MINIMIZATION OF TWO BAR BRACKET OPTIMIZATION PROBLEM THROUGH DERIVATIVE FREE METHODS

Sadaf, S.[1*], M. F. Tabassum[2], A. Sana[2], N. A. Shahid[3], N. A. Chaudhry[4], M. Saeed[2]

[1] Department of Mathematics, Lahore Garrison University, Lahore, Pakistan.

[2] Department of Mathematics, University of Management and Technology, Lahore, Pakistan.

[3] Department of Mathematics, Lahore Leads University, Kamahan Campus, Lahore, Pakistan.

[4]Department of Mathematics, UCEST, Lahore Leads University, Lahore, Pakistan.

*Corresponding Author: zsadaf9@gmail.com , +92-345-4493533

Abstract: *The purpose of this exploration is to construct optimization models of two bar bracket alongside stress constraints. The derivative free methods Nelder-Meed Method, Hooke-Jeeve Method and Multidirectional Search Methods are used to solve model. Penalty functions are used to eliminate constraints. Then constrained optimization model is changed into unconstrained model. Finally discuss the comparison and efficiency of these methods through MATLAB Programming.*

Keywords*: derivative free methods, penalty function, unconstrained optimization, structural optimization problem*

1. INTRODUCTON

In real world problems decision making is very important, when many possible solutions are there and we choose best one through optimization. Scientific optimization has vital part in choice sciences and in dissection of physical framework.

Mathematical optimization is maximization or minimization of objective function subject to constraints. Real world problems are often nonlinear and large. Financial problems have hundreds of thousands of variables and constraints. Many engineering problems can be solved more accurately as their discretization become finer and problem size larger.

Following are basic techniques of optimization [1].

- Derivative based
- Derivative free
- Evolutionary

The main techniques of optimization, namely, derivative based method and derivative free method (direct search) are being used frequently. Among the direct search methods we focused on Nelder-Mead (NM) method [2, 6], Hooke-Jeeves (HJ) method and Multi-Directional Search (MDS) Method. Hooks-Jeeves and Nelder-Mead utilized before this technique. At last the greater part of the direct search technique are not difficult to utilize, straightforward and simple to make them into unconstrained optimization problems by using penalty function. For the constrained optimization problems

The original constrained problem is transformed into unconstrained optimization problems by using penalty function [8-10]. The direct search technique is utilized by the certainty the choice making procedure is focused around exclusively on function esteem data.

It shall assume throughout that an initial estimation of the solution is available. This initial estimate may or may not be feasible. We discuss algorithms that generate a sequence of points. Approximate stationary points of an associated unconstrained function called a penalty function [3].

The performance of Nelder-Mead method and Hooke-Jeeves methods vary as the nature of the feasible region and the response surface of the objective function changes.

All the methods are designed for unconstrained optimization problems. These methods can be applied by transforming

The structures of the penalty function along with the rules for updating the penalty parameters at the close of each unconstrained minimization stage define the particular method. The penalty function is exact if only one unconstrained minimization is required.

2. MATERIAL AND METHODS

2.1 Nelder- Meed Simplex Method

Reflection: Reflection take place when $x^G \geq x^R > x^B$.

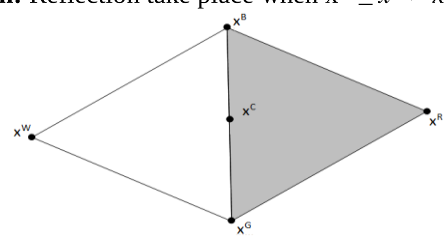


Fig 1: Reflection for Nelder- Meed Simplex Method

Expansion: Expansion take place when $x^G \geq x^B > x^E$.

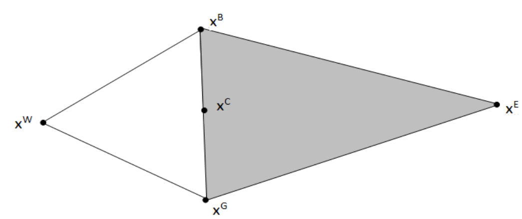


Fig 2: Expansion for Nelder- Meed Simplex Method

Outer contraction: Outside contraction take place when $x^W \geq x^R > x^G$.

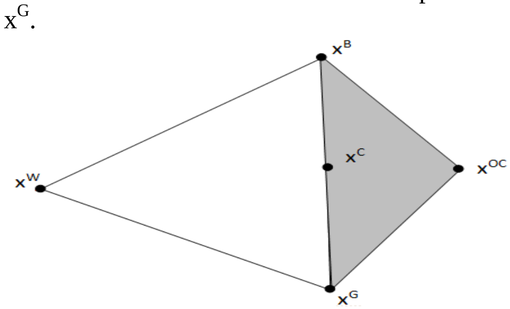


Fig 3: Outer contraction for Nelder- Meed Simplex Method

Inner contraction: Inside contraction takes place when $x^R \geq x^W$.

