BIASES IN GLS ESTIMATORS FOR DYNAMIC PANEL DATA MODELS ALLOWING CROSS- SECTIONAL HETEROSCEDASTICITY

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ABSTRACT: *The inclusion of lagged dependent variable in the list of explanatory variables introduces the specific estimation problems even the generalized least squares estimator for the dynamic panel data models allowing cross sectional heteroscedasticity becomes biased and inconsistent. In this study, the analytical expressions for the inconsistency have been derived in the first order autoregressive case. A comparison between asymptotic bias and small sample simulated bias has also been carried out. The analytical biases emerged coincident with or a little above the small sample simulated biases. The closeness of the two types of biases mainly depends on coefficient of lagged dependent variable (γ) and the number of cross sectional units N.*

Keywords: Bias estimation; Bias corrected estimator; Heteroscedastic *GLS*
JEL Classification: C13; C23

1. INTRODUCTION

In case of dynamic panel data models (*DPDM*s), the estimators become biased and inconsistent customarily as a consequence of the inclusion of lagged dependent variable (see Baltagi [8], Kiviet [17]). That is why, that the estimation of *DPDM*s has been a challenge for researchers, and a range of different estimators has been developed (see Anderson and Hsiao [1-2], Arellano and Bond [3], Arellano and Bover [4], Kiviet [17], Blundell and Bond [10], Judson and Owen [16], Hansen [14], Bun and Carree [11-12]). In all these considerations, the error component model commonly assumes that the regression disturbances are homoscedastic in terms of common variance across time and cross sections. But in case of panel data where the cross sectional units may be of different sizes and different variation fashions, this assumption may be a restrictive one. Therefore, in most of the practical situations, we have to face the case of heteroscedasticity. Mazodier and Trognon [19], Baltagi [6-7], Baltagi and Griffin [9], Randolph [20], Li and Stengos [18], Roy [21] and Aslam [5] have discussed the estimation of (static) panel data models with heteroscedastic errors. Most of these researchers have considered the case of cross sectional heteroscedasticity which is a case of more practical orientation and interest. The generalized least squares (*GLS*) estimation remained a common and straight forward solution to this problem due to its attractive properties. Taylor [22] found the feasible *GLS* estimator more efficient than least squares dummy variable (*LSDV*) estimator for all but the fewest degrees of freedom and its variance never more than 17% above Cramer Rao bound. But in the meanwhile, the correlation between the lagged dependent (explanatory) variable and the error term renders the *GLS* estimator biased and inconsistent in case of *DPDM*s also. In the circumstances that a typical set of panel data has a relatively large number of cross sections and a relatively small number of time periods, the bias is very serious. Therefore, we are led to be interested in the calculation of this bias.

In this study, we consider the *GLS* estimation of *DPDM*s with cross sectional heteroscedasticity coming into models through the remainder error term. In Section 2, the *GLS* estimator of heteroscedastic first order *DPDM* is given. The inconsistency of the estimator is calculated in section 3. In section 4, we consider the case of inclusion of exogenous regressors in the model with the special reference of the inconsistency of *GLS* estimator. In section 5, Monte Carlo evidences are compiled. The section 6 concludes the article.

2. The *GLS* Estimation of Heteroscedastic Dynamic Panel Data Model

Initially, we consider an autoregressive random effect *DPDM*, i.e.

$$\left.\begin{aligned} y_{it} = \gamma\, y_{i,t-1} + u_{it} \quad \text{with} \quad u_{it} = \mu_i + v_{it} \\ i = 1,2,\dots,N; \qquad t = 1,2,\dots,T \end{aligned}\right\} \tag{1}$$

where the dependent variable y_{it} is determined by its one period lagged value $y_{i,t-1}$ and the error term u_{it} which is assumed to be the sum of random unit effect $\mu_i \sim IID(0,\sigma_\mu^2)$ and the remainder error component v_{it} is assumed to be independently distributed with mean zero and variance σ_i^2. Obviously the cross sectional heteroscedasticity is assumed coming into model through the remainder error term v_{it}.

Stacking the observations over time, we get,

$$\left.\begin{aligned} y_i = \gamma y_{i,-1} + u_i \quad \text{with} \quad u_i = \mu_i e_T + v_i \\ i = 1,2,\dots,N \end{aligned}\right\} \tag{2}$$

where

$y_i = (y_{i1},\dots,y_{iT})'$, $\qquad y_{i,-1} = (y_{i0},\dots,y_{i,T-1})'$,

$v_i = (v_{i1},\dots,v_{iT})'$ and $e_T = (1,\dots,1)'$:a $T \times 1$ vector.

Further stacking over cross sections, we have

$$\left.\begin{aligned} & y = \gamma y_{-1} + u \\ & \quad \text{with} \quad u = (I_N \otimes e_T)\mu + v \end{aligned}\right\} \tag{3}$$

where

$y = (y_1', \ldots, y_N')'$, $y_{-1} = (y_{1,-1}', \ldots, y_{N,-1}')'$, $\mu = (\mu_1, \ldots, \mu_N)'$ and $v = (v_1', \ldots, v_N')'$.

