SEMI CONNECTEDNESS IN IRRESOLUTE TOPOLOGICAL GROUPS

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***ABSTRACT**: In this paper, we will continue the study of irresolute topological groups. We will investigate semi connectedness for irresolute topological groups. It is shown that every semi connected component of identity of an irresolute topological group G is equivalent to G. Also, a subgroup containing semi open neighbourhood of identity of a semi connected irresolute topological group G is equal to G.*

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1 INTRODUCTION

A topologized group is a triple $(G,*,\tau)$ such that $(G,*)$ is a group and (G,τ) is a topological space. If both the multiplication $m: G \times G \to G$ and the inversion $i: G \to G$ mappings of G are continuous, then $(G,*,\tau)$ is called a topological group. If only m is continuous, then $(G,*,\tau)$ is called a paratopological group. In case that m is separately continuous, then $(G,*,\tau)$ is called a semi topological group. If left translations $l_x: G \to G$ defined by $l_x(y) = x * y$ for all $y \in G$ are continuous, $(G,*,\tau)$ is called a left semi topological group. Right semi topological groups can be defined similarly. It is known that weaker and less restrictive assumptions can be used to characterize a group topology. In the literature, a lot of research work has been done in this line. In a series of papers, by celebrated mathematicians like A.V. Arhangel'skii, M. Tkachenko, *Ljubiša* D.R. Kočinac, T.Banakh and Ravsky, on topologized groups emphesisis its importance in the literature. A structure of topologized groups, defined with less restrictive conditions upon continuity is the focus of this paper.

s-topological group [1,2,3] is a generalization of topological groups as well as irresolute topological groups [4].These two topologized groups were obtained by keeping the group and topological structure on a set unaltered, but weakening the continuity conditions in the sense of Levine [5]. In 2014 [2,3] and in 2015 [4], the authors have presented many interesting basic properties of s-topological groups and irresolute topological groups.

The notion of connectedness is a basic, useful and fundamental notion in topological spaces. Das [6] in 1974 defined the weaker form of connectedness and called it semi connected space. Many mathematicians studied semi connected spaces rigorously. In this paper, we investigate some important results related to semi connectedness and semi components in irresolute topological groups .

2 Preliminaries

Throughout this paper, X and Y are always topological spaces on which no separation axioms are assumed. For a subset A of a space X, the symbols $Int(A)$ and $Cl(A)$ are used to denote the interior of A and the closure of A. If $f: X \to Y$ is a mapping between topological spaces X and Y and B is a subset of Y, then $f^{-1}(B)$ denotes the pre image of B. Our other topological notation and terminology are standard as in [7]. If $(G,*)$ is a group, then e denotes its identity element, and for a given $x \in G$, $l_x: G \to G$, $y \mapsto x * y$, and $r_x: G \to G$, $y \mapsto y * x$, denote the left and the right translation by x, respectively. The operation $*$ we call the multiplication mapping $m: G \times G \to G$, and the inverse operation $x \to x^{-1}$ is denoted by i.

In 1963, N. Levine [5] defined semi open sets in topological spaces. Since then many mathematicians explored different concepts and generalized them by using semi open sets [8,9, 10, 11]. A subset A of a topological space X is said to be *semi open*, if there exists an open set U in X such that $U \subset A \subset Cl(U)$, or equivalently, if $A \subset Cl(Int(A))$. $SO(X)$ denotes the collection of all semi open sets in X, whereas $SO(X,x)$ represents the collection of all semi open sets in X containing x. The complement of a semi open set is said to be semi closed, the semi closure of $A \subset X$, denoted by $sCl(A)$, is the intersection of all semi closed subsets of X containing A [12], [13]. Let us mention that $x \in sCl(A)$, if and only if for any semi open set U containing x, $U \cap A \neq \varphi$.

Clearly, every open (closed) set is semi open (semi closed). It is known that the union of any collection of semi open sets is again a semi open set, while the intersection of two semi open sets need not be semi open. The intersection of an open set and a semi open set is semi open. If $A \subset X$ and $B \subset Y$ are semi open in spaces X and Y, then $A \times B$ is semi open in the product space $X \times Y$. Basic properties of semi open sets are given in [5], and of semi closed sets and the semi closure in [12,13].

