ANALYSIS AND COMPARISON OF COPE'S PERFORMANCE FOR VARIOUS NETWORK TOPOLOGIES

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ABSTRACT: *COPE is an architecture for wireless mesh networks that improves the overall throughput of a network using opportunistic coding. COPE's performance has been analyzed by different researchers and modifications to this architecture have been proposed that further improved the throughput of a network. Two simple network topologies were analyzed using COPE in research work presented in [1]. In this research paper, this idea has been extended to three other network topologies. Coding gains for these networks are mathematically formulated and COPE's performance is analyzed when applied to these networks. On the basis of this analysis, the gains of these networks, which are obtained as a result of opportunistic coding, are compared with each other. Simulation results also help to analyze the performance of networks in COPE and non-COPE cases.*

Keywords: COPE, Opportunistic Coding, Throughput, Wireless Networks

INTRODUCTION:

Network coding was introduced in [2] which revolutionized the operation of networks. Due to the broadcast nature of wireless medium, network coding became very useful. COPE, introduced in [3, 4], is one of the first practical systems which uses network coding for wireless networks. COPE is a new forwarding architecture in which network coding and physical layer broadcast properties are properly exploited. This system greatly enhances the throughput of a wireless network by introducing a coding shim between the IP and MAC layers. Using this coding shim, coding opportunities are identified and thus multiple packets can be forwarded in a single transmission.

COPE's outstanding performance has attracted many researchers and a large number of attempts have been made to model COPE system and to analyze its performance. In [5, 6], network coding was analyzed for practical wireless networks. COPE's network coding opportunity analysis for multihop wireless networks is presented in [7] by deriving the probability density function of overhearing probability between two neighboring nodes. In [8], loop coding was proposed in which coded packets are temporarily stored by receivers for future decoding. Joint network coding and scheduling schemes are presented in [9] in order to optimize network throughput.

In [1], two simple network topologies i.e. Alice-and-Bob wireless network and Cross network are considered and COPE's performance curve is analyzed by examining one coding structure. In this paper, this idea is extended to three other network topologies i.e. "X" topology, chain topology and wheel topology. Throughput gains of these network topologies have been mentioned in [4] and it has been proved that in the absence of opportunistic learning, maximum achievable coding gain using COPE is 2 [4]. This finding will also be used in the analysis of different topologies in this work. This paper is organized as follows: Coding gains of three topologies are analyzed for Routing (non-COPE) case and Coding (COPE) case in next section. Then COPE's performance is compared for these topologies on the basis of this analysis, and the paper is concluded in last section.

PERFORMANCE ANALYSIS OF COPE:

In this Section, Non-COPE and COPE cases for three different network topologies are analyzed. For this analysis, it is assumed that all the nodes in the network are sharing a lossless wireless channel and the total bandwidth is 1.

"X" Topology:

A network based on "X" topology is shown in Fig. 1. In this network topology, node 1 and node 2 are connected through Link 'A' whereas node 3 and node 4 form Link 'B'. Router 'R' acts as a central node or a relay node. In the normal routing conditions, node 1 transmits its packets to 'R' which forwards it to node 2. Similarly, node 3 sends its packets to 'R' which then forwards it to node 4. But, with coding, 'R' receives packets from node 1 and node 2 and retransmits a single packet to both node 3 and node 4. Thus, fewer transmissions are required.

Non-COPE case: If the load offered to the network is small, each node gets sufficient bandwidth for transferring its packets. Relay node has a bandwidth requirement equal to the sum of transmission rates at the rest of the nodes [1]. The total throughput of the system is always equal to the bandwidth of the relay node, as shown in equation (1).

$$\begin{aligned} BW_1 + BW_2 &= BW_A = 1/4 \\ BW_3 + BW_4 &= BW_B = 1/4 \qquad \textbf{(1)} \\ BW_R &= 1/2 \end{aligned}$$

In this case, the total throughput of the system is equal to 1/2.