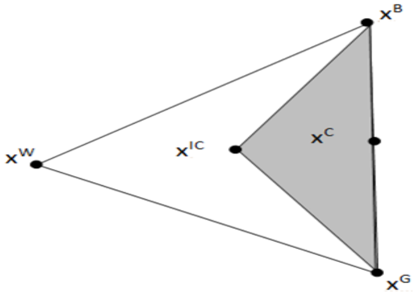


Fig 4: Inner contraction for Nelder- Meed Simplex Method

Shrink: At the point when all the function values are greater than the function value best case scenario point then shrink exist

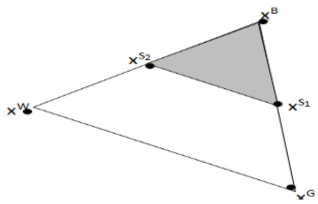


Fig 5: Shrink for Nelder- Meed Simplex Method

2.2 Hook- Jeeves Method

This method works with two types of moves [4].

- Exploratory move
- Pattern move

Exploratory move: The move which is performed at the current base point x_c to investigate the conduct of the objective function in the area of x_e is called an exploratory move [5].

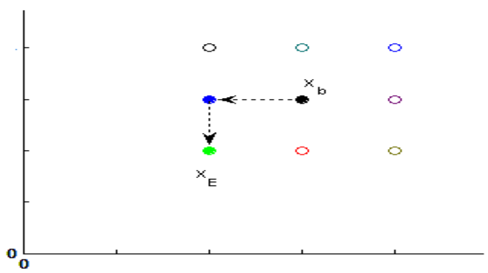


Fig 6: Exploratory move for Hook- Jeeves Method

Pattern move: A fruitful exploratory move gives two focuses. One of these is beginning base point x_b and the other point is x.

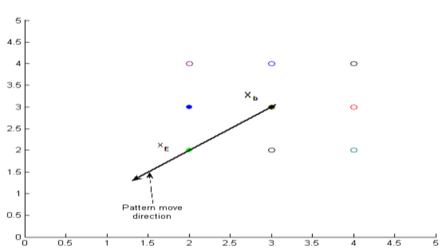


Fig 7: Pattern move for Hook- Jeeves Method

2.3 Multi-directional search Method

Any iteration in multi-directional search calculation we take N + 1 point. Which characterize in deteriorate simplex [7]. The system utilizes the accompanying operations:-

- reflection
- expansion
- inner Contraction

Reflection: In the wake of reflecting the first simplex through the best point give another simplex.

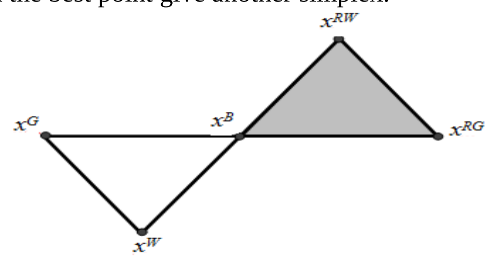


Fig 8: Reflection for Multi-directional search Method

Expansion: We expand the reflected simplex by doing the length of two times of each edge along reflected simplex for this one of the reflected point < best point.

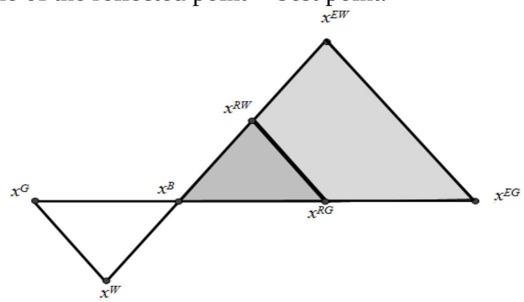


Fig 9: Expansion for Multi-directional search Method

Inner contraction: If the function value of the reflect point ≥ function value of best point then inner contraction has done at the best point by doing half the length of each edging.