Recall that a set $U \subset X$ is a semi neighbourhood of a point $x \in X$, if there exists $A \in SO(X)$ such that $x \in A \subset U$. A set $A \subset X$ is semi open in X, if and only if A is a semi neighbourhood of each of its points. If a semi neighbourhood U of a point x is a semi open set, we say that U is a semi open neighbourhood of x.

Definition 1[5] Let X and Y be topological spaces. A mapping $f: X \to Y$ is semi continuous, if for each open set V in Y, $f^{-1}(V) \in SO(X)$

Clearly, continuity implies semi continuity, the converse need not be true. Notice that a mapping $f: X \to Y$ is semi continuous, if and only if for each $x \in X$ and each neighbourhood V of $f(x)$ there is a semi open neighbourhood U of x with $f(U) \subset V$.

In [14], Kempisty defined quasi continuous mappings: a mapping $f: X \to Y$ is said to be quasi continuous at a point $x \in X$, if for each neighbourhood U of x and each neighbourhood W of $f(x)$ there is a nonempty open set

$V \subset U$, such that $f(V) \subset W$. f is quasi continuous, if it is quasi continuous at each point (see also [15]). Neubrunnová in [16] proved that semi continuity and quasi continuity coincide.

Definition 2 A mapping $f: X \to Y$ between topological spaces X and Y is called:

1. semi open [17], if for every open set A of X, the set $f(A)$ is semi open in Y;
2. pre semi open [9], if for every semi open set A of X, the set $f(A)$ is semi open in Y;
3. irresolute [9], if for every semi open set B in Y, the set $f^{\leftarrow}(B)$ is semi open in X;
4. semi homeomorphism [9], if it is bijective, pre semi open and irresolute.

Definition 3 A topological space (X, τ), is said to be semi compact ([18]; [19]; [20]) ; if every semi open cover of X has a finite subcover.

Definition 4 A subset A of a group G is symmetric, if $A = A^{-1}$.

Definition 5 Two non-null subsets A, B of a topological space (X, τ) are said to be semi separated [6], if and only if $A \cap B = \emptyset$, where $\emptyset$ denotes the null set.

Definition 6 [6] In topological space (X, τ), a set which cannot be expressed as the union of two semi separated sets is said to be a semi connected set. The topological space (X, τ) is said to be semi connected, if and only if X is semi connected.

Definition 7 A subspace (Y, τ_Y) of a topological space (X, τ_X) is semi connected [6], if it is semi connected in the subspace topology. i.e if there do not exist disjoint semi open sets U and V of Y, such that $Y = U \cup V$.

Definition 8 Let (X, τ) be a topological space and $x \in X$. The semi component [21] of x, denoted by $SC(x)$, is the union of all semi connected subsets of X containing x. Further if $E \subset X$ and if $x \in E$, then the union of all semi connected sets containing x and contained in E is called the semi component of E corresponding to x.

Definition 9 A space G is totally semi disconnected, if the singletons are the only semi connected subsets of G. Equivalently, a space G is totally semi disconnected, if each one-point subset in G is its only semi connected component. Of course, every discrete space is totally semi disconnected.

Definition 10 Semi component of an identity element e of an irresolute topological group $(G, *, \tau)$ is the largest semi connected subset of G that contains the identity element e of the group G.

Definition 11 [10] Let A be a subset of a space X. Then a point $x \in A$ is said to be a semi isolated point of A, if there is a semi open set U such that $U \cap A = \{x\}$.

Definition 12 [10] A set A is said to be semi discrete, if each point of A is semi isolated.

Definition 13 A triple $(G, *, \tau)$ is an irresolute topological group [21] with a group $(G, *)$ and a topology τ such that for each $x, y \in G$ and each semi open neighbourhood W of $x * y^{-1}$, there are semi open neighbourhoods U of x and V of y such that $U * V^{-1} \subset W$.

