MHD MIXED CONVECTION BOUNDARY LAYER FLOW THROUGH POROUS MEDIUM DUE TO POROUS NON-LINEARLY SHRINKING SURFACE

Farooq Ahmad[1,a], Sajjad Hussain[2], Jawdat Alebraheem[3]
& A. M. Alanbari [4]
f.ishaq@mu.edu.sa[1,a], s.nawaz@mu.edu.sa[2], j.alebraheem@mu.edu.sa[3],
301105736@mu.edu.sa[4]
[1,2,3,4]Mathematics Department, Majmaah University, College of Science, Alzulfi, KSA
([1, 2] Punjab Higher Education Department, College wing, Lahore, Pakistan)
a: Correspondence author, f.ishaq@mu.edu.sa

ABSTRACT: *This study intends to investigate the steady, incompressible flow and heat transfer for viscous electrically conducting fluid. The fluid flows through porous medium due to a non- linearly porous shrinking surface. A magnetic field is applied normal to the surface. The coupled non linear partial differential equations used for the mathematical model of the problem have been transformed to the ordinary differential form. The resulting equations are then solved numerically by employing Mathematica software. The results for velocity and temperature distribution reveal the flow kinematics under the effects of key physical parameters of the problem namely non linear shrinking parameter s, magnetic parameter M, porosity parameter K, mixed convection parameter λ and Prandtl number P_r. The results are presented in graphical form.*

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Key Words: MHD Mixed Convection, Flow through porous surface, Non-linearly shrinking surface.

1. INTRODUCTION

The convective heat transfer in fluids flows through porous medium has numerous applications. The earth mantle is an example of porous medium to yield the geothermal energy through convection process. The well known Darcy's law describes the flow of fluid through porous media. This law states that fluid flow through porous media experiences a resistance. Several industrial problems such as heat drums to generate underground chemicals, solar power collectors, heat exchangers with porous materials, the techniques for thermal insulation and petroleum reservoir modeling involve the fluid flow with convective heat transfer through porous media. This research area has gained the attraction of researchers. Ojjela and Kumar [1] considered the unsteady two-dimensional incompressible MHD flow and heat transfer of a micropolar fluid in a porous medium between parallel plates with chemical reaction, Hall and ion slip effects. Hsu and Cheng [2] reported mixed convection flow in porous medium about a vertical plate. Tien and Vafai [3] investigated the free convection boundary layer flow in a porous medium, owing to combined heat and mass transfer. Combined heat and mass transfer in fluid-saturated porous media finds applications in a variety of engineering processes such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes, geothermal and geophysical engineering, moisture migration in a fibrous insulation and nuclear waste disposal, and others. Bejan and Khair [4] investigated the free convection boundary layer flow in a porous medium, owing to combined heat and mass transfer. Takhar et al [5] analyzed mixed convention of an incompressible viscous fluid in a porous medium past a vertical plate. Several authors [6-11] have presented various aspects of the convective heat flow problems through porous media. El-Hakiem [12] discussed the effect of magnetic field and double dispersion on mixed convection heat and mass transfer in non-Darcy porous medium.

The fluid flow due to shrinking surface behaves differently as compared to the forward stretching flow, this was first observed by Wang [13]. Physically, the vorticity generated due to the shrinking of the sheet is not confined within the boundary layer, and the steady flow exists only when adequate suction on the boundary is imposed as reported by Miklavčič and Wang [14]. Goldstein [15] remarked that the shrinking flow is a backward flow. Recently, Sajjad[16] considered MHD forced convective boundary layer flow of micropolar fluids past a shrinking porous sheet prescribed with variable heat flux and heat source. Ahmad and Hussain [17] studied convective heat transfer for MHD micropolar fluids flow through porous media over a stretching surface. Rosca [18] investigated the forced convection boundary flow past a permeable shrinking surface. Bagh et al [19] examined the magnetohydrodynamic fluid flow through porous media over an exponentially shrinking porous sheet with convective heat transfer.

This work examines the MHD mixed convection boundary layer flow through porous media due to porous non-linearly shrinking surface. The numerical solution of the problem describes the effects of physical parameters of the study on fluid velocity and heat function.

2. MATHEMATICAL ANALYSIS

The fluid flow is steady, incompressible and two-dimensional. The fluid is viscous and electrically conducting. The fluid flows over a porous shrinking surface. The surface coincides with the plane $y = 0$, and the flow is bounded in the region $y > 0$. A magnetic field of variable magnetic strength is applied perpendicular to the surface.

