ESTIMATION MODELING OF HETEROGENEOUS TISSUE PERMITTIVITY FOR DEVELOPING TISSUE PHANTOMS

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ABSTRACT *This paper presents a theoretical model to determine absolute permittivity values of glandular breast tissue with close approximation. Experimental permittivity values are compared with Debye 1st order/ single pole theoretical results to find errors/ mismatching for 1GHz to 6 GHz Ultra-Wide Band (UWB) frequencies. Least square fitting is employed for error reduction. Projected estimation model has the capacity to lessen the gap 19.28% and 41.20% for healthy and malignant breast tissues respectively. Further, a methodology has been developed and proposed for predicting absolute permittivity values for experimental breast phantoms for other frequencies of 7GHz, 8GHz, 9GHz and 10GHz which are not available so far for normal and tumor containing cases for breast tissue cancer. This research is targeted to help researchers to obtain optimum breast tissue permittivity values for making real life-like phantoms of breast cancer research.*

Keywords: Tissue permittivity, glandular, malignant breast tissue, least squares.

I. INTRODUCTION

Human breast is a heterogeneous biological tissue comprising primarily of adipose and glandular tissue layers apart from skin in the outer. In Fig.1 an illustration of human breast tissues, including glandular, adipose and fibrous tissues are shown [1]. Breasts can have varying percentage distribution of adipose and glandular content giving the difference in physical appearances. Cancer in breasts shoots up in the mammary ducts. In early phase, it is termed as tumor but later it can keep on adding more layers of cells, turning into malignant. Every tissue of breast has their individual dielectrics which are used by the scientists to make synthetic models of breasts called breast phantoms. A true copy of the breast is a heterogeneous phantom in which instead of considering individual tissue layers, a heterogeneous combination of tissues is considered. This heterogeneous tissue structure could have varying percentages of adipose and glandular portions along with tumor and skin with their respective dielectric properties. These dielectric properties [2] describe the behavior of electric and magnetic fields when they pass through them.

Therefore, it is very necessary to develop phantoms with acceptable values. In literature, Debye and Cole-Cole designs [3-5] are used as the foundation models in finding dielectric values for making of human breast phantoms [6-18] and for performing the computational analysis using FDTD. As indicated in [6] Debye model requires least computation overhead. Lazebnik et al [6] performed experimental characterization of normal and malignant breast tissues for UWB (0.5-20GHz) for dielectric properties. In the same study they performed fitting of single and double pole Debye model with highly accurate Cole-Cole models. These researches have been considered as a milestone in breast cancer detection for UWB and their

results have been extensively used by many later coming researches for the making of breast phantoms as in [6-18]. However, gap still subsists among analytical and practical measures of Debye results and practically obtained values.

In this letter we have attempted to decrease the gap between theoretical and practical dielectric values used for making the phantoms. We have considered the work carried out by Lazebnik et al. [6] and experimental phantom model presented in [13] for 1.0 GHz to 6 GHz center frequencies. Although the dielectric values in [6] and [13] are comparable, but yet there are gaps which requires to be crammed to develop a pragmatic phantom.

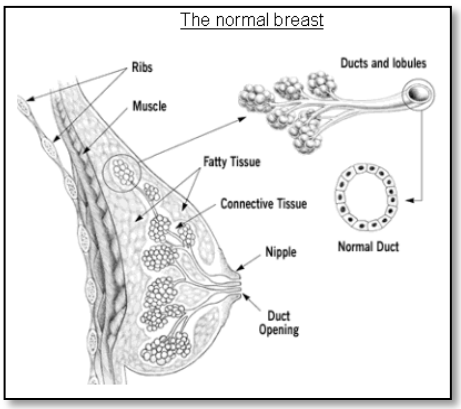

Fig. 1. Different types of breast tissues [1]

II. SINGLE RELAXATION DEBYE MODEL

Lazebnik *et al.* [6] analytically specified UWB dielectric characteristics of wide quantity of non-malign and malign breast tissue samples were obtained from the surgeries of cancer patients. They fitted single pole Cole-Cole model to data samples with single pole and two pole Debye models. The single pole Debye model that was used was,

$$\varepsilon^*(\omega) = \varepsilon'(\omega) - j\varepsilon''(\omega) \quad (1)$$

$$\varepsilon^*(\omega) = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)}{1 + (j\omega\tau)^{1-\alpha}} - j\frac{\sigma_s}{\omega\varepsilon_0} \qquad (\alpha = 0) \quad (2)$$