Lemma 14 Let $f: X \to Y$ be a given mapping. Then f is irresolute, if and only if for every $x \in X$ and every semi open set $V \subset Y$ containing $f(x)$, there exists a semi open set U in X such that $x \in U$ and $f(U) \subset V$.

Lemma 15 [4] If $(G, *, \tau)$ is an irresolute topological group, then:

1. $A \in SO(G)$, if and only if $A^{-1} \in SO(G)$.
2. If $A \in SO(G)$, and $B \subset G$, then $A * B$ and $B * A$ both are semi open in G.

Lemma 16 [4] For irresolute topological group $(G, *, \tau)$, every left and right translations are semi homeomorphism.

Lemma 17 [4] Let U be any symmetric semi open neighbourhood of e in an irresolute topological group $(G, *, \tau)$. Then the set $L = \bigcup_{n=1}^{\infty} U^n$ is a semi open and a semi closed subgroup of G.

Lemma 18 [4] Let A and B be subsets of an irresolute-topological group G.Then:

1. $sCl(A) * sCl(B) \subset sCl(A * B)$;
2. $(sCl(A))^{-1} = sCl(A^{-1})$;
3. $(sInt(A))^{-1} = sInt(A^{-1})$.

Lemma 19 Semi closed subspace of semi compact space is semi compact.

Lemma 20 [21] Let $f: (X, \tau_X) \to (Y, y)$, be open, semi continuous and $A \subset X$ *be* open. Then $f(A)$ is semi connected, if A is semi connected.

Lemma 21 [20] If $f: X \to Y$ is a semi homeomorphism, then $f(sClB) = sCl(f(B))$ for all $B \subset Y$.

Lemma 22 [20] If $f: X \to Y$ is a semi homeomorphism, then $f(sIntB) = sInt(f(B))$ for all $B \subset Y$.

Lemma 23 [21] Let A be semi connected subspace of (X, τ). Let B be subspace of G such that $A \subset B \subset sCl(A)$. Then B is semi connected.

Lemma 24 [21] If the topological space (X, τ) is separated by semi open sets C and D, and if Y is semi connected subspace of X, then Y lies entirely within either C or D.

Lemma 25 semi homeomorphic image of a semi connected space is semi connected.

Lemma 26 [4] Let $A \subset Y \subset X$ where X is a topological space and Y is a subspace. Let $A \in SO(X)$. Then $A \in SO(Y)$.

Lemma 27 [22] A function $f: (X, \tau_X) \to (Y, \tau_Y)$ is pre semi open, if and only if $f^{\leftarrow}(sCl(B) \subset sCl(f^{\leftarrow}(B))$ for every subset B of Y.

Lemma 28 [23] Let (X, τ_X) be a topological space and $A \subset X_O \in PO(X)$, then $X_O \cap sCl_X(A) = sCl_{X_O}(A)$.

Theorem 29 If a function

$f:(X,\tau_X)\to(Y,\tau_Y)$

is pre semi open and $Y_O\in PO(Y)$, Then the function $f_o: X_O\to Y_O$ defined by $f_o(x)=f(x)$

forall $x\in X_O=f^{\leftarrow}(Y_O)$, is pre semi open.

Proof. Let B be any subset of Y_O. Since $B\subset Y_O\in PO(Y)$, therefore by Lemma 28, $sCl_{Y_O}(B)=Y_O\cap sCl_Y(B)$. Therefore, by Lemma 27, we obtain $f_o{}^{\leftarrow}(sCl_{Y_O})(B))=$
$f_o{}^{\leftarrow}(Y_O\cap sCl_Y(B))=$
$f^{\leftarrow}(Y_O)\cap f_o{}^{\leftarrow}(sCl_Y(B))=$
$X_O\cap f^{\leftarrow}(sCl_Y(B))\subset X_O\cap sCl_X(f^{\leftarrow}(B))=sCl_{X_O}f_o{}^{\leftarrow}(B)$.
This proves that $f_o: X_O\to Y_O$ is pre semi open.