Under the above assumptions the equations of motion become as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} \quad (2)$$
$$=\nu(\frac{\partial^2 u}{\partial y^2})-\frac{\sigma B^2(x)}{\rho}u-\frac{\upsilon}{K'}u+g\beta(T-T_\infty),$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha(\frac{\partial^2 T}{\partial y^2}), \quad (3)$$

Where $\underline{V}=V(u,v)$ is velocity and T is fluid temperature, υ and α are kinematic viscosity and thermal diffusivity respectively, and the relation for electromagnetic body force as given by Rossow [20] is $\underline{J}\times\underline{B}=\underline{E}\sigma(\underline{V}\times\underline{B})\times\underline{B}=-\sigma B^2\underline{V}$,

Where $B(x)$ is a magnetic field strength, $\underline{J}$ is the current density, $\underline{E}$ is electric field intensity and σ is the electrical conductivity, g is acceleration due to gravity, β is coefficient of thermal expansion.

The boundary conditions are:

$$u=-c(x+a)^n,\quad v=v_w, \quad \text{at} \quad y\to 0,\ ,$$
$$T=T_\infty+\sqrt{\frac{c(n+1)}{\upsilon}}\frac{K}{q_0(x+a)^{2n-1}},$$
$$u\to 0,\ T\to T_\infty,\ \text{as} \quad y\to\infty, \quad (4)$$

Where a, c and n are constants, and $c > 0$. $v_w=-\frac{1}{2}\sqrt{c(n+1)\upsilon}(x+a)^{\frac{n-1}{2}}f_w$.

Using similarity transformations:

$$u=c(x+a)^n f'(\eta),$$
$$v=-\frac{1}{2}\sqrt{n(n+1)\upsilon}((x+a)^n[f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)],$$
$$T-T_\infty=\sqrt{\frac{\upsilon}{c(n+1)}}\frac{q_0}{K}(x+a)\theta(\eta), \quad (5)$$
$$\eta=\sqrt{\frac{n(1+n)}{\upsilon}}(x+a)^{(n-1)/2}y,$$

where the primes denote differentiation with respect to η.

The equation of continuity (1) is identically satisfied.

Substituting (5) in to equations (2) and (3), we have

$$f'''+\frac{1}{2}ff''-sf'^2-Mf'-\frac{1}{K}f'+\lambda\theta=0, \quad (6)$$

$$\theta''+\Pr(\frac{1}{2}f\theta'-mf'\theta)=0. \quad (7)$$

The corresponding boundary conditions are:

$$f(0)=f_w,\quad f'(0)=-1,\quad \theta(0)=1, \quad (8)$$
$$f'(\infty)\to 0,\quad \theta(\infty)\to 0,$$

Where

$$s=\frac{n}{n+1},\qquad m=\frac{2n-1}{n+1},\qquad M=\frac{\sigma B_0^2}{\rho n(n+1)},$$
$$\lambda=\frac{g\beta}{c}\sqrt{\frac{\upsilon}{c(n+1)}}\frac{q_0}{k},\ K=\frac{K'c}{\upsilon},\ \text{and}\quad \Pr=\frac{\upsilon}{\alpha}.$$

3. RESULTS AND DISCUSION

The set of coupled non- linear equations (6) and (7) together the boundary conditions (8) do not lend themselves to analytical solution. Mathematica software has been employed to obtain numerical solutions for parametric study of the problem. Several computations have been made for representative values of the parameters that influence the nature of flow and heat transfer.

Fig.1 shows that the magnitude of horizontal velocity f' increases with increase in magnetic field strength, the situation for flow due to shrinking sheet is reverse as that for stretching sheet where usually the Lorentz force opposes the fluid flow. Also, the boundary layer decreases with an increase in the values of *M*. Fig.2 presents the effect of suction parameter f_w on horizontal velocity f'. It is noticed that velocity increases with increase in suction, the result corresponds to the physical phenomenon and because of increase in the speed the boundary layer decreases.

Fig.3 shows that velocity f' increases with increase in the porosity parameter *K*. The non-linearity parameter *s* causes to reduce the flow and the boundary layer thickness increases as shown in fig.4. The fig.5 demonstrates the effect of convection parameter λ on f', the velocity increases with an increase in the values of λ.