Here ω is angular frequency, τ defines the relaxation time and σ_s is the static conductivity, which is usually assumed zero value [19]. $\varepsilon''(\omega)$ is related to effective conductivity $\sigma(\omega)$as

$$\varepsilon''(\omega) = \frac{\sigma(\omega)}{\omega\varepsilon_0} \quad (3)$$

The tissue composition used in this research study [6] was taken as a normal sample and enumerated in provisions of proportions of adipose, fibro-connective and glandular

tissues. The tissues composition of each malignant sample was distinguished in terms of invasive and non-invasive carcinomas. In this research study database of normal samples was divided into three groups based on the adipose tissue content. The first order Debye results for 2.5GHz to 6 GHz fitted for Cole-Cole models in [6] are tabulated in Table I and Table II for Group 2 and Group 6 correspondingly.

TABLE I
DEBYE ABSOLUTE PERMITTIVITY FOR GROUP 2 (WITHOUT TUMOR) [6]

Frequency (GHz)	Absolute Permittivity (F/m)
1.0	52.3225
1.5	47.6168
2.0	43.9304
2.5	42.6987
3.0	46.8714
3.5	40.2419
4.0	39.1464
4.5	37.9174
5.0	36.7554
5.5	35.9417
6.0	33.8071

TABLE II
DEBYE ABSOLUTE PERMITTIVITY FOR GROUP 4 (WITH TUMOR) [6]

Frequency (GHz)	Absolute Permittivity (F/m)
1.0	64.8817
1.5	60.8881
2.0	57.7744
2.5	55.9542
3.0	54.4583
3.5	53.5972
4.0	52.6758
4.5	51.8671
5.0	51.1332
5.5	50.3421
6.0	49.7088

III. INVESTIGATIONAL PHANTOM OF BREAST

Porter, et al. [13] made heterogeneous phantoms from layers of fat, skin, gland and tumor. Measurements of relative permittivity and conductivity in case of individual homogeneous phantom layers were carried out for UWB band. In this research a laboratory method was discussed to how to combine these individual phantom layers but no method was described to how to combine permittivity values of the resultant heterogeneous model. The concluding heterogeneous breast phantom consisted of 2mm layer of skin. This was all-encompassing a combination of fat, gland and tumors. It had a radius of 0.065 m with 0.015 m tumor inside the gland, few in quantity. If two such tumors are considered then this makes 46% presence of tumors in this breast phantom. Different layers of a normal breast and tumor containing breast with their absolute permittivity, as taken from [13] are shown in Table III.

TABLE III
EXPERIMENTAL VALUE OF ABSOLUTE PERMITTIVITY AT 1.0- 6 GHZ UWB CENTER FREQUENCY [13]

Frequency (GHz)	Absolute Permittivity Values For:			
	Skin (F/m)	Fat/Adipose (F/m)	Gland (F/m)	Tumor (F/m)
1.0	40.9982	15.0269	35.1843	59.6813
1.5	39.6579	14.5495	34.5244	57.6145
2.0	38.6749	14.3134	33.7749	57.0301
2.5	37.0304	14.0032	32.0156	55.0363
3.0	36.0450	13.8057	31.8353	54.0447
3.5	35.0755	13.5092	31.0644	53.0739
4.0	34.0918	13.2136	30.0805	52.1176
4.5	33.1548	13.0188	29.0991	50.1760
5.0	32.2248	12.5399	28.6372	47.2758
5.5	31.2832	12.0598	28.1822	46.3276
6.0	30.3816	11.0765	27.2660	45.4251

As mentioned earlier that this research study in [13] has no information of dielectric properties for heterogeneous breast phantom. This is a complex problem.

Perez [19] provided the formulations which could be used to find the effective permittivity for a multilayer medium. We have used this methodology to determine effective permittivity of heterogeneous breast model. This simplifies the complex problem by instead of solving each layer individually but taking it as a whole. As shown in Perez [19] in this examination it is taken as an assumption that if for any incident signal (on the breast) is a plane wave (in the far field of the transmitting antenna) and it is normal to the dielectric surface then this electric field ($\hat{E}$) vector is parallel to the interface. The effective permittivity is then given as,

$$\varepsilon_{eff} = \xi_1\varepsilon_1 + \xi_2\varepsilon_2 + \xi_3\varepsilon_3 + \xi_3\varepsilon_3 \quad (4)$$