Theorem 30 If $f:(X,\tau_X)\to(Y,\tau_Y)$ is irresolute mapping and X_O is an open subset of X, then $f|_{X_O}: X_O\to Y$ is irresolute.

Proof. Let f be irresolute mapping, then for any semi open set V in Y, $f^{\leftarrow}(V)$ is semi open in X. Hence $(f|_{X_O})^{\leftarrow}(V)=f^{\leftarrow}(V)\cap X_O$ is semi open in X. By Lemma 26, $(f|_{X_O})^{\leftarrow}(V)$ is semi open inX_O. This proves that $f|_{X_O}$ is irresolute.

Theorem 31 If a function

$f:(X,\tau_X)\to(Y,\tau_Y)$

Is semi homeomorphism and X_O is an open subset of X, then the function $f: X_O\to Y_O$ defined by $f_o(x)=f(x)$for all $x\in X_O=f^{\leftarrow}(Y_O)$, is semi homeomorphism.

Proof. Proof follows from Theorem 29 and Theorem 30.

3 Irresolute Topological Groups and Semi connectedness

In this section, we continue the study of irresolute topological groups, then we will present some results on semi connectedness in the presence of irresolute topological groups. The concept of semi connectedness in topological spaces was initially defined and investigated by Das [6] in 1974. With the help of this concept, in this section we will explore the basic properties of irresolute topological groups and see when irresolute topological group terms semi connected irresolute topological group.

We know that a topological group is termed connected, if it satisfies the following equivalent conditions:

1. It is connected as a topological space.
2. and the connected component of the identity element equals the whole group.

For a locally connected topological group, being connected is equivalent to having no proper open subgroup.

Theorem 32 Let $(G,.,\tau)$be an irresolute topological group, then for each $x,y\in G$and for every semi open neighbourhood W containing $x.y$, there exist semi open neighbouhoods U of x and V of y such that

1. $U.V\subseteq W$;
2. $V^{-1}\subset W$.

Proof.1. Let $x,y\in G$ and W be an semi open neighbourhood of $x.y.l_x(y)=x.y\in W$. By Lemma 16, $l_x: G\to G$ $(r_x: G\to G)$ is irresolute, there exists semi neighbourhood V containing y, such that $l_x(V)=x.V\subseteq W$. Let $U\in SO(G,x)$ such that $r_y(U)\subseteq W$, This implies $U.y\subseteq W$. Now $x.V\subseteq W$, for all $x\in U$ gives $x\in\bigcup_{x\in U}(x.V)\subseteq W$ or $U.V\subseteq W$.
2. Let W be an semi open neighbourhood of y^{-1}. Since inverse mapping $i: G\to G$ is irresolute, by (Theorem 3.1, [4]), there exists a semi open neighbourhood V containing y, such that $i(V)\subseteq W$. This implies that $V^{-1}\subset W$

Theorem 33 Let $(G,.,\tau)$ be an irresolute topological group and Ha subgroup of G. If H contains a non empty semi open set, then H is semi open in G.

Proof. Let U be a non empty semi open subset of G with $U\subset H$. For any $h\in H$, the set $h.U$ is semi open in G and is a subset of H. Therefore, the subgroup $H=\bigcup_{h\in H}(h.U)$ is semi open in G as the union of semi open sets.

Theorem 34 Let $(G,.,\tau)$ be an irresolute topological group. Then every semi open subgroup of G is semi closed in G.

Proof. Let H be a semi open subgroup of G. Then by Lemma 15, every left coset$x.H$ of H is semi open. Thus, $Y=\bigcup_{x\in G\setminus H}(x.H)$ is also semi open as a union of semi open sets. Hence, $H=G\setminus Y$ is semi closed.

Theorem 35 A subgroup H of an irresolute topological group $(G,.,\tau)$ is semi discrete, if and only if it has a semi isolated point.