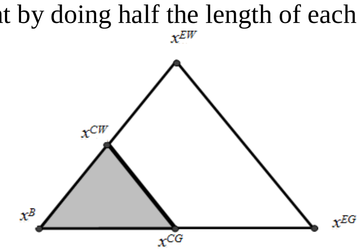


Fig 10: Inner contraction for Multi-directional search Method

3. STRUCTURAL OPTIMIZATION PROBLEM

Consider a two bar truss. The bars 1 and 2 have same length. We have to minimize the mass under stress constraint. The design variables are the cross sectional areas A_1, A_2 . The objective function is

$$f(A) = (A_1 + A_2)\rho l$$

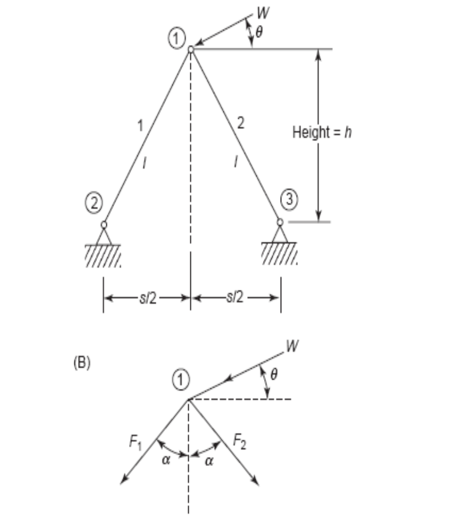
In this problem

x_3 = Outer diameter of Bar 1
x_4 = Inner diameter of Bar 1
x_5 = Outer diameter of Bar 2
x_6 = Inner diameter of Bar 2

And

x_1 = height of bracket
x_2 = span of bracket

Here due to symmetry



The two-bar bracket. (A) The structure. (B) Free body diagram for node 1.

$$x_3 = x_5 \text{ and } x_4 = x_6$$

For Bar 1

$$\text{Outer Radius } r_o = \frac{x_3}{2}$$

$$\text{Inner Radius } r_i = \frac{x_4}{2}$$

$$A_o = \pi\frac{x_3^2}{4}$$

$$A_i = \pi\frac{x_4^2}{4}$$

$$A_1 = \pi\frac{x_3^2}{4} - \pi\frac{x_4^2}{4}$$

$$A_1 = \frac{\pi}{4}(x_3^2 - x_4^2)$$

For Bar 2

$$\text{Outer Radius } r_o = \frac{x_5}{2}$$

$$\text{Inner Radius } r_i = \frac{x_6}{2}$$

$$A_o = \pi\frac{x_5^2}{4}$$

$$A_i = \pi\frac{x_6^2}{4}$$

$$A_2 = \pi\frac{x_5^2}{4} - \pi\frac{x_6^2}{4}$$

$$A_2 = \frac{\pi}{4}(x_5^2 - x_6^2)$$

$$M = \rho \times \text{material volume}$$

$$M = \rho \times [\, l(A_1 + A_2)]$$

Putting the value of A_1 and A_2 in above formula

$$M = \rho\left[\sqrt{(0.5\,s)^2 + h^2}\;\left[\frac{\pi}{4}(x_3^2 - x_4^2) + \frac{\pi}{4}(x_5^2 - x_6^2)\;\right]\right]$$

$$M = \rho\left[\sqrt{(0.5x_2)^2 + x_1^2}\;\left[\frac{\pi}{4}(x_3^2 - x_4^2) + \frac{\pi}{4}(x_5^2 - x_6^2)\;\right]\right]$$

Which is our objective function i.e. total mass of two bar bracket

$$M = \rho\frac{\pi}{4}\sqrt{(0.5x_2)^2 + x_1^2}\;\;(x_3^2 - x_4^2 + x_5^2 - x_6^2)$$

Now we find the Bar stresses

From the last Figure we have

For horizontal forces

$$-F_1\sin\alpha + F_2\sin\alpha = w\cos\theta$$

$$-F_1\cos\alpha - F_2\cos\alpha = w\sin\theta$$

$$\sin\alpha = \frac{0.5\,s}{l}$$

$$\cos\alpha = \frac{h}{l}$$

$$l^2 = (0.5s)^2 + h^2$$

Putting the values of sinα and cosα in above horizontal forces

$$-F_1\frac{0.5s}{l} + F_2\frac{0.5\,s}{l} = w\cos\theta$$

$$-F_1 0.5s + F_2 0.5s = wl\cos\theta$$

$$-F_1 + F_2 = \frac{wl}{0.5s}\cos\theta \qquad (1)$$

$$-F_1\frac{h}{l} - F_2\frac{h}{l} = w\sin\theta$$

$$-F_1 h - F_2 h = wl\sin\theta$$

$$-F_1 - F_2 = \frac{wl}{h}\sin\theta \qquad (2)$$

From (1) and (2)