In Equations (4), ξ_1, ξ_2 , $\xi_3$and ξ_4 are the volume fractions of the constituent layers of skin,adipose, glandular and tumor. Likewise ε_1 , ε_2 , ε_3 and ε_4 are the absolute permittivity values of each layers of skin, adipose, glandular and tumor. In varied literature on breast cancer detection and phantom making [6-18] different percentages of skin, adipose , fat and tumor have been taken. For our purpose we took skin as 20%, adipose as 50% of breast, glandular is 50%. In this 50% glandular part, it is taken that 40% is malignant. This makes out of 50% glandular portion 30% is normal and 20% is malignant. We took two cases here, first is without tumor and second is with tumor. Table IV and Table V shows absolute effective permittivity values obtained from Equation 4. The former is without tumor and latter is with tumor for the breast.

Comparing Table I with Table IV and Table II with Table V it is quite obvious that gap exists between Debye fitted models proposed in [6] and an experimental breast phantom model obtained from [13].

Our contribution in this research is to reduce this gap by statistically modeling these two results using least square fitting criteria. This is presented in the following section.

TABLE IV
ABSOLUTE EFFECTIVE PERMITTIVITY FOR HETEROGENEOUS NORMAL BREAST FOR EXPERIMENTAL DATA IN [13]

Frequency (GHz)	Absolute Effective Permittivity
1.0	30.2998
1.5	29.5586
2.0	28.9164
2.5	27.6148
3.0	27.2683
3.5	26.6000
4.0	25.8226
4.5	25.0861
5.0	24.5255
5.5	23.9656
6.0	23.0322

TABLE V
ABSOLUTE EFFECTIVE PERMITTIVITY FOR HETEROGENEOUS TUMOR CONTAINING BREAST FOR EXPERIMENTAL DATA IN [13]

Frequency (GHz)	Absolute Effective Permittivity
1.0	35.1992
1.5	34.1766
2.0	33.5674
2.5	32.2189
3.0	31.7102
3.5	31.0019
4.0	30.2301
4.5	29.3015
5.0	28.2532
5.5	27.5947
6.0	26.6640

IV. ERROR CORRECTION FOR DEBYE MODEL

This section has discussed how we have obtained analytical model using least square fit from [6] and [13].

In this continuation, at first statistical correlation between the absolute permittivity results of Table I with Tale IV and Table II with Table V for normal and malignant breast respectively was performed. For two sets of data x and y correlation coefficient, can be found using Pearson correlation equation [20] as Pearson correlation coefficient, Γ_{xy} as,

$$\Gamma_{xy} = \frac{Cov(x,y)}{\sigma_x \sigma_y} \quad (5)$$

Where $Cov(x,y)$ describes the covariance of x and y, σ_x is the standard deviation in x, σ_y is the standard deviation in y. The first task was to perform a correlation test to obtain correlation matrix Γ as,

$$\Gamma = \begin{bmatrix} \Gamma_{xx} \Gamma_{xy} \\ \Gamma_{yx} \Gamma_{yy} \end{bmatrix} \quad (6)$$

Correlation results are illustrated in the correlation matrix, Γ_1, as in Equation (7) and Γ_2 as in Equation (8). Equation (7) is for the correlation between absolute permittivity results of Table I and Table IV for the normal breast case. Equation (8) is for the correlation between absolute permittivity results of Table II and Table V for the malignant breast case. The detail analytical derivation to obtain the results in Equation (7) and Equation (8) are shown in Appendix A.

$$\Gamma_1 = \begin{bmatrix} 1 & 0.9483 \\ 0.9483 & 1 \end{bmatrix} \quad (7)$$

$$\Gamma_2 = \begin{bmatrix} 1 & 0.9465 \\ 0.9465 & 1 \end{bmatrix} \quad (8)$$

We fitted this data sets and estimated a polynomial equaton of the regression curve using least square fiting criteria. The final fitted model is shown in Equation. (9) for normal breast case and Equation (10) for malignant breast data.

$$P_1(x) = 0.3952x + 10.1788 \quad (9)$$

Where where x is the Debye model value in Table I. $P_1(x)$ is the best fitted value and new realistic value relative to the experimental reaults in Table IV.