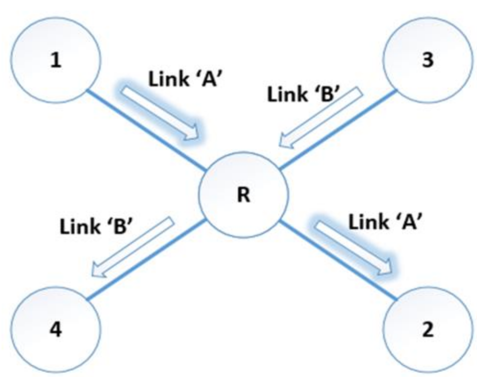

Fig. 1 "X" Topology

If the load offered to the network increases beyond 1/2, and eventually becomes very large, the system allocates the channel fairly among all nodes, as given by equation (2).

$$BW_A = BW_B = BW_R = 1/3 \quad \textbf{(2)}$$

COPE case: When coding is incorporated at the relay node, every packet transmitted by 'R' creates a throughput of 2 packets. All the nodes get their required bandwidth and the system's throughput increases linearly as the offered load is increased until the total bandwidth is consumed. In this case,

$$BW_A = BW_B = BW_R = 1/3 \quad \textbf{(3)}$$

Since 'R' forwards two packets during every transmission, the maximum throughput of this system is 2/3. When offered load is further increased beyond the handling capacity of system, packets will be lost but the bandwidth allocated to nodes will remain same and total throughput will remain 2/3.

It has been seen that "X" topology requires 4 transmissions in non-COPE case but requires only 3 transmissions in COPE case, we can easily find out the coding gain. Coding gain is defined as the ratio of the number of transmissions required by the non-COPE or the non-coding approach, to the minimum number of transmissions required by the COPE or coding approach to transfer the same set of packets from one node to another node [4]. Considering the "X" topology, coding gain comes out to be 4/3 or 1.33.

Infinite Chain Topology:

This topology is an extension of Alice-and-Bob network topology (shown in Fig. 2) where N = 2, as there are two links in the topology. It has already been discussed in [1, 4] that Alice-and-Bob network requires 4 transmissions in non-COPE case (see Fig. 2 (a)) but when COPE case is considered, only 3 transmissions are required (see Fig. 2 (b)). The coding gain again comes out to be 4/3 or 1.33. In case of an infinite chain topology, as shown in Fig. 3, the number of links is infinite. Therefore, it has to be analyzed in a different way from "X" topology. In non-COPE case, the total number of transmissions required to deliver a packet from one node to another node will be two times the number of links in between them (e.g. 4 in case of Alice-and-Bob). Therefore, if the number of links is N, the number of transmissions required will be 2N. Now in COPE case, every intermediate node on the path between two nodes can transmit the data simultaneously to their neighbors on either side by combining the two packets through coding and sending them in opposite directions. So, N+1 transmissions are required in COPE case. Hence the coding gain comes out to be 2N / (N+1) in case of infinite chain topology. It will be seen later that infinite chain's coding gain approaches 2 as the chain length increases.

Infinite Wheel Topology:

Consider the infinite wheel topology shown in Fig. 4 with N nodes placed at a uniform distance on its circumference. One node is present at the center of the circle. It has been assumed that all nodes can hear a transmission from one node except its diametrically opposed node. Every node can communicate with its diametrically opposed node through the central node only as this is geographically the shortest path between two opposite nodes. This makes it a two-hop transmission. It can be seen that infinite wheel topology is very similar to the "X"