In order to find the *GLS* estimator of the regression coefficient γ , we need the inverse of the variance-covariance matrix of u . So, under assumptions of the model (1), we have

$$\left.\begin{aligned} \Omega &= Var(u) \\ &= \sigma_\mu^2 I_N \otimes J_T + diag(\sigma_i^2) \otimes I_T \\ &= diag(T\sigma_\mu^2 + \sigma_i^2) \otimes \frac{J_T}{T} \\ &\qquad + diag(\sigma_i^2) \otimes \left(I_T - \frac{J_T}{T} \right) \\ &= diag\left(A_i\right) \end{aligned}\right\} \tag{4}$$

where

$$\left.\begin{aligned} A_i &= \sigma_\mu^2 J_T + \sigma_i^2 I_T \\ &= \left(T\sigma_\mu^2 + \sigma_i^2\right)\frac{J_T}{T} + \sigma_i^2\left(I_T - \frac{J_T}{T} \right) \\ \text{and} \quad & J_T = e_T e_T' \end{aligned}\right\} \tag{5}$$

Following Baltagi and Griffin [9], Roy [21] and Baltagi [8], we used a simple trick devised by Wansbeek and Kapteyn [23-24] to derive Ω^{-1}. Under this derivation we have

$$\left.\begin{aligned} \Omega^{-1} &= diag\left(\frac{1}{T\sigma_\mu^2 + \sigma_i^2} \right) \otimes \frac{J_T}{T} \\ &\qquad + diag\left(\frac{1}{\sigma_i^2} \right) \otimes \left(I_T - \frac{J_T}{T} \right) \\ &= diag\left(A_i^{-1}\right) \end{aligned}\right\} \tag{6}$$

where

$$A_i^{-1} = \left(\frac{1}{T\sigma_\mu^2 + \sigma_i^2} \right)\frac{J_T}{T} + \frac{1}{\sigma_i^2}\left(I_T - \frac{J_T}{T} \right)$$

$$= \frac{1}{\sigma_i^2}\left(I_T - \frac{\sigma_\mu^2}{T\sigma_\mu^2 + \sigma_i^2} J_T \right)$$

$$= \begin{bmatrix} q_i + \sigma_i^{-2} & q_i & \cdots & q_i \\ q_i & q_i + \sigma_i^{-2} & \cdots & q_i \\ \vdots & \vdots & \ddots & \vdots \\ q_i & q_i & \cdots & q_i + \sigma_i^{-2} \end{bmatrix}$$

$$\text{with} \quad q_i = \frac{-\sigma_\mu^2}{\sigma_i^2\left(T\sigma_\mu^2 + \sigma_i^2\right)}.$$

Therefore, running *GLS* on (3), we have *GLS* estimator

$$\hat{\gamma}_{GLS} = \left(y_{-1}'\Omega^{-1}y_{-1}\right)^{-1}\left(y_{-1}'\Omega^{-1}y\right). \tag{7}$$

Following Roy [21], we have its easily manageable version

$$\left.\begin{aligned} \hat{\gamma}_{GLS} &= \left(\sum_{i=1}^{N} y_{i,-1}' A_i^{-1} y_{i,-1} \right)^{-1} \\ &\qquad \times \left(\sum_{i=1}^{N} y_{i,-1}' A_i^{-1} y_i \right) \end{aligned}\right\} \tag{8}$$

3. Bias of *GLS* Estimator

In order to find the bias term, we use the probability limit. Thus, using (2) in (8) and applying the probability limit, we have:

$$\operatorname*{plim}_{N\to\infty}\left(\hat{\gamma}_{GLS} - \gamma\right) = \frac{\operatorname*{plim}_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} B_i}{\operatorname*{plim}_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} C_i} \tag{9}$$

where

$$B_i = y_{i,-1}' A_i^{-1} u_i \quad \text{and} \quad C_i = y_{i,-1}' A_i^{-1} y_{i,-1}.$$

Now the numerator in (9) can be simplified as

$$\operatorname*{plim}_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} B_i$$

$$= \operatorname*{plim}_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N}\left\{ \sum_{t=1}^{T} y_{i,t-1}\left(\frac{u_{it}}{\sigma_i^2} + \sum_{s=1}^{T} q_i u_{is} \right) \right\}$$

$$= \sum_{t=1}^{T}\left\{ \operatorname*{plim}_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} y_{i,t-1}\left(\frac{u_{it}}{\sigma_i^2} + \sum_{s=1}^{T} q_i u_{is} \right) \right\}$$

$$= \sum_{t=1}^{T} \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} E\left\{ y_{i,t-1}\left(\frac{u_{it}}{\sigma_i^2} + \sum_{s=1}^{T} q_i u_{is} \right) \right\}$$

$$= \sum_{t=1}^{T}\left\{ \begin{aligned} & \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \frac{1}{\sigma_i^2} E\left(y_{i,t-1}u_{it}\right) \\ & + \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} q_i E\left(y_{i,t-1}\sum_{s=1}^{T} u_{is} \right) \end{aligned} \right\} \tag{10}$$

To evaluate $E\left(y_{i,t-1}u_{it}\right)$ and $E\left(y_{i,t-1}\sum_{s=1}^{T} u_{is} \right)$, the backward substitution in (1) gives $y_{it}=\gamma^n y_{i,t-n}+\sum_{j=0}^{n-1}\gamma^j u_{i,t-j}$.