Similarly we have,

$$P_2(x) = 1.6035x + 5.2916 \quad (10)$$

Where where x is the Debye model value in Table II. $P_1(x)$ is the best fitted value and new realistic value relative to the experimental reaults in Table V. Therefore $P_1(x)$ and $P_2(x)$ is the anticipated equations for absolute permittivity value. In both cases x is the Debye model value in Table I and Table II respectively.The regression curve of Equation (9) and Equation (10) is shown in Fig. 2 and Fig. 3 respectively for normal and malignant breast cases. These curves fit very closely with the desired absolute permittivity values in Table IV and Table V.

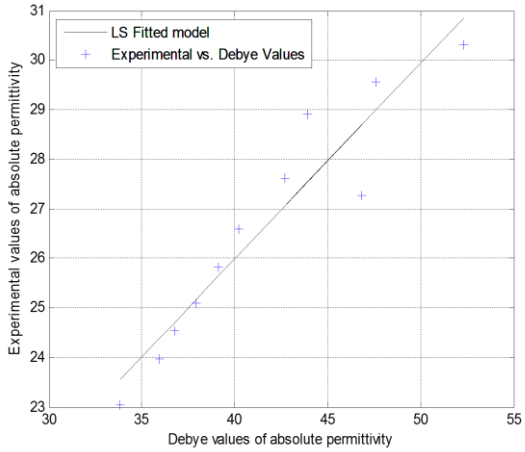

Fig. 2. Least square regression curve for normal heterogeneous breast tissue

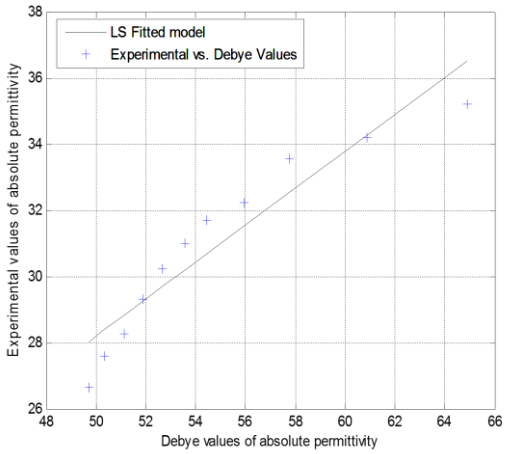

Fig. 3. Least square regression curve for malignant heterogeneous breast tissue

TABLE VI

COMPARISON OF NEW AND OLD DEBYE MODEL VALUES FOR NORMAL BREAST

FREQUENCY (GHz)	OLD DEBYE VALUE (A)	EXPERIMENTAL VALUE (B)	ERROR % $E_1 = \frac{\lvert A-B\rvert}{B} \times (100)$	NEW DEBYE VALUE (C)	NEW ERROR % $E_2 = \frac{\lvert C-B\rvert}{B} \times (100)$
1.0	52.3225	30.2998	42.0903	30.8566	1.8378
1.5	47.6168	29.5586	37.9240	28.9969	1.9000
2.0	43.9304	28.9164	34.1767	27.5400	4.7596
2.5	42.6987	27.6148	35.3263	27.0533	2.0332
3.0	46.8714	27.2683	41.8231	28.7023	5.2591
3.5	40.2419	26.6000	33.8997	26.0823	1.9458
4.0	39.1464	25.8226	34.0358	25.6494	0.6705
4.5	37.9174	25.0861	33.8401	25.1637	0.3095
5.0	36.7554	24.5255	33.2737	24.7045	0.7299
5.5	35.9417	23.9656	33.3209	24.3829	1.7414
6.0	33.8071	23.0322	31.8717	23.5393	2.2019

TABLE VII

COMPARISON OF NEW AND OLD DEBYE MODEL VALUES FOR NORMAL BREAST

FREQUENCY (GHz)	OLD DEBYE VALUE (A)	EXPERIMENTAL VALUE (B)	ERROR % $E_1 = \frac{\lvert A-B\rvert}{B} \times (100)$	NEW DEBYE VALUE (C)	NEW ERROR % $E_2 = \frac{\lvert C-B\rvert}{B} \times (100)$
1.0	64.8817	35.1992	45.7486	36.5104	3.7251
1.5	60.8881	34.1766	43.8698	34.2791	0.3001
2.0	57.7744	33.5674	41.8991	32.5395	3.0620
2.5	55.9542	32.2189	42.4191	31.5226	2.1611
3.0	54.4583	31.7102	41.7715	30.6868	3.2271
3.5	53.5972	31.0019	42.1576	30.2057	2.5680
4.0	52.6758	30.2301	42.6110	29.6909	1.7834
4.5	51.8671	29.3015	43.5065	29.2391	0.2127
5.0	51.1332	28.2532	44.7458	28.8291	2.0384
5.5	50.3421	27.5947	45.1856	28.3871	2.8716
6.0	49.7088	26.6640	46.3595	28.0333	5.1354