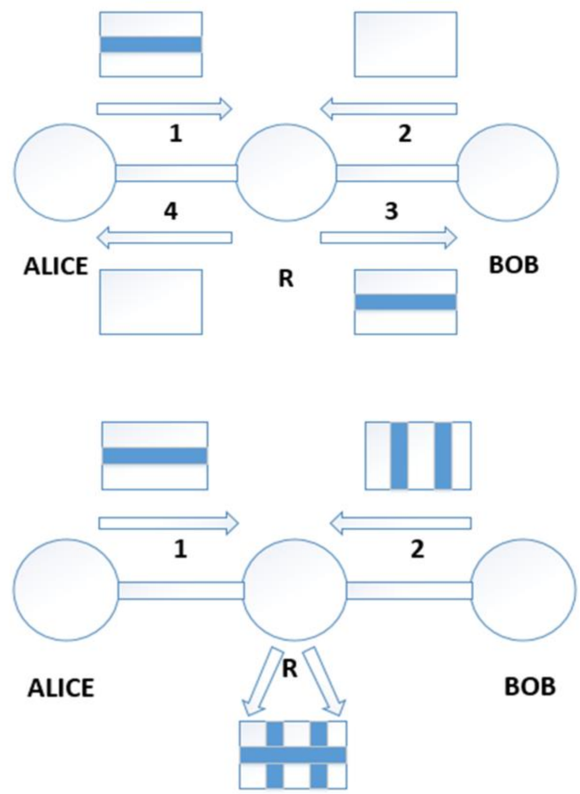

Fig. 2 Alice-and-Bob Network in a) Routing and b) COPE cases

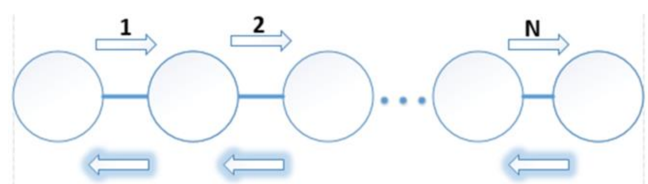

Fig. 3 Infinite Chain Topology

topology with a larger number of nodes. In non-COPE case, if node 1 tries to communicate with its diametrically opposed node i.e. node (N/2) + 1, the number of transmissions required are 4. This is so, because node 1 sends its data to 'R' which forwards it to node (N/2) + 1. Then node (N/2) + 1 replies to 'R' which forwards its reply to node 1. Thus, 4 transmissions are required. Similarly, for every pair of diametrically opposed nodes, 4 transmissions are required. It has already been assumed that there are N nodes in the wheel, where N is an even number. So, there are N/2 pairs of diametrically opposed nodes. Therefore, total 4x(N/2) or 2N transmissions are required in non-COPE case.

Now consider the COPE case. In this scenario, all N nodes will send their packets to the relay node. Thus, N transmissions are made. After coding, 'R' will forward the coded packet to all N nodes. This will add 1 to the total number of transmissions. Thus, N+1 transmissions are required in COPE case. Hence, the coding gain comes out to be 2N / (N+1), which is the same as that of infinite chain topology. This gain also approaches 2 when N becomes larger.

SIMULATION RESULTS:

In this section, simulations are carried out for "X" topology, infinite chain and infinite wheel cases and results have been analyzed in order to verify COPE's effectiveness using the coding gains of these topologies, as coding gain is an important parameter to prove the usefulness of COPE.

"X" Topology:

Coding gain for "X" topology was calculated in last section and it came out to be 1.33. This gain has been plotted in Fig. 5 using MATLAB. As the number of nodes is fixed in case of "X" topology i.e. 4, there is only one point in graph (corresponding to N=4 only).

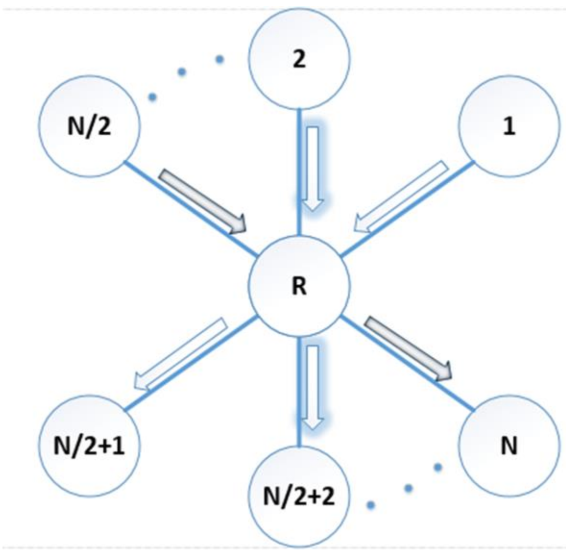

Fig. 4 Infinite Wheel Topology

Infinite Chain Topology:

Coding gain for infinite chain topology has been calculated in last section. This gain is plotted in Fig. 6 for different chain lengths. Results show that as the chain length increases from 10 to 1000, the coding gain approaches 2. This confirms the analysis presented in last section.

Infinite Wheel Topology:

The graph of coding gain for different values of N has been plotted in Fig. 7. As the coding gain for both, infinite chain topology and infinite wheel topology, came out to be same, their plots are also the same. Similarly, this graph also shows that increasing the number of nodes in an infinite wheel topology results in an approximate coding gain of 2.