Assuming the stochastic initial value and stationarity of y_{it} and $|\gamma|<1$, n may be infinitely large, for which we have

$$\left.\begin{aligned} y_{it} &= \sum_{j=0}^{\infty}\gamma^j u_{i,t-j} = \frac{\mu_i}{1-\gamma}+\sum_{j=0}^{\infty}\gamma^j v_{i,t-j} \\ \Rightarrow y_{i,t-1} &= \frac{\mu_i}{1-\gamma}+\sum_{j=0}^{\infty}\gamma^j v_{i,t-1-j}\end{aligned}\right\} \qquad (11)$$

So, using $u_{it}=\mu_i+v_{it}$ and (11), we get

$$E\left(y_{i,t-1}u_{it}\right)=\frac{\sigma_\mu^2}{1-\gamma}$$

and

$$\left.\begin{aligned} &E\left(y_{i,t-1}\sum_{s=1}^{T}u_{is}\right) \\ &= \frac{1}{1-\gamma}\left\{T\sigma_\mu^2+\left(1-\gamma^{t-1}\right)\sigma_i^2\right\}\end{aligned}\right\} \qquad (12)$$

The simplification of (10) gives

$$\operatorname*{plim}_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}B_i=\frac{1-\gamma^T}{(1-\gamma)^2}\bar{S}, \qquad (13)$$

where

$$\left.\begin{aligned}\bar{S} &= \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\frac{\sigma_\mu^2}{T\sigma_\mu^2+\sigma_i^2} \\ &= \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T+\sigma_i^2/\sigma_\mu^2}\end{aligned}\right\} \qquad (14)$$

The similar manipulation gives the denominator of (9) as

$$\left.\begin{aligned} &\operatorname*{plim}_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}C_i \\ &= \frac{T}{1-\gamma^2}+\frac{2\gamma\left(1-\gamma^T\right)}{\left(1-\gamma^2\right)\left(1-\gamma\right)^2}\bar{S}\end{aligned}\right\} \qquad (15)$$

So (9) becomes

$$\left.\begin{aligned} &\operatorname*{plim}_{N\to\infty}\left(\hat{\gamma}_{GLS}-\gamma\right) \\ &= \frac{1+\gamma}{T}\left(\frac{1-\gamma}{(1-\gamma^T)\bar{S}}+\frac{2\gamma}{T(1-\gamma)}\right)^{-1}\end{aligned}\right\} \qquad (16)$$

From (14), it is obvious that $0<\bar{S}<0.5$ for all (positive) values of σ_μ^2 and σ_i^2 and for $T\geq 2$. So if $\gamma>0$, the bias in (16) is always positive for all values of T. Just for the sake of convenience, we considered the different values of $\bar{S}$ instead of σ_μ^2 and σ_i^2 and found that the graphs of asymptotic bias (16) vs. T or γ reflected no substantial pattern of variation for different values of $\bar{S}$ except that the skewness and $\bar{S}$ moved in opposite direction. Therefore, in Fig. 1 and 2, we select a particular value $\bar{S}=0.25$ (say) just for the graphical interpretation.

Fig. 1 shows that the bias approaches to zero with the increasing value of T. In Fig. 2, It is also prominent that bias initially increases as the value of γ increases, but when γ reaches to its highest value 1 i.e. a situation of non-stationarity, the biases suddenly reduces to zero. This incident flashes in the form of strong (negative) skewness.

Here it is also mentionable that computing the bias in this way is just considering the approximation of $E\left(B_i\right)/E(C_i)$. Whereas the standard Hurwicz [15] bias of the estimator of lagged dependent variable coefficient in the first-order autoregressive model (for a particular value of i) is given by $E\left(B_i/C_i\right)$ that, to the order of T^{-1} , is approximated as

$$E\left(\frac{B_i}{C_i}\right)=\frac{E(B_i)}{E(C_i)}\left\{1-\frac{Cov(B_iC_i)}{E(B_i)E(C_i)}+\frac{Var(B_i)}{E^2(B_i)}\right\}.$$

Obviously this standard bias becomes smaller than our consideration given in (16) in terms of the second and third terms.

4. Inclusion of Exogenous Variables

In model (1), if we assume to include the K additional exogenous regressors, the model becomes

$$\left.\begin{aligned} & y_{it}=\gamma\, y_{i,t-1}+x_{it}\beta+u_{it} \\ &\quad \text{with}\quad u_{it}=\mu_i+v_{it} \\ & i=1,2,\ldots,N; \qquad t=1,2,\ldots,T\end{aligned}\right\} \qquad (17)$$

where x_{it} is a $1\times K$ vector of additional K pure exogenous variables and β is a $K\times 1$ vector of coefficients. Again the error components μ_i and v_{it} are assumed as above.

Stacking the observations over time, we get,

$$\left.\begin{aligned} & y_i=\gamma y_{i,-1}+x_i\beta+u_i \\ &\quad \text{with}\quad u_i=\mu_i e_T+v_i, \\ &\qquad\qquad i=1,2,\ldots,N\end{aligned}\right\} \qquad (18)$$

where $x_i=(x'_{i1},\ldots,x'_{iT})'$: a $T\times K$ matrix.

Further stacking over cross sections, we have

$$\left.\begin{aligned} & y=\gamma y_{-1}+x\beta+u \\ &\quad \text{with}\quad u=(I_N\otimes e_T)\mu+v\end{aligned}\right\} \qquad (19)$$

where $x=(x'_1,\ldots,x'_N)'$: an $NT\times K$ matrix.