Using [21] and [22], Appendix B is given to exemplify the valuation of slope and intercept values in the projected model equations, i.e. Equation (9) and Equation (10). Using the proposed model equations new values for absolute permittivity were evaluated for error calculations. The results are shown in Table VI and Table VII again for normal and malignant breast case respectively separately.

Table VI and Table VII show that percentage errors of normal and malignant heterogeneous breast tissue respectively in the absolute permittivity values for Debye [6], experimental [13] and proposed analytical models. As obvious from the tabulated results, new error E_2 is far less than E_1 both in normal and malignant case at all the frequencies. Average error for normal breast case is 21.41% by using the Debye model fitting presented in [6]. But average error for normal breast case is 2.12% by using our corrected model proposed by us in Equation (9). Therefore the gap is reduced approximately by 19.28%. Similarly average error for malignant breast case is 43.66% by using the Debye model fitting presented in [6]. But average error for malignant breast case is 2.46% by using our corrected model proposed by us in Equation (10). Therefore the gap is reduced approximately by 41.20%.

V. PREDICTING ABSOLUTE PERMITTIVITY FOR OTHER FREQUENCIES

For our study we only had frequency values 1-6 GHz from experimentally obtained breast phantoms. In order to predict experimental absolute permittivity values beyond 6 GHz we can make use of the linearity of the polynomials $P_1(x)$ and $P_2(x)$ for normal and tumor containing breast models respectively. This could be done easily by extending the curves in Fig.2 and Fig.3in their normal linear propagation direction.

Fig.4 shows extended curve of Fig. 2 for normal breast case using $P_1(x)$. We have permittivity and conductivity curves in [6] for frequencies 0.5 GHz to 20 GHz. Using both conductivity and permittivity values, an absolute permittivity of 31.4006 corresponds to 7 GHz. Likewise absolute permittivity values of 31.0483, 30.3644 and 29.7321 corresponds to 8 GHz, 9GHz and 10 GHz respectively.

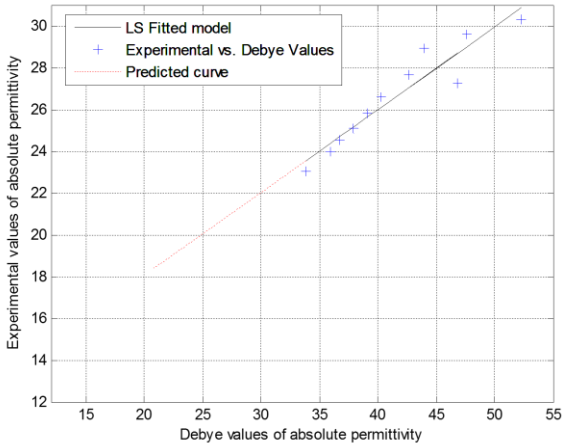

Fig. 4. Predicted value curve for normal breast

Now using Fig. 4 and $P_1(x)$ of predicted value curve of normal breast, absolute permittivity values of 31.4006, 31.0483, 30.3644 and 29.7321 corresponds to 22.5883 , 22.4491, 22.1788 and 21.9289. These are the predicted values of absolute permittivity for experimental breast phantom for 7 GHz, 8GHz, 9Ghz and 10GHz respectively.

A similar analysis cold be done for tumor containing breast. Fig.5 shows extended curve of Fig. 3 for tumor containing breast case using $P_2(x)$. We have permittivity and conductivity curves in [6] for frequencies 0.5 GHz to 20 GHz. Using both conductivity and permittivity values, an absolute permittivity of 48.5077 corresponds to 7 GHz. Likewise absolute permittivity values of 47.6759, 46.8721 and 46.0977 corresponds to 8 GHz, 9GHz and 10 GHz respectively.