Proof. Suppose that $x\in H$ and x is semi isolated in the relative topology of $H\subset G$. That is, there is a semi open neighbourhood U of e in G, such that $(x.U)\cap H=\{x\}$. Then for arbitrary $y\in H$, we have
$(y.U)\cap H=(y.U)\cap(y.x^{-1}.H)=y.x^{-1}((x.U)\cap H)=\{y\}$. Thus every point of H is semi isolated, so that H is indeed semi discrete. If H is semi discrete, then by definition all of its point are semi isolated.

Theorem 36 Suppose that $(G,.,\tau)$ is irresolute topological group and K be semi compact semi open neighbourhood of $e\in G$. Then there exists a semi open semi compact subgroup H of G such that $H\subseteq K$.

Proof. Since K is semi open neighbourhood of itself, therefore by (Theorem 3.26, [2]), there exists a semi open neighbourhood V of e such that $KV\subset K$. Then $V\subset K$ and let us assume that$V=V^{-1}$. Clearly $VV\subset KV=K$. By induction $V^n\subset K,\forall n\in\nabla$, Then $H=\cup\{V^n: n\in\nabla\}$ is semi open subgroup of G and $H\subseteq K$. Then set H is also semi closed in G, since every semi open subgroup of an irresolute topological group is semi closed. Hence by Lemma 19, H is semi compact.

Theorem 37 Let G be an irresolute topological group. Then semi interior of a symmetric subset $A$of G is again symmetric.

Proof. Let A be a subset of an irresolute-topological group G.Then by Lemma 18, $(sInt(A))^{-1}=sInt(A^{-1})$,

$A^{-1}=A$ Therefore,

$(sInt(A))^{-1}=sInt(A)$.

Theorem 38 Let G be an irresolute topological group. Then semi closure of a symmetric subset A of G is again symmetric.

Proof. Let A be a subset of an irresolute topological group G. Then by Lemma 18, $(sCl(A))^{-1}=sCl(A^{-1})$, Since

$A^{-1}=A$, Therefore, $\left(sCl(A)\right)^{-1}=sCl(A)$.

Definition 39 A semi topological group with respect to irresoluteness [24] is a group G endowed with a topology such that for each a ∈ G, the translations $l_a: G\to G(r_a: G\to G), l_a(x)=a.x,\ r_a(x)=x.a$ are irresolute, and such that the inverse mapping $i: G\to G, i(x)=x^{-1}$ is irresolute. Since every irresolute topological group is semi topological group with respect to irresoluteness, and by (Theorem 4.8, [24]), we have:

Theorem 40 Let $(G,.,\tau)$ be a semi topological group with respect to irresoluteness and A,B be subsets of G. Then:

1) if A is semi compact and B is finite, then $A.B$ and $B.A$ are semi compact.
2) if A is semi Lindelöf and B is countable, then $A.B$ and $B.A$ are semi Lindelöf.

Corollary 41 Let $(G,.,\tau)$ be an irresolute topological group and A,B be subsets of G. Then:

1) if A is semi compact and B is finite, then $A.B$ and $B.A$ are semi compact.
2) if A is semi Lindelöf and B is countable, then $A.B$ and $B.A$ are semi Lindelöf.

And by (Theorem 4.9,[24]), we have:

Theorem 42 Let G be a semi topological group with respect to irresoluteness, where all the translations are also open. If the semi component $SC(e)$ of identity e is open, then $SC(e)$ is semi closed normal subgroup.

Corollary 43 Let G be an irresolute topological group, where all the translations are also open. If the semi component $SC(e)$ of identity e is open, then $SC(e)$ is semi closed normal subgroup.

Theorem 44 Let $(G,.,\tau)$be an irresolute topological group. Then semi interior of any invariant subgroup of G is an irresolute topological invariant subgroup again.

Proof. It follows as the proof of

(Theorem 4.6,[24]).

Theorem 45 Let G be an irresolute topological group. Then semi closure of any invariant subgroup of G is an irresolute topological invariant subgroup again.

Proof. It follows asthe proof of

(Theorem 4.5,[24]).