Therefore, using partitioned regression technique (see Greene, [13]), we have the *GLS estimators*

$$\left.\begin{aligned} {}_X\hat{\gamma}_{GLS} &= \left(y'_{-1}\Omega^{-\frac{1}{2}}M\Omega^{-\frac{1}{2}}y_{-1}\right)^{-1} \\ &\quad\times\left(y'_{-1}\Omega^{-\frac{1}{2}}M\Omega^{-\frac{1}{2}}y\right) \\ {}_X\hat{\beta}_{GLS} &= -\left(x'\Omega^{-1}x\right)^{-1}\left(x'\Omega^{-1}y_{-1}\right)\hat{\gamma}_{GLS} \\ &\quad+\left(x'\Omega^{-1}x\right)^{-1}\left(x'\Omega^{-1}y\right) \end{aligned}\right\} \quad (20)$$

where

$$M = I_{NT} - \Omega^{-\frac{1}{2}}x(x'\Omega^{-1}x)^{-1}x'\Omega^{-\frac{1}{2}} \quad (21)$$

and

$$\left.\begin{aligned} \Omega^{-\frac{1}{2}} &= diag\left(\frac{1}{\sqrt{T\sigma_\mu^2+\sigma_i^2}}\right)\otimes\frac{J_T}{T} \\ &\quad+diag\left(\frac{1}{\sigma_i}\right)\otimes\left(I_T-\frac{J_T}{T}\right) \end{aligned}\right\} \quad (22)$$

Simply using (19), we get

$$\left.\begin{aligned} {}_X\hat{\gamma}_{GLS}-\gamma &= \left(y'_{-1}\Omega^{-\frac{1}{2}}M\Omega^{-\frac{1}{2}}y_{-1}\right)^{-1} \\ &\quad\left(y'_{-1}\Omega^{-\frac{1}{2}}M\Omega^{-\frac{1}{2}}u\right) \\ {}_X\hat{\beta}_{GLS}-\beta & \\ &= -\left(x'\Omega^{-1}x\right)^{-1}\left(x'\Omega^{-1}y_{-1}\right) \\ &\quad\times\left({}_X\hat{\gamma}_{GLS}-\gamma\right) \\ &\quad+\left(x'\Omega^{-1}x\right)^{-1}\left(x'\Omega^{-1}u\right) \end{aligned}\right\} \quad (23)$$

Taking plims as $N\to\infty$ and using the exogenioty of *X*, it can easily be seen that

$$\left.\begin{aligned} &\operatorname*{plim}_{N\to\infty}\frac{1}{N}\left(y'_{-1}\Omega^{-\frac{1}{2}}M\Omega^{-\frac{1}{2}}u\right) \\ &= \operatorname*{plim}_{N\to\infty}\frac{1}{N}\left(y'_{-1}\Omega^{-1}u\right) \\ &= \operatorname*{plim}_{N\to\infty}\frac{1}{N}\left(\sum_{i=1}^{N}B_i\right) \end{aligned}\right\} \quad (24)$$

and (23) becomes

$$\left.\begin{aligned} &\operatorname*{plim}_{N\to\infty}\left({}_X\hat{\gamma}_{GLS}-\gamma\right) \\ &= \operatorname*{plim}_{N\to\infty}\left(\frac{1}{N}y'_{-1}\Omega^{-\frac{1}{2}}M\Omega^{-\frac{1}{2}}y_{-1}\right)^{-1} \\ &\quad\times\operatorname*{plim}_{N\to\infty}\frac{1}{N}\left(\sum_{i=1}^{N}B_i\right) \end{aligned}\right\} \quad (25)$$

$$\left.\begin{aligned} &\operatorname*{plim}_{N\to\infty}\left({}_X\hat{\beta}_{GLS}-\beta\right) \\ &= -\operatorname*{plim}_{N\to\infty}\left(x'\Omega^{-1}x\right)^{-1}\left(x'\Omega^{-1}y_{-1}\right) \\ &\quad\times\operatorname*{plim}_{N\to\infty}\left({}_X\hat{\gamma}_{GLS}-\gamma\right) \end{aligned}\right\} \quad (26)$$

Clearly, the numerator (24) in bias expression for coefficient of lagged dependent variable remains unchanged by the inclusion of exogenous variables in the model. This is because of the assumption of uncorrelatedness of included exogenous variables and error term. Whereas the inclusion of M reduces the denominator and consequently the bias of coefficient of lagged dependent variable became larger than that of the model without inclusion of exogenous variables. In (25), the bias of ${}_X\hat{\beta}_{GLS}$ depends on the correlation between x and y_{-1}.

5. Monte Carlo Study

For the study and comparison of performance of the estimators we use Monte Carlo evidences. We have made use of the simulation design mainly same as used by Bun and Carree [11-12]. According to this design:

(i) The y_{it} 's have been generated by the equation $y_{it} = \gamma\, y_{i,t-1} + \mu_i + v_{it}$, with $i = 1,2,\ldots,N;\quad t = 1,2,\ldots,T+10$, $\mu_i \sim IIND(0,\sigma_\mu^2)$ and $v_{it}\ (i = 1,2,\ldots,N)$ independently normally distributed with mean zero and variance σ_i^2, choosing $y_{i1} = 0$ and all possible combinations of $\gamma = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and $\sigma_\mu^2 = 0.01, 0.1, 1.0$. The first 10 cross sections were discarded so that the actual samples contain *NT* observations (*see* also Arellano and Bond, [3]).

(ii) Regarding the variance structure of remainder error component v_{it} allowing cross sectional heteroscedasticity, we follow Bun and Carree [11-12] with specification $Var(v_{it}) = \sigma_i^2 \sim \chi^2(1)$. In this specification, it is ensured that $\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2 \approx 1$.