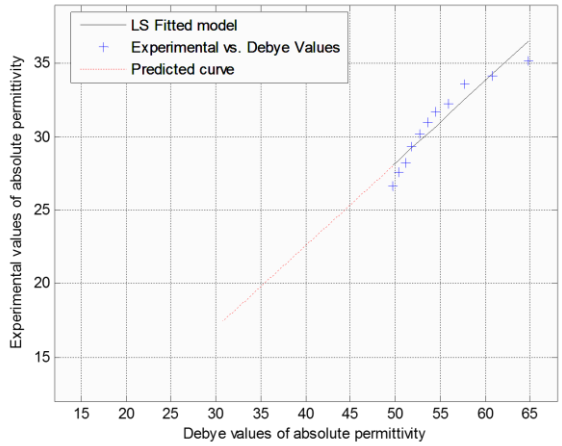

Fig. 5.: Predicted value curve for tumor containing breast

Now using Fig. 5 and $P_2(x)$, absolute permittivity values of 48.5077, 47.6759, 46.8721 and 46.0977 corresponds to 27.3617, 26.8970, 26.4479 and 26.0153. These are the predicted values of absolute permittivity for experimental tumor containing breast phantom for 7 Ghz, 8GHz, 9Ghz and 10GHz respectively.

VI. RELATIONSHIP BETWEEN TWO MODELS

A similar linear regression model can be developed between the two models $P_1(x)$ and $P_2(x)$ obtained earlier. This model is useful to predict normal breast absolute permittivity values from tumor containing breast; which could consequently give the soft boundaries of absolute permittivity values for benign tumors. After correlating the two models we obtain a new linear regression model $P_3(x)$ as follows,

$$P_3(x) = 1.0998x + 1.6406 \qquad (11)$$

In this equation 'x' is the Debye parameter obtained from $P_1(x)$. For any value of 'x', we can get a corresponding value in $P_3(x)$ which is absolute permittivity without tumor ,i.e. $P_2(x)$. This means that at respective frequency points statistically there is a difference of approximately 1.6406 between absolute permittivity of normal and tumor containing breast. So it means that keeping on the safe side, for benign case we could make an assumption that any value of absoluter permittivity greater than ½ the value of 1.6406 than normal value should be taken seriously as an abnormal permittivity value and could lead to cancerous cells development.

VII. CONCLUSION

This paper proposes an estimation model to approximate breast tissue permittivity values with high accuracy. This model has shown its capability to reduce the gap among experimental and analytical 1st order Debye model for 1.0 GHz - 6 GHz UWB frequency. The model has competently abridged this gap by decreasing the relative error to roughly 19.28% and 41.20% for normal and malignant breast cases respectively, showing its efficiency. Therefore the proposed models could be very useful to determine actual breast tissue permittivity values directly for early detection of breast cancer. We have also formulated procedure for predicting absolute permittivity values for experimental breast phantoms. This was done for 7GHz, 8GHz, 9GHz and 10GHz, which were not mentioned or stated in [13] using our proposed models in Equations (9) and (10). Lastly we have attempted to develop soft boundaries between normal, benign and malignant tumors in terms of absolute permittivity values, which are essential to develop threshold values for early detection of breast tumors. For this purpose we have proposed another model equation which can be further worked upon to more maturity level in the future study.

APPENDIX A

Table I and Table IV has provided the data set of absolute permittivity values as *x* and *y* respectively for normal breast case and in Table II and Table V for malignant case. Let *n* is the length of dataset which is equal to 8 (for all the frequencies). Correlation coefficient is given by,

$$\Gamma_{xy} = \frac{Cov(x,y)}{\sigma_x \sigma_y} \qquad A.1$$

$$\Gamma_{xx} = \frac{Cov(x,x)}{\sigma_x \sigma_x} \qquad A.2$$

$$\Gamma_{yy} = \frac{Cov(y,y)}{\sigma_y \sigma_y} \qquad A.3$$

Where

$$Cov(x,y) = \sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i \qquad A.4$$

Standard deviation of *x* ,

$$\sigma_x = \sqrt{\sum_{i=1}^{n} {x_i}^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2} \qquad A.5$$

Standard deviation of *y* ,

$$\sigma_y = \sqrt{\sum_{i=1}^{n} {y_i}^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2} \qquad A.6$$

$$Cov(x,x) = \sum_{i=1}^{n} x_i x_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i \qquad A.7$$

$$Cov(y,y) = \sum_{i=1}^{n} y_i y_i - \frac{1}{n}\sum_{i=1}^{n} y_i \sum_{i=1}^{n} y_i \qquad A.8$$

Substituting the values of data set in *x* and *y* Equations($A.4$) to ($A.8$), we obtain the following results in two cases. Case 1 is for normal case and in Case 2 we get for malignant breast case.