Theorem 46 Let $(G,.,\tau)$ be an irresolute topological group, C the semi component of e, and U any semi open neighbourhood of e. Then $C\subset\bigcup_{n=1}^{\infty}V^{n.}$ In particular, if G is semi connected. then $G=\bigcup_{n=1}^{\infty}U^{n}$.

Proof. Let V be a symmetric semi open neighbourhood of e, By (Theorem 3.14, [2]) $V^{-1}=V\subset U$. Now V is symmetric semi open neighbourhood of e, therefore by Theorem 17, $L=\bigcup_{n=1}^{\infty}V^{n}$ is semi open as well as semi closed subgroup of G. Since C is semi connected, we have $C\subset\bigcup_{n=1}^{\infty}V^{n}\subset\bigcup_{n=1}^{\infty}U^{n}$, Now, if G is semi connected $G=\bigcup_{n=1}^{\infty}U^{n}$.

Theorem 47 Let $(G,.,\tau)$ be a semi connected irresolute topological group and H be a subgroup which contains a semi open neighbourhood of identity, then $H=G$. In particular, a semi open subgroup of a semi connected irresolute topological group G equals G.

Proof. Let U be a semi open neighbourhood of identity that is contained in H and $h\in H$ be arbitrary. The set $U.h=\{u.h \mid u\in U\}$is semi open by Lemma 15, and is contained in H. Hence the set $L=\bigcup_{h\in H}^{\infty}U.h$is semi open as the union of semi open sets and it is contained in H. Since U contains the identity element, $H\subset L$, and we conclude that H is semi open and semi closed as well by Theorem 34. As G is semi connected, therefore, $G=\bigcup_{h\in H}^{\infty}U.h=H$. This completes the proof.

Theorem 48 Let $(G,.,\tau)$ be an irresolute topological group and H be a semi connected component of e, then H is semi closed, and a normal subgroup of G.

Proof. By the Lemma16, $l_a: G\to G$ is a semi homeomorphism for every $a\in G$. This implies $l_a(H)=a.H$. Since H is semi connected, therefore by Lemma 25, $a.H$ is semi connectd. Similarly $l_{a^{-1}}(a.H)=a.H.a^{-1}$. is semi connected. Since $e\in a.H.a^{-1}$ and H is the biggest semi connected subset of G containing e, therefore $a.H.a^{-1}\subset H$ and $H\subset a.H.a^{-1}$. This implies $H=a.H.a^{-1}$. By Lemma 23, $sCl(H)$ is semi connected. Since H is largest semi connected subset, therefore $sCl(H)\subseteq H$. This implies $sCl(H)=H$. Thus H is semi closed invariant subgroup of G.

Theorem 49 Let U be a symmetric semi open neighbourhood of the identity element e of a semi connected irresolute topological group $(G,.,\tau)$. Then $G=\bigcup_{n=1}^{\infty}U^{n}$.

Proof. By Lemma 17, the set $L=\bigcup_{n=1}^{\infty}U^{n}$is semi open and semi closed subgroup of G. Since $(G,.,\tau)$ is semi connected, therefore $L=G=\bigcup_{n=1}^{\infty}U^{n}$.

Theorem 50 Let G be an irresolute topological group and let H be a subgroup of G. If H, G/H are semi connected, then G is semi connected.

Proof. Assume that $G=U\cup V$, where U and V are disjoint novoid semi open sets. Since$H$ is semi connected, each coset of H is either a subset of U or a subset of V by Lemma 24 and is semi connectd by Lemma 25. Thus the relation

$G/H=\{xH : xH\subset U\}\cup\{xH: xH\subset V\}$
$=\{xH : x\in U\}\cup\{xH : x\in V\}$.

It expresses G/H as the union of disjoint nonvoid semi open sets. This contradicts the hypothesis that G/H is semi connected.

Theorem 51 Let G be semi connected semi T_O irressolute topological group and H is a totally semi disconnected normal subgroup. Then $sCl(H)$ is contained in the centre of G.