(iii) To study the behavior of different measures varying with *T* and *N*, we consider the sample sizes (*T*,*N*): (3,20), (3,50), (3,100), (3,200), (3,400), (3,60), (6,60) , (10,60) , (20,60) , (40,60). This selection is considered under the idea that for a specific value of *T*, the cases with increasing *N* and for a specific value of *N*, the cases with increasing *T* could be considered.

(iv) Using all possible combinations of γ, σ_μ^2, N and T for a set of 10,000 replications, the simulated bias *(SB)* and analytical theoretical bias *(TB)* of $\hat{\gamma}_{GLS}$ have been calculated and compiled in Tables 1 and 2.

To observe the patterns of closeness of the two biases (*SB* and *TB*) a number of graphs, based on Tables 1 and 2, were constructed. A selection has been presented in Fig. 3 and Fig. 4. (Two additional tables and some additional graphs may be obtained from corresponding author). From these presentations (Tables and including all omitted Graphs) we observed the following:

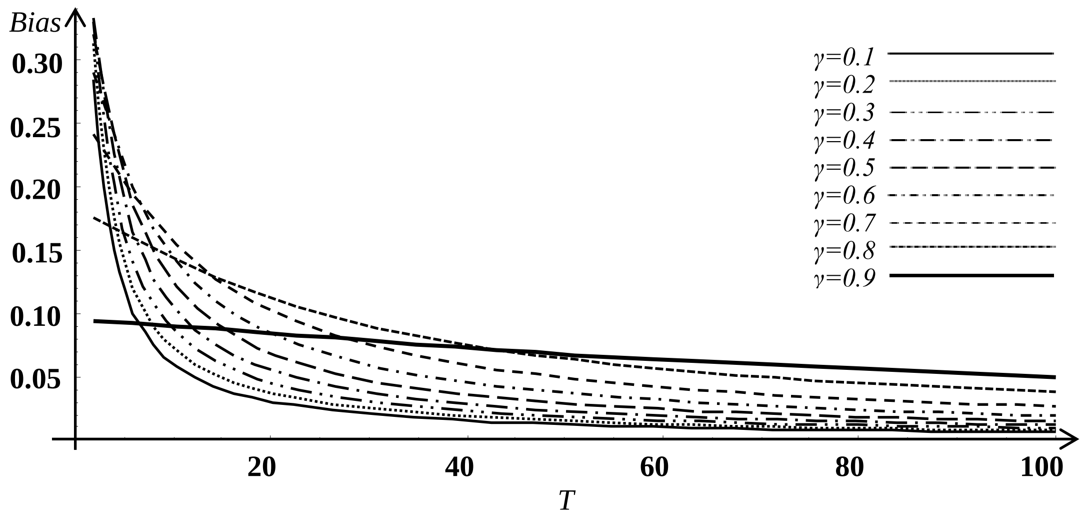

Fig. 1: Graph of *Bias* vs *T* for different values of γ and $\bar{S} = 0.25$.

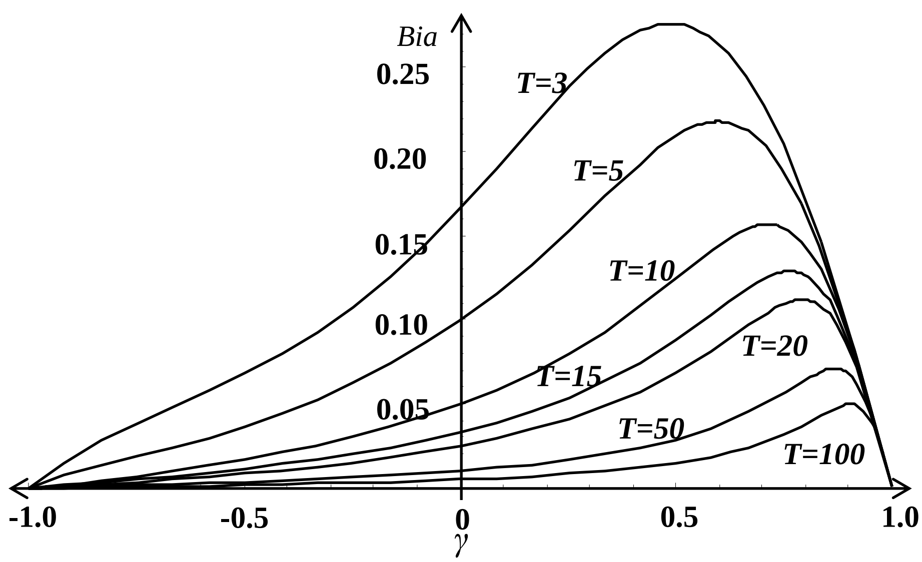

Fig. 2: Graph of *Bias* vs γ for different values of *T* and $\bar{S} = 0.25$.