Case 1:

$$Cov(x,y) = 127.7536 \qquad A.9$$

$$\sigma_x = 17.9786 \qquad A.10$$

$$\sigma_y = 7.4953 \qquad A.11$$

$$Cov(x,x) = \ 323.2302 \qquad A.12$$

$$Cov(y,y) = 56.1523 \qquad A.13$$

Putting the results in Equations $(A.9)$ to $(A.13)$ in (A.1) to (A.3) correlation coefficients are acquired, as,

$$\Gamma_{xx} = 1, \Gamma_{yy} = 1$$
$$\Gamma_{xy} = 0.9483, \Gamma_{yx} = 0.9483$$

Hence correlation coefficient matrix becomes

$$\Gamma_1 = \begin{bmatrix} \Gamma_{xx} \Gamma_{xy} \\ \Gamma_{yx} \Gamma_{yy} \end{bmatrix}$$

Which is

$$\Gamma_1 = \begin{bmatrix} 1 & 0.9483 \\ 0.9483 & 1 \end{bmatrix} \qquad A.14$$

Case 2:

$$Cov(x,y) = \ 124.4651 \qquad A.15$$

$$\sigma_x = \ 14.9258 \qquad A.16$$

$$\sigma_y = \ 8.8102 \qquad A.17$$

$$Cov(x,x) = \ 222.7801 \qquad A.18$$

$$Cov(y,y) = \ 77.6187 \qquad A.19$$

Using the results in Equations $(A.15)$ to $(A.19)$ in (A.1) to (A.3) we acquire coefficient of correlation, as,

$$\Gamma_{xx} = 1, \Gamma_{yy} = 1$$
$$\Gamma_{xy} = 0.9465, \Gamma_{yx} = 0.9465$$

Hence correlation coefficient matrix becomes

$$\Gamma_2 = \begin{bmatrix} \Gamma_{xx} \Gamma_{xy} \\ \Gamma_{yx} \Gamma_{yy} \end{bmatrix}$$

Which is

$$\Gamma_2 = \begin{bmatrix} 1 & 0.9465 \\ 0.9465 & 1 \end{bmatrix} \qquad A.20$$

APPENDIX B

Table II and Table V has provided the required data values of absolute permittivity values as *x* and *y* respectively for normal breast case and in Table II and Table V for malignant case. Let *n* is the length of dataset which is equal to 8 (for all the frequencies).

Slope and intercept for linear regression is,
$slope, m \rightarrow$

$$m = \frac{Cov(x,y)}{{\sigma_x}^2} \qquad B.1$$

Using results from Appendix A, we again make two cases as *Case 1* and *Case 2* for normal and malignant heterogeneous breast respectively as follows.

Case 1:

From *Case 1* of Appendix A, slope *m* is found to be equal to,

$$m = 0.3952 \qquad B.2$$

The intercept term, b, is,

$$b = \hat{y} - m\hat{x} \qquad B.3$$

Where

$$\hat{y} = mean\ value\ of\ (y) = 26.6082 \qquad B.4$$

And

$$\hat{x} = mean\ value\ of\ (x) = 41.5682 \qquad B.5$$

Putting Equations $(B.2)$, $(B.4)$ and $(B.5)$ in $(B.3)$, intercept, b is found as

$$b = 10.1788 \qquad B.6$$

Hence the model equation for normal heterogeneous breast case is

$$P_1(x) = mx + b \qquad B.7$$

$$P_1(x) = 0.3952x + 10.1788 \qquad B.8$$

Case 2:

From *Case 2* of Appendix A, slope *m* is found to be equal to,

$$m = 0.5587 \qquad B.9$$

The intercept term, b, is given as,

$$b = \hat{y} - m\hat{x} \qquad B.10$$

Where

$$\hat{y} = mean\ value\ of\ (y) = 30.9016 \qquad B.11$$

And

$$\hat{x} = mean\ value\ of\ (x) = 54.8437 \qquad B.12$$

Putting Equations $(B.9)$, $(B.11)$ and $(B.12)$ in Equation $(B.10)$, we get the value of intercept , b, as

$$b = 0.2610 \qquad B.13$$

Hence the model equation for malignant heterogeneous breast case is

$$P_2(x) = mx + b \qquad B.14$$
$$P_2(x) = 0.5587x + 0.2610 \qquad B.15$$

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