Fig.6 and fig.7 respectively presents the curves for temperature distribution under the effects of suction parameter f_w and Prandtle number P_r. It is noticed that temperature decreases with an increase in the values of these parameters. The situation has relevance to the physical nature of these parameters. Fig.8 shows that temperature decreases with an increase in the values of parameter *m*. The magnetic field has no significant effect on temperature function.

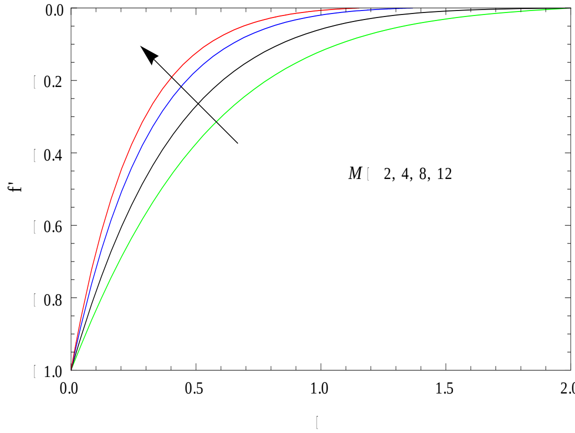

Fig.1: The horizontal velocity curves against various values of *M*

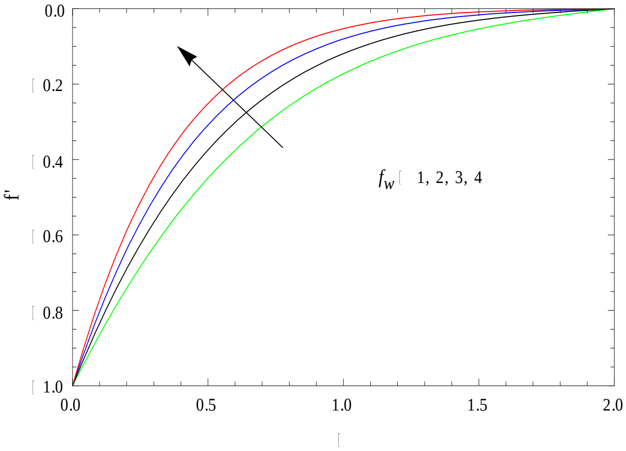

Fig.2: The horizontal velocity curves againstvarious values of f_w.

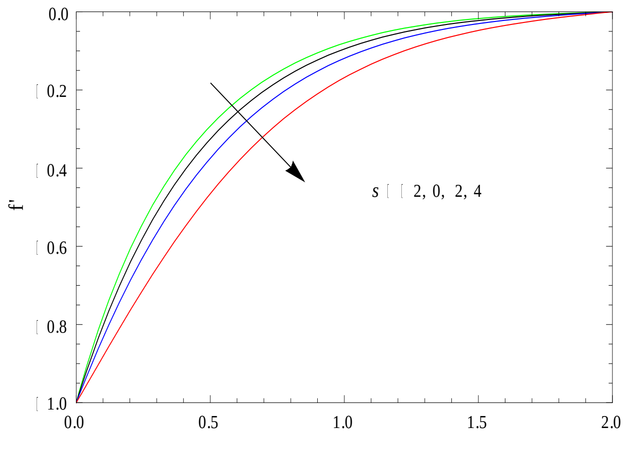

Fig.3: The horizontal velocity curves against various values of *s*.

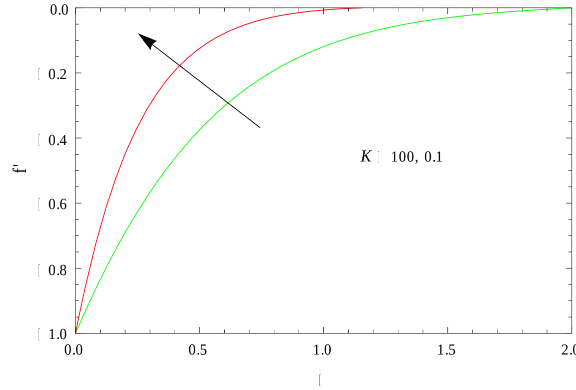

Fig.4: The horizontal velocity curves against various values of *K*.