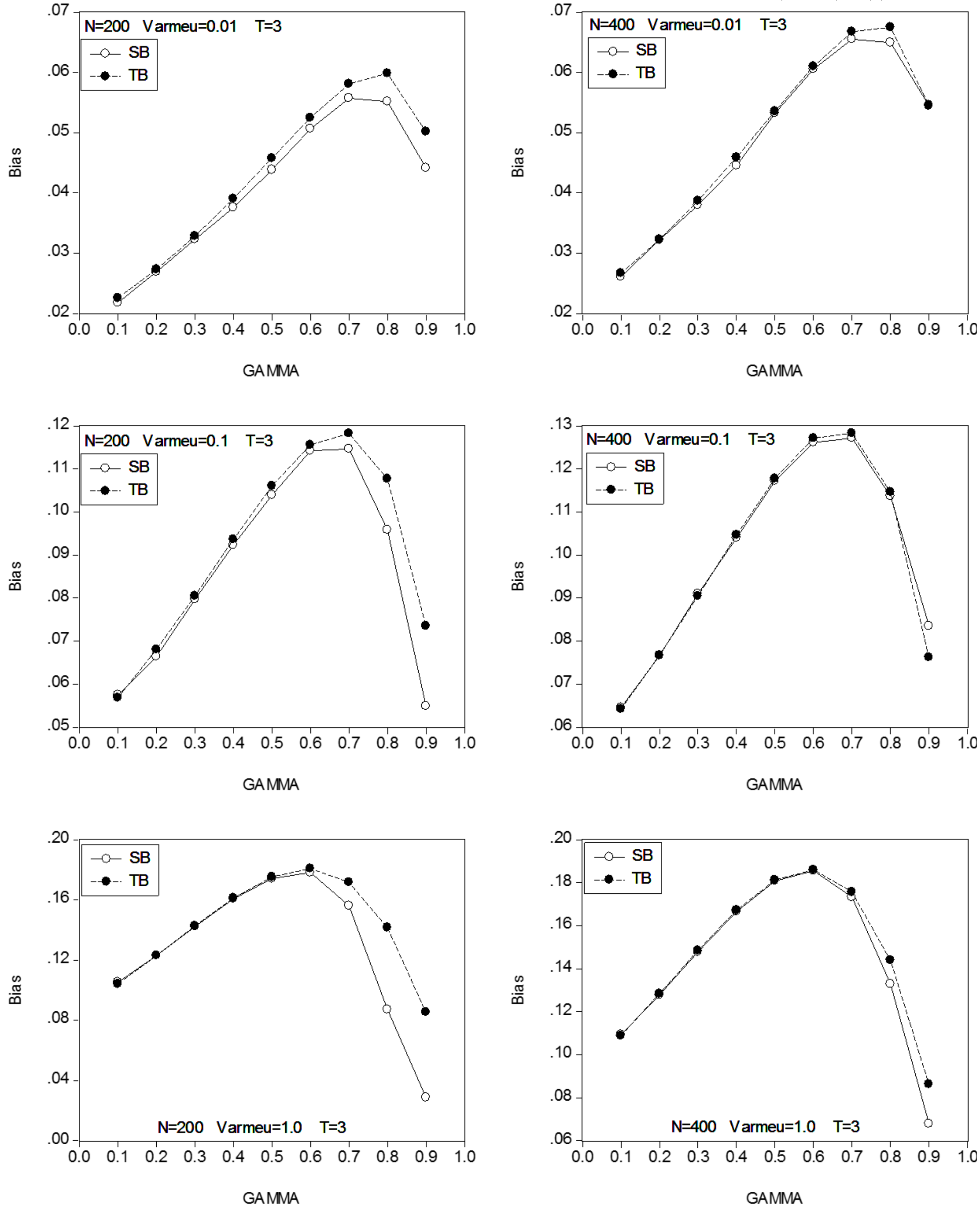

***Fig 3*: Graphs of *Biases* (*SB, TB*) *vs* γ (For *N*=400, *T*=3 and σ_{μ}^{2}=0.01, 0.1, 1.0)**

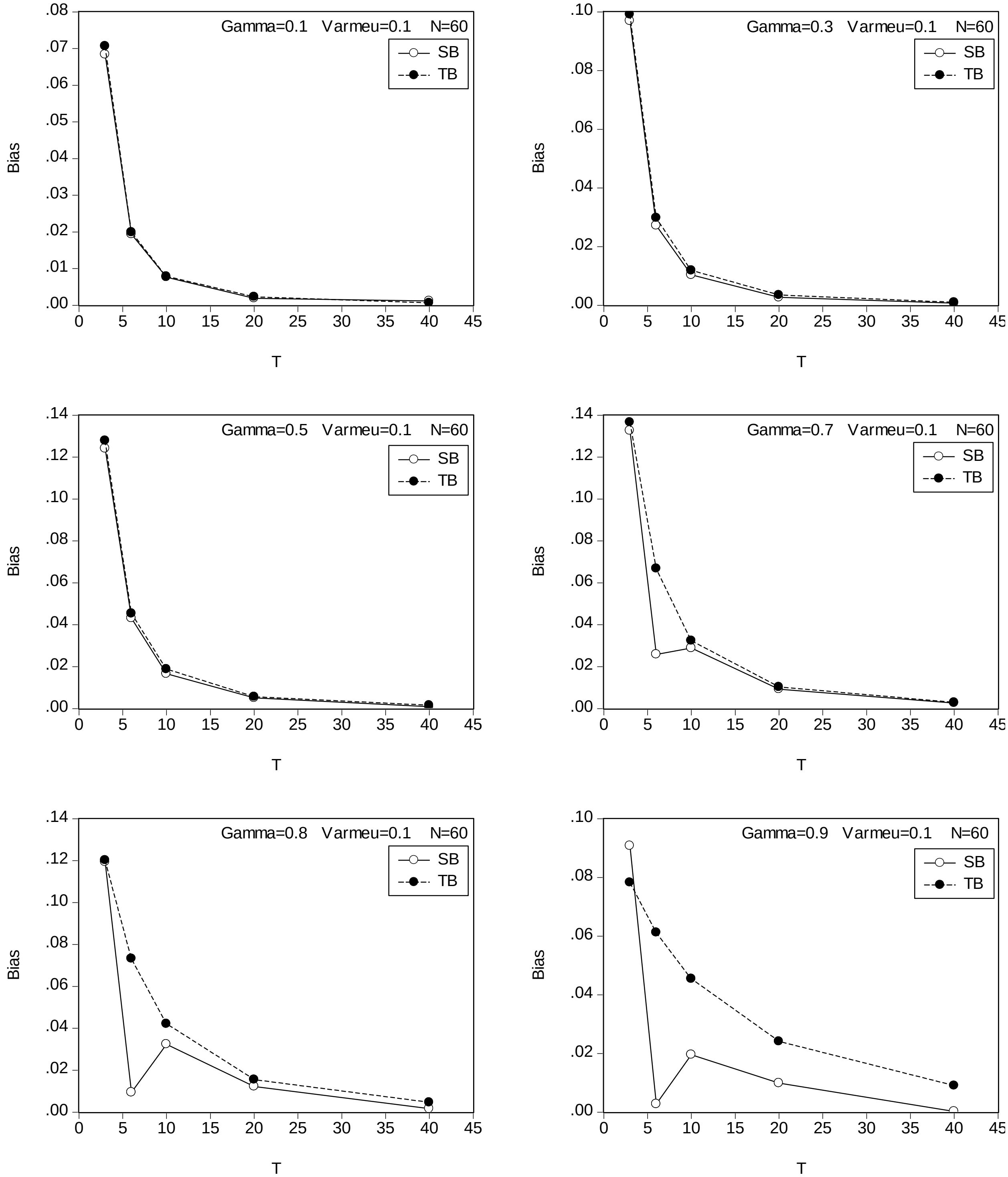

***Fig 4*: Graphs of *Biases* (*SB, TB*) *vs T* for *N*=60, σ_{μ}^{2} =0.1, γ =0.1, 0.3, 0.5, 0.7, 0.8, 0.9**

***Table 1*: Biases *vs* γ for *T*=3; *N*=20, 50, 100, 200, 400**

σ_μ^2	γ	*N=20*		*N=50*		*N=100*		*N=200*		*N=400*	
		TB	***SB***	***TB***	***SB***	***TB***	***SB***	***TB***	***SB***	***TB***	***SB***
0.01	**0.1**	0.03794	0.03314	0.02251	0.02241	0.02468	0.02412	0.02258	0.02178	0.02669	0.02611
	0.2	0.04570	0.04350	0.02724	0.02612	0.02985	0.03025	0.02732	0.02690	0.03226	0.03220
	0.3	0.05455	0.04562	0.03274	0.02943	0.03584	0.03494	0.03284	0.03231	0.03869	0.03799
	0.4	0.06420	0.05635	0.03894	0.03455	0.04256	0.04045	0.03906	0.03759	0.04589	0.04458
	0.5	0.07407	0.06484	0.04565	0.04068	0.04978	0.04702	0.04578	0.04385	0.05356	0.05326
	0.6	0.08293	0.07081	0.05237	0.04701	0.05690	0.05429	0.05251	0.05067	0.06103	0.06059
	0.7	0.08831	0.07651	0.05797	0.05238	0.06262	0.05929	0.05812	0.05575	0.06681	0.06556
	0.8	0.08535	0.07438	0.05974	0.05266	0.06388	0.05926	0.05987	0.05517	0.06754	0.06498
	0.9	0.06379	0.06330	0.05010	0.04585	0.05253	0.05110	0.05018	0.04416	0.05461	0.05447
0.1	**0.1**	0.07699	0.07174	0.05791	0.05662	0.06277	0.06230	0.05680	0.05750	0.06416	0.06452
	0.2	0.09167	0.08516	0.06934	0.06609	0.07506	0.07319	0.06804	0.06644	0.07668	0.07660