Proof. Let us fix an element $a\in H$. Then by Lemma 16, the right translation $r_{ax^{-1}}: G\to G$ defined by $r_{ax^{-1}}(x)=xax^{-1}$ carries G into H semi homeomorphically. By Lemma 25, the image of G that is H is semi connected. Therefore H must be a singleton. Hence $a=xax^{-1}$for all $x\in G$. This proves that H is contained in the centre of G. But $H=sCl(H)$ by Theorem 48.

Theorem 52 Let K be a discrete invariant subgroup of a semi connected irresolute topological group G. Then every element of K commutes with every element of G, that is, K is contained in the center of the group $(G,.,\tau)$.

Proof. If $K=\{e\}$, there is nothing to prove. Suppose, therefore, that the subgroup K is not trivial. Take an arbitrary element $x \in K$ distinct from the identity element e of G. Since the group K is discrete, we can find an open neighbourhood U of x in G such that $U \cap K=\{x\}$. Then by definition of an irresolute topological group, there exists a symmetric semi open neighbourhood V of e and a semi open neighbourhood $V.x$ of x in G such that $(V.x).V^{-1} \subset U$. Let $y \in V$ be arbitrary. Since K is an invariant subgroup of G, we have $y.K=K.y$.This implies that $y.x \in K.y$. This implies that $y.x.y^{-1} \in K$.It is also clear that $y.x.y^{-1} \in Vx.V^{-1} \subset U$. Therefore, $y.x.y^{-1} \in U \cap K=\{x\}$, that is, $y.x.y^{-1}=x$. This implies that $y.x=x.y$, for each $y \in V$. Since the group G is semi connected, Theorem 49, implies that V^n, with $n \in N$, covers the group G. Therefore, every element $g \in G$ can be written in the form $g=y_1.y_2 \dots\dots y_n$, where, $y_1.y_2 \dots\dots y_n \in V$ and $n \in N$. Since x commutes with every element of V, we have:

$$
\begin{aligned}
g.x &= y_1.y_2 \dots\dots y_n.x \\
&= y_1.y_2 \dots\dots x.y_n \\
&= y_1.x.y_2 \dots\dots y_n \\
&= x.y_1..y_2 \dots\dots y_n \\
&= x.g
\end{aligned}
$$

We have thus proved that the element $x \in K$ is in the center of group G. Since x is an arbitrary element of K, we conclude that the center of G contains K.

Theorem 53 Let $(G,.,\tau)$be an irresolute topological group and let C be the semi component of identity in G. Then for all $a \in G,\ aC=Ca$ is the semi component of a.

Proof. The mapping x⟼ax is semi homeomorphism of G by Lemma 16. C is a normal subgroup of G, by Theorem 48. Hence $Ca=aC$ is semi component of a.

Theorem 54 Let G be a semi connected irresolute topological group and e its identity element. If U is any semi open neighborhood of e, then G is generated by U.

Proof. Let U be a semi open neighborhood of e. For each $n \in N$, we denote by U^n the set of elements of the form $u_1 \dots u_n$where $u_1 \dots u_n$ each $u_i \in U$. Let $W=\bigcup_{n=1}^{\infty} U^n$. Since G is semi connected and W is semi open and semi closed, we must have $W=G$. This means that G is generated by U. Since each U^n is semi open, we have that W is a semi open set. We now see that it is also semi closed. Let g be an element of semi closure of W. That is, $g \in sCl(W)$. Since gU^{-1} is a semi open neighborhood of g, it must intersect W. Thus, let $h \in W \cap gU^{-1}$. Since $h \in g.U^{-1}$, then $h=g.u^{-1}$ for some elements $u \in U$. Since $h \in W$, then $h \in U^n$for some$n \in N$, i.e., $h=u_1 \dots u_n$ with each $u_i \in U$. We then have $g=u_1 \dots u_n$.u i.e., $g \in U^{n+1} \subseteq W$. Hence, W is semi closed. Since G is semi connected and W is semi open and semi closed, we must have $W=G$. This means that G is generated by U.

Theorem 55 An irresolute topological group G, having a semi open subgroup, is not semi connected.

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