$$F_1 = -0.5\,wl\left[\frac{\sin\theta}{h} + \frac{2\cos\theta}{s}\right]$$

$$F_2 = -0.5wl\left[\frac{\sin\theta}{h} - \frac{2\cos\theta}{s}\right]$$

Put the design variable x_1 and x_2, we have

$$F_1 = -0.5\,wl\left[\frac{\sin\theta}{x_1} + \frac{2\cos\theta}{x_2}\right]$$

$$F_2 = -0.5wl\left[\frac{\sin\theta}{x_1} - \frac{2\cos\theta}{x_2}\right]$$

In this problem bar stresses are involved, which are denoted by σ_1 for bar 1 and σ_2 for bar 2

$$\sigma_1 = \frac{F_1}{A_1}$$

$$\sigma_1 = \frac{2W\sqrt{(0.5x_2)^2 + x_1^2}}{\pi(x_3^2 - x_4^2)}\left[\frac{\sin\theta}{x_1} + \frac{2\cos\theta}{x_2}\right]$$

$$\sigma_2 = \frac{F_2}{A_2}$$

$$\sigma_2 = \frac{2W\sqrt{(0.5x_2)^2 + x_1^2}}{\pi(x_5^2 - x_6^2)}\left[\frac{\sin\theta}{x_1} - \frac{2\cos\theta}{x_2}\right]$$

Now constraints are

Stress constraint for bar 1 is

$$\frac{2W\sqrt{(0.5x_2)^2 + x_1^2}}{\pi(x_3^2 - x_4^2)}\left[\frac{\sin\theta}{x_1} + \frac{2\cos\theta}{x_2}\right] \le \sigma_a$$

Stress constraint for bar 2 when in tension is

$$\frac{2W\sqrt{(0.5x_2)^2 + x_1^2}}{\pi(x_5^2 - x_6^2)}\left[\frac{\sin\theta}{x_1} - \frac{2\cos\theta}{x_2}\right] \le \sigma_a$$

Stress constraint for bar 2 is

$$\frac{-2W\sqrt{(0.5x_2)^2 + x_1^2}}{\pi(x_5^2 - x_6^2)}\left[\frac{\sin\theta}{x_1} - \frac{2\cos\theta}{x_2}\right] \le \sigma_a$$

W= 25000N and $\theta = 45°$

TARGET

Our target is to minimize the total mass of two bar bracket

Objective function

Minimize total mass of bars

$$M = \rho\frac{\pi}{4}\sqrt{(0.5x_2)^2 + x_1^2}\;\;(x_3^2 - x_4^2 + x_5^2 - x_6^2)$$

Subject to constraints

$$\frac{2W\sqrt{(0.5x_2)^2 + x_1^2}}{\pi(x_3^2 - x_4^2)}\left[\frac{\sin\theta}{x_1} + \frac{2\cos\theta}{x_2}\right] \le \sigma_a$$

(stress constraint for bar 1)

$$\frac{2W\sqrt{(0.5x_2)^2 + x_1^2}}{\pi(x_5^2 - x_6^2)}\left[\frac{\sin\theta}{x_1} - \frac{2\cos\theta}{x_2}\right] \le \sigma_a$$

(stress constraint for bar 2 when bar is in tension)

$$\frac{-2W\sqrt{(0.5x_2)^2 + x_1^2}}{\pi(x_5^2 - x_6^2)}\left[\frac{\sin\theta}{x_1} - \frac{2\cos\theta}{x_2}\right] \le \sigma_a$$

(stress constraint for bar 2)

$$x_{il} \le x_i \le x_{iu}\ ; i = 1 \text{ to } 6$$

Our final model becomes

$$M = 6.08375\sqrt{(0.5x_2)^2 + x_1^2}(x_3^2 - x_4^2 + x_5^2 - x_6^2) +$$

$$1000max\left[\begin{array}{l}\left(\frac{15923.56688\sqrt{(0.5x_2)^2+x_1^2}}{(x_3^2-x_4^2)}\left[\frac{1}{\sqrt{2}x_1} + \frac{\sqrt{2}}{x_2}\right] - 250\right), \\ \left(\frac{15923.56688\sqrt{(0.5x_2)^2+x_1^2}}{(x_5^2-x_6^2)}\left[\frac{1}{\sqrt{2}x_1} - \frac{\sqrt{2}}{x_2}\right] - 250\right), \\ -\left(\frac{15923.56688\sqrt{(0.5x_2)^2+x_1^2}}{(x_5^2-x_6^2)}\left[\frac{1}{\sqrt{2}x_1} - \frac{\sqrt{2}}{x_2}\right] - 250\right)\end{array}\right]$$

4. RESULTS AND DISCUSSION

The result which obtained from two bar bracket by applying NM method is that, by taking the initial guess from 1 to 12. Many points are being checked between these ranges. We get many solutions; it does not show consistent performance but got a convergent point. So final solution which is feasible as it satisfy all constraints. The function value is 7.97×10^7 at the points (0.9820,-0.4575, 0.4176, -0.3191, -0.1836, and 0.8936).