	0.3	0.10753	0.09937	0.08202	0.07943	0.08859	0.08606	0.08052	0.07970	0.09046	0.09099
	0.4	0.12344	0.11504	0.09529	0.09156	0.10261	0.10028	0.09362	0.09237	0.10467	0.10399
	0.5	0.13734	0.12830	0.10785	0.10319	0.11561	0.11324	0.10606	0.10393	0.11779	0.11723
	0.6	0.14591	0.13233	0.11737	0.11116	0.12501	0.12262	0.11559	0.11427	0.12714	0.12620
	0.7	0.14412	0.13298	0.11987	0.11442	0.12652	0.12386	0.11831	0.11468	0.12836	0.12723
	0.8	0.12501	0.12126	0.10882	0.10588	0.11341	0.11178	0.10773	0.09585	0.11466	0.11373
	0.9	0.07996	0.09329	0.07396	0.08282	0.07574	0.08372	0.07353	0.05484	0.07621	0.08352
1.0	**0.1**	0.11229	0.11070	0.10770	0.10855	0.10876	0.10901	0.10421	0.10541	0.10878	0.10937
	0.2	0.13230	0.12870	0.12707	0.12508	0.12828	0.12675	0.12308	0.12300	0.12830	0.12789
	0.3	0.15289	0.14206	0.14713	0.14399	0.14847	0.14721	0.14272	0.14232	0.14849	0.14782
	0.4	0.17187	0.16003	0.16583	0.16161	0.16724	0.16554	0.16118	0.16069	0.16725	0.16658
	0.5	0.18581	0.17367	0.17992	0.17441	0.18129	0.17887	0.17535	0.17425	0.18131	0.18080
	0.6	0.18998	0.17852	0.18480	0.18014	0.18601	0.18332	0.18075	0.17821	0.18602	0.18560
	0.7	0.17858	0.16974	0.17471	0.17023	0.17562	0.17270	0.17165	0.15617	0.17563	0.17335
	0.8	0.14568	0.13980	0.14348	0.14069	0.14400	0.13591	0.14172	0.08710	0.14401	0.13288
	0.9	0.08667	0.09008	0.08600	0.09029	0.08616	0.07672	0.08546	0.02878	0.08616	0.06780