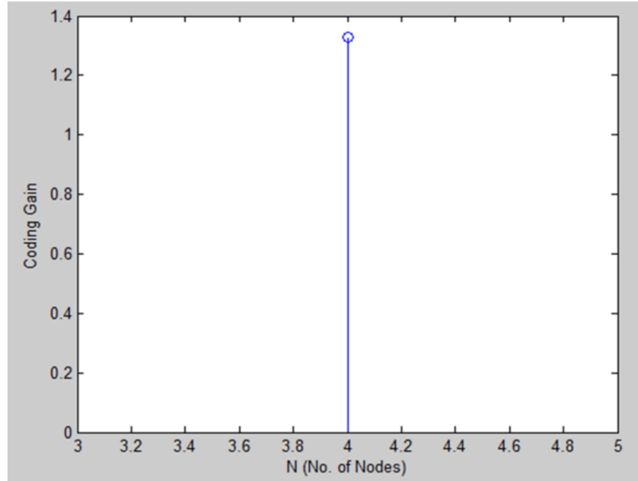

Fig. 5 Coding Gain for "X" Topology

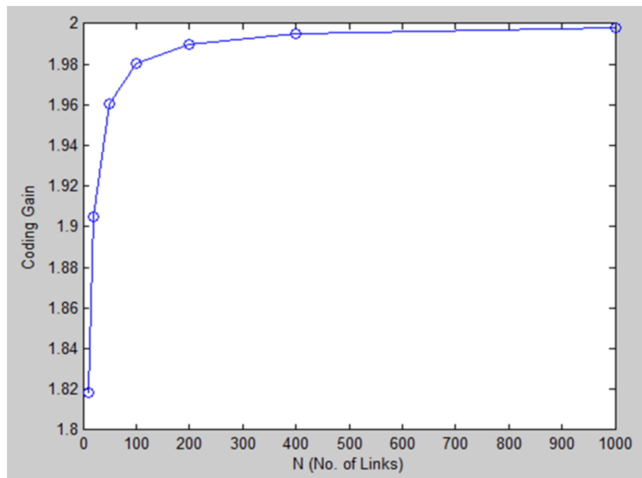

Fig. 6 Coding Gain for Infinite Chain Topology

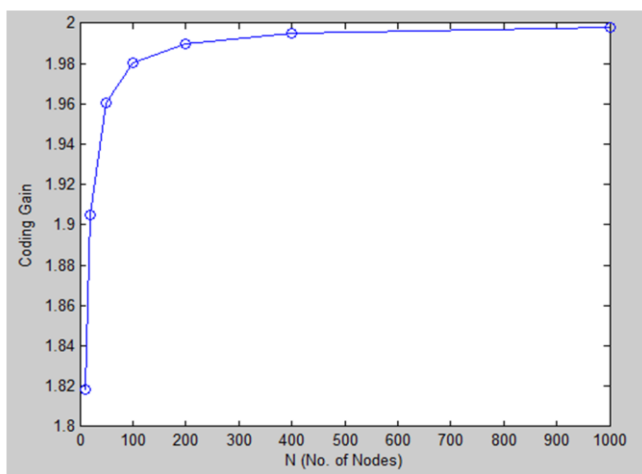

Fig. 7 Coding Gain for Infinite Wheel Topology

The approximate coding gains of different networks, which were under discussion in this paper and previous work ([1, 4]), have been summarized in Table I. Simulation results prove the theorem introduced in [4] which stated that COPE's maximum coding gain is 2, which is achievable as N grows larger i.e. for larger networks. Thus, for larger networks, COPE's performance is improved than in case of smaller networks. These gains have been calculated in absence of opportunistic listening but if opportunistic listening is introduced, the results are even better.

Table I: Theoretical Gains for Different Network Topologies

Topology	Coding Gain
Alice-and-Bob	1.33
"X"	1.33
Cross	1.6
Infinite Chain	2
Infinite Wheel	2

CONCLUSION:

In this paper, COPE's performance has been analyzed for various different network topologies using theoretical analysis and coding gains for these topologies are also calculated and compared. These gains show that coding introduces much improvement in the performance of wireless mesh networks. COPE shows better performance in case of larger networks as compared to the smaller networks. Simulation results proved that a maximum coding gain of 2 can be achieved for larger networks in the absence of opportunistic listening. This analysis gives an insight into the performance of networks in routing and coding cases.

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