Table-1: Result of two bar by applying Nelder-Mead Method

Initial guess	Function value	Final point	Function value	Iteration Count
(12, 1.5, 1, 7.5, 1.5, 8)	$-4.2762 X 10^{10}$	(0.9820, -0.4575 0.4176, -0.3191, -0.1836, 0.8936)	$7.97 X 10^7$	95

The result which obtained from a two bar bracket by applying HJ method is that, by taking the initial guess from 1 to 12 . Many points are being checked between these ranges. HJ searches local optimum. We get many solutions; it does not show consistent performance but final solution which is feasible as it satisfies all constraints. The function value is -6.321$\times 10^8$ at the points (363, 352.5, 0, 358.5, 0.5, and 359)

Table-2: Result of two bar by applying Hooke-Jeave's Method

Initial guess	Function value	Final point	Function value	Iteration Count
(12, 1.5, 1, 7.5, 1.5, 8)	$-4.2753 X 10^{10}$	(363, 352.5, 0, 358.5, 0.5,359)	-6.321$\times 10^8$	25

The result which obtained from two truss bar bracket by applying MDS method is that, by taking the initial guess from -1 to 20.many points are being checked between this ranges.MDS searches the local optimum. We get many solutions; it does not show consistent performance but got a convergent point. So final solution which is feasible as it satisfy all constraints. The function value is-3.299177$\times 10^7$ at the points (12.898,-0.021, 1.195, 18.652, 1.315, 8.54

Table-3: Result of two bar by applying MDS Method

Initial guess	Function value	Final point	Function value	Iteration count
(10, -0.1, 2, 20, 1.5, 9)	$-4.2753 X 10^7$	(12.898, -0.021, 1.195, 18.652, 1.315, 8.548)	$-3.2991 X 10^7$	100

Nelder and Meed Search method is better than the other two methods because it gives global optimum while other two methods give local optimum. But Hooks and Jeeves gives less number of iterations other than two methods.

5. CONCLUSION

We have applied MDS method, HJ method and NM method to solve two bar bracket optimization problem. These methods are also implemented in MATLAB on formulated problems by many times by choosing different step size and initial guess. We conclude that NM method gives optimum solution but its convergence is very far. While Hooks and Jevees gives optimum solution in less no. of iterations other two methods.

REFRENCES

[1] R. Hooke and T. A. Jeeves, Direct Search Solution of Numerical and Statistical Problems, Journal of the Association for Computing Machinery, 8 (1961), pp. 212-229.

[2] C. T. Kelley, Detection and Remediation of Stagnation in the Nelder – Meed Algorithm using a Sufficient Decrease Condition, SIAM J. O PTIM., 10: 142-147 (1999).

[3] Tabassum, M. F., M. Saeed , A. Sana, Nazir Ahmad, Solution of 7 Bar Tress Model using Derivative Free Methods, *Proceedings of the Pakistan Academy of Sciences*, 52(3): 265-271 (2015).

[4] M. Marazzi and J. Nocedal, Wedge Trust Region Methods for Derivative Free Optimization, Math. Program, 91: 289-305 (2002)

[5] R. Hooke and T. A. Jeeves, Direct Search Solutions of Numerical and Statistical problems, Journal of the Association for Computing Machinery, 8: 26-37 (1961)

[6] J. A. Nelder and R. Mead, A Simplex Method for Function Minimization, Compute. J., 7:308-313 (1965)

[7] Virginia J. Torczon, Multi-Directional Search: A Direct Search Algorithm for Parallel Machines, TR 90-7, Rice University, Houston Texas ,May 1989

[8] Tabassum, M.F., M.Saeed, Nazir Ahmad and A.Sana, Solution of War Planning Problem Using Derivative Free Methods. Sci.Int., 27(1): 395-398 (2015)

[9] Ali J., M. Saeed, N. A. Chaudhry and M. F. Tabassum. New Mathematical Models of N-Queens Problem and its Solution by a Derivative-Free Method of Optimization, Sci.Int., 27(3): 2005-2008 (2015).

[10] Tabassum, M. F., M. Saeed, N. A. Chaudhry, A. Sana and Z. Zafar., Optimal Design of Oxygen Production System by Pattern Search Methods. Pakistan Journal of Science, 67(4): 371-376 (2015).