Table 2: Biases *vs* γ for *N*=60; *T*=3, 6, 10, 20, 40

σ_μ^2	γ	*T=3*		*T=6*		*T=10*		*T=20*		*T=40*	
		TB	***SB***	***TB***	***SB***	***TB***	***SB***	***TB***	***SB***	***TB***	***SB***
	0.1	0.03422	0.03261	0.00985	0.00872	0.00366	0.00368	0.00119	0.00106	0.00040	0.00080
	0.2	0.04127	0.04045	0.01205	0.01144	0.00448	0.00397	0.00146	0.00119	0.00049	0.00037
	0.3	0.04934	0.04622	0.01484	0.01290	0.00554	0.00523	0.00180	0.00079	0.00060	0.00124
	0.4	0.05822	0.05638	0.01843	0.01531	0.00694	0.00622	0.00226	0.00211	0.00076	0.00061
0.01	**0.5**	0.06743	0.06376	0.02311	0.01992	0.00886	0.00721	0.00290	0.00209	0.00097	0.00048
	0.6	0.07591	0.07202	0.02911	0.02518	0.01164	0.01010	0.00385	0.00224	0.00129	0.00088
	0.7	0.08155	0.07728	0.03634	0.02457	0.01577	0.01315	0.00541	0.00414	0.00183	0.00218
	0.8	0.07992	0.07439	0.04329	0.01295	0.02172	0.01736	0.00831	0.00625	0.00288	0.00195
	0.9	0.06114	0.06289	0.04276	0.00507	0.02741	0.01769	0.01404	0.00821	0.00573	0.00061
	0.1	0.07067	0.06838	0.01991	0.01937	0.00788	0.00762	0.00228	0.00191	0.00064	0.00118
	0.2	0.08430	0.08239	0.02429	0.02326	0.00965	0.00993	0.00280	0.00234	0.00079	0.00137
	0.3	0.09917	0.09696	0.02976	0.02713	0.01190	0.01036	0.00346	0.00269	0.00098	0.00070
	0.4	0.11428	0.11020	0.03668	0.03609	0.01485	0.01328	0.00434	0.00417	0.00123	0.00019
0.1	**0.5**	0.12786	0.12413	0.04538	0.04309	0.01886	0.01661	0.00557	0.00510	0.00158	0.00095
	0.6	0.13688	0.13210	0.05584	0.04541	0.02449	0.02204	0.00737	0.00615	0.00210	0.00184
	0.7	0.13662	0.13270	0.06679	0.02582	0.03239	0.02887	0.01028	0.00927	0.00296	0.00259
	0.8	0.12016	0.11934	0.07325	0.00940	0.04215	0.03236	0.01548	0.01218	0.00464	0.00171
	0.9	0.07824	0.09081	0.06120	0.00276	0.04548	0.01958	0.02408	0.00992	0.00901	0.00027
	0.1	0.11036	0.10832	0.02984	0.03052	0.01114	0.01042	0.00291	0.00297	0.00075	0.00075
	0.2	0.13011	0.12703	0.03629	0.03657	0.01363	0.01297	0.00357	0.00401	0.00092	0.00029
	0.3	0.15048	0.14810	0.04425	0.04205	0.01678	0.01587	0.00442	0.00321	0.00113	0.00070
	0.4	0.16934	0.16419	0.05414	0.05178	0.02089	0.01932	0.00554	0.00448	0.00142	0.00036
1.0	**0.5**	0.18335	0.17780	0.06614	0.06036	0.02642	0.02516	0.00709	0.00669	0.00183	0.00181
	0.6	0.18782	0.18372	0.07967	0.04134	0.03401	0.03172	0.00937	0.00847	0.00243	0.00144
	0.7	0.17698	0.17388	0.09185	0.01474	0.04421	0.03909	0.01303	0.01157	0.00343	0.00191
	0.8	0.14478	0.14110	0.09456	0.00351	0.05535	0.02575	0.01939	0.01162	0.00536	0.00052
	0.9	0.08640	0.08654	0.07119	0.00071	0.05459	0.00666	0.02891	0.00317	0.01031	0.00005

a) The pattern of closeness of *SB* and *TB* for different values of σ_μ^2 is no more different (Fig. 3).

b) The intimacy of *SB* and *TB* increases with the increasing values of *N* for any specific value of σ_μ^2 (Fig. 3).

c) The closeness of *SB* and *TB* is no more sensitive to *T*, whereas the proximity of the *SB* and *TB* decreases as value of γ increases (Fig. 4).

6. CONCLUSION

In this study we presented expressions for the asymptotic biases of the *GLS* estimator of the first order Dynamic Panel Data Models. These biases (*TB*) have been shown coincident with or a little above the estimates (*SB*) obtained by the Monte Carlo experiments. The intimacy of the *Theoretical bias* and *simulated bias* mainly depends on γ and *N*.

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