPTA's (higher learning institutions), which offer mathematical education programs, specifically, 71 (64.0 %) from UPSI and 40 (36.0 %) from UKM. From this number, 22 (19.8%) are males, and the remainder, 89 (80.2%), are females. Majority of the respondents consist of 94 (84.7%) Malays; 13 (11.7%) Chinese; 1 (0.9%) Indian; and 3 (2.7 %) from Bumiputra Sabah/Sarawak.

The instrument used in this research was the Fraction Knowledge Test (FKT). It was used to measure the respondents' understanding of fractions. This instrument (FKT) is a set of written tests consisting of 16 items regarding fractions for measuring the mathematical conceptual knowledge of the trainee teachers. The instrument used is a modification from a research conducted by [21], who studied about the understanding of fractions concept among the trainee teachers. In this study, the written test (FKT) used gives an overview of the knowledge and skill of the respondents about fractions using the four mathematical operations, i.e., addition, subtraction, multiplication, and division. FKT Cronbach alpha is 0.65.

4. RESULTS AND DISCUSSIONS

This study was conducted to find out the mathematical conceptual knowledge of fractions of trainee teachers' in the Higher Learning Institutions in Malaysia.

4.1 Conceptual Knowledge Level

Table. 1. Conceptual knowledge level

Variable	Mean	s.d	Level
Conceptual knowledge	2.82	0.47	Moderate

The purpose of this analysis is to determine the level of conceptual knowledge of fractions of the trainee teachers. Based on table 1, the level of conceptual knowledge is moderate (mean = 2.82). The findings were similar to [1,22], which state that teachers have procedural knowledge but the conceptual knowledge on basic concepts in fractions is lacking.

4.2 Analysis of Fraction Knowledge Test (FKT) of Trainee Teachers in Fractions

The trainee teachers' conceptual knowledge in solving problems relating to fractions is arranged according to items. The conceptual knowledge is arranged on the basis of the highest frequency from the answers given by the respondents in the study.

On the basis of Table 2, item 2 (Fig. 1) has the highest mean (3.37) in the FKT given. A total of 105 (76.6%) trainee teachers gave the correct answers and reasons. On the basis of the answers given by the respondents, it can be concluded that most of the respondents were able to give correct answers and reasons accurately based on the diagram given. Apart from that, 32 respondents (13.1%) gave wrong answers for this item. Most of the respondents obtained a 0 score on this item because they gave a wrong answer for the last question which was $\frac{3}{4}$. They simplified the fractions $\frac{9}{16}$ as $\frac{3}{4}$.

Meanwhile, item 6 mean is the lowest mean (1.54) among the 16 items given. 81 (59.1%) trainee teachers gave the wrong answer and reasons. Based on the answer given by the respondents, it was concluded that they were unable to find the fractions between the two fractions given.

Based on the 16 items in FKT, the items were collected according to categories of procedural knowledge in fractions. There were two categories; category 1 consisted of ADD, SUB, MUL, and DIV components, and category 2 consisted of CON and EQU components. Category 1 involved the four mathematical operations, which are addition, subtraction, multiplication, and division, based on the algorithm which is in fractions. Category 2 involved the basic concepts in fractions and equivalent fractions which refer to general attributes on fractions.

Table. 2. FKT trainee teachers score percentage based on items

Item	% Score					Mean
	0	1	2	3	4	
Item 1	32.8	0	0	24.1	43.1	2.45
Item 2	13.1	0	0	10.2	76.6	3.37
Item 3	0	8.8	19.0	26.3	46.0	3.09
Item 4	12.4	0	0	24.1	63.5	3.26
Item 5	8.0	0	0	40.9	51.1	3.28
Item 6	59.1	0.7	0	8.8	31.4	1.54
Item 7	16.8	0	0	16.8	66.4	3.16
Item 8	57.7	0	0	6.6	35.8	3.00
Item 9	2.2	0	0	75.9	21.9	3.15
Item 10	2.2	0	0	81.8	16.1	3.09
Item 11	2.9	0	0	81.8	15.3	3.07
Item 12	8.0	0	0	83.9	8.0	2.84
Item 13	5.1	0	0	94.2	0.7	2.85
Item 14	8.8	0	0	91.2	0	2.74
Item 15	2.2	0	0	97.8	0	2.93
Item 16	3.6	0	0	96.4	0	2.89

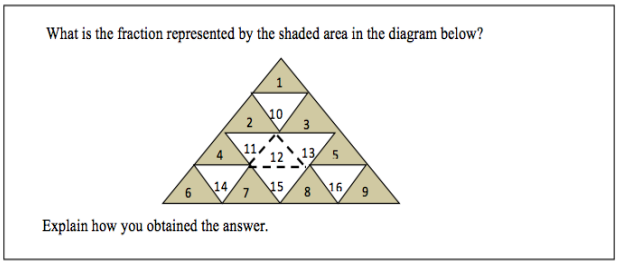

Fig.1. Item 2

ANOVA one way test was carried out and there was a significant difference between the two categories [$F(1,820) = 18.26$, $p < 0.05$]. The details also revealed that the trainee teachers were more capable of solving items that involved algorithm in fractions as compared to items that involved general attributes on fractions. It was also found that category 1 obtained a higher min (mean = 2.96) significantly compared to category 2 (mean = 2.72)

Find the fractions in the two following fractions:

$\frac{3}{4}$ and $\frac{4}{5}$

Explain how you obtain the answer.

Solution:

$$\frac{3\times5}{4\times5} = \frac{15}{20} \quad \frac{4\times4}{5\times4} = \frac{16}{20}$$

(The process is continued by multiplying the numerator and denominator with 2)

$$\frac{15\times2}{20\times2} = \frac{30}{40} \quad \frac{16\times2}{20\times2} = \frac{32}{40}$$

(after the second process, the answer for this item is obtained, that is, the fraction between $\frac{30}{40}$ and $\frac{32}{40}$ is $\frac{31}{40}$

Fig 2. Item 6

Apart from that, 16 items in FKT were also collected according to conceptual knowledge categories in fractions. The first category, which is the CON (item 1, 2, 7 and 8), refers to general knowledge (basic concept) about fractions. The second category consists of EQU components (item 3, 4, 5 and 6), ADD (item 9 and 10), and SUB (item 11 and 12). It refers to basic knowledge about calculation in fractions. The last category, which is the third category, consists of MUL component (item 13 and 14) and DIV (item 15 and 16) that are considered to be the most difficult in fractions.

The results from one way ANOVA analysis showed that there was a significant difference between the category [$F(2,819) = 9.71$, $p < 0.05$]. The details also revealed that the trainee teachers have general knowledge (basic concept) about fractions (category 1) is the lowest compared to the three categories. It was also found that the second category had a higher mean (mean = 2.85), which was the difficult knowledge in fractions topic significantly higher compared to the first category (mean = 2.65). This finding is different from the [23], who stated that problem solving, which involved the operation of division, is the most difficult

knowledge in fractions. This is because of the low basic concept understanding among the trainee teachers.

5. CONCLUSION

The trainee teachers' mathematical conceptual knowledge of fraction is at a moderate level. The study also found that the trainee teachers have better procedural knowledge in algorithm operations in fractions compared to general attributes in fractions. Besides, it was also found that the trainee teachers have excellent conceptual knowledge in the basic calculation of fractions as compared to basic concept knowledge and knowledge that involved multiplication and division operations.

6. REFERENCES

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* For correspondence; Tel. + (60) 196546250, E-mail: mazlini@fsmt.upsi.edu.my