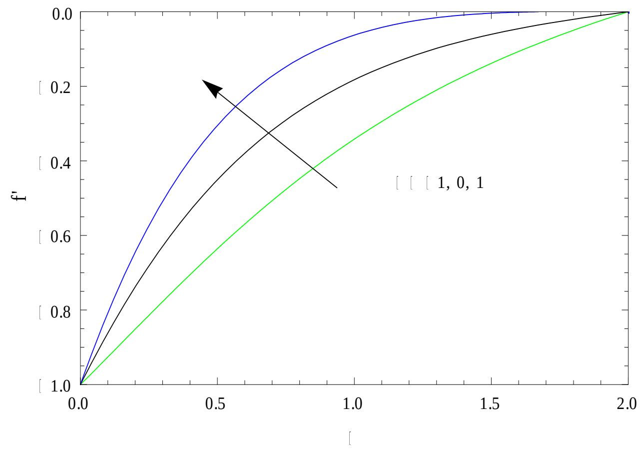

Fig.5: The horizontal velocity curves against various values of λ.

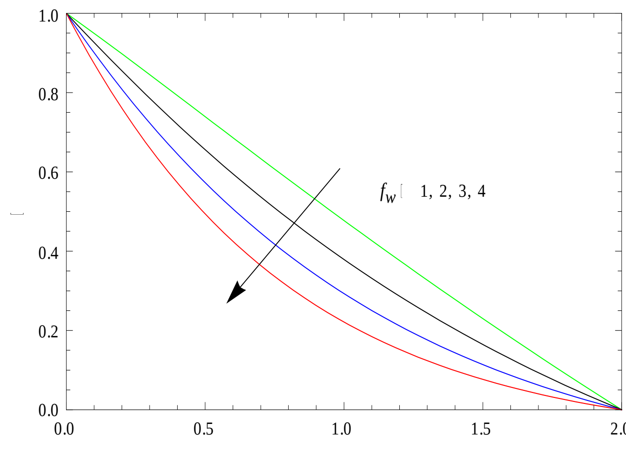

Fig.6: The Temperature function against various values of f_w.

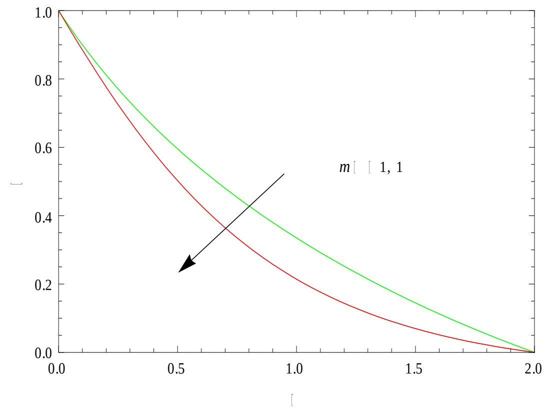

Fig.7: The Temperature function against various values of *m*.

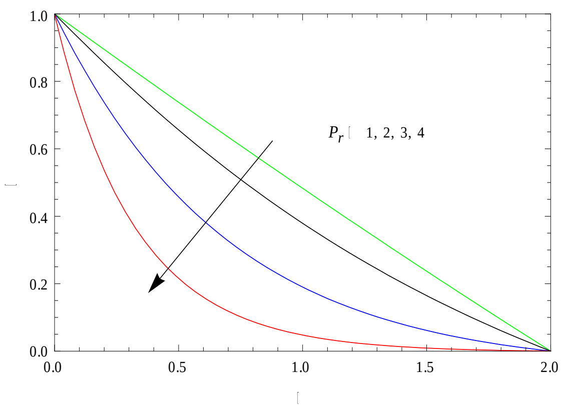

Fig.8: The Temperature function against various values of p_r. 2

4. CONCLUSION

From the above given results and discussuion, we conclude that:

- The magnitude of horizontal velocity f′ increases with increase in magnetic field strength M.
- The boundary layer decreases with increase in the values of M.
- The velocity f′ increases with increase in the porosity parameter K.
- The non-linearity parameter s causes to reduce the flow and makes the boundary layer thickness increase.
- The velocity f′ increases with increase in the values of convection parameter λ.
- Under the effects of suction parameter s and Prandtle number Pr, it is noticed that temperature decreases with increase in the values of these parameters.
- The temperature decreases with increase in the values of non- linearity parameter m.
- The magnetic field has no significant effect on temperature function.

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