COMPUTING THE TOPOLOGICAL INDICES FOR CERTAIN FAMILIES OF GRAPHS

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ABSTRACT. *There are certain types of topological indices such as degree based topological indices, distance based topological indices and counting related topological indices etc. Among degree based topological indices, the so-called atom-bond connectivity (ABC), geometric arithmetic (GA) are of vital importance. These topological indices correlate certain physico-chemical properties such as boiling point, stability and strain energy etc. of chemical compounds. In this paper, we compute formulas of General Randić index ($R_\alpha(G)$) for different values of α , First zagreb index, atom-bond connectivity (ABC) index, geometric arithmetic GA index, the fourth ABC index (ABC_4) , fifth GA index (GA_5) for certain families of graphs.*

Key words: Atom-bond connectivity (ABC) index, Geometric-arithmetic (GA) index, ABC_4 index, GA_5 index.

1. INTRODUCTION AND PRELIMINARY RESULTS

Cheminformatics is a new subject which relates chemistry, mathematics and information science in a significant manner. The primary application of cheminformatics is the storage, indexing and search of information relating to compounds. Graph theory has provided a vital role in the aspect of indexing. The study of Quantitative structure-activity (QSAR) models predict biological activity using as input different types of structural parameters of molecules. Topological indices are very interesting class of these parameters. Topological indices can be derived from graph representation based on only nodes(atoms) and edges(chemical bonds).

A graph can be recognized by a numeric number, a polynomial, a sequence of numbers or a matrix which represents the whole graph, and these representations are aimed to be uniquely determined for that graph. A topological index is a numeric quantity associated with a graph which characterize the topology of graph and is invariant under graph automorphism. There are some major classes of topological indices such as distance based topological indices, degree based topological indices and counting related polynomials and indices of graphs. Among these classes, degree based topological indices are of great importance and play a vital role in chemical graph theory and particularly in chemistry. In more precise way, a topological index is a function "Top" from '$\sum$' to the set of real numbers, where '$\sum$' is the set of finite simple graphs with property that Top(G) = Top(H) if both G and H are isomorphic. Obviously, the number of edges and vertices of a graph are topological indices.

A few networks such as hexagonal, honeycomb, and grid networks, for instance, bear resemblance to atomic or molecular lattice structures. These networks have very interesting topological properties which have been studied in different aspects in [13, 15, 16, 19]. The hexagonal and honeycomb networks have also been recognized as crucial evolutionary biology, in particular for the evolution of cooperation, where the overlapping triangles are vital for the propagation of cooperation in social dilemmas. Relevant research that applies this theory and which could benefit further from the insights of the new research is found in [11, 14, 17, 18, 20, 23].

A graph G is connected if there exist a connection between any given pair of vertices of graph G. Throughout in this article, G is connected graph with vertex set V(G) and edge set E(G), d_u is the degree of vertex $u \in V(G)$ and

$$S_u = \sum_{v \in N_G(u)} d_v$$

where $N_G(u) = \{v \in V(G): uv \in E(G)\}$.

The general Randić index was proposed by Bollobás and Erdös [3], and Amic et al. [1] independently, in 1998. The definition is as follows:

Definition 1.1. The general Randić index is defined as

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha.$$

First Zagreb index denoted by $M_1(G)$ was introduced about forty years ago by Ivan Gutman and Trinajstić. The formal definition is given as:

Definition 1.2. Consider a graph G, the first Zagreb index is defined as

$$M_1(G) = \sum_{uv \in E(G)} (d_u + d_v)$$

Atom-bond connectivity index denoted by ABC(G) was introduced by Estrada et al. in [6]. The definition is as follows:

Definition 1.3. For a graph G, the ABC index is defined as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}} \quad .$$

An important connectivity topological descriptor is geometric-arithmetic GA index which was introduced by Vukičević et al. in [22]. The formal definition is given by:

Definition 1.4. Consider a graph G, then its GA index is defined as

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}.$$

The fourth version of ABC index was introduced by Ghorbani et al. [9] in 2010. The definition is given as:

Definition 1.5. For a graph G, the ABC_4 index is defined as

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}} \quad .$$

Recently fifth version of GA index is proposed by Graovac et al. [10] in 2011. The definition is as follows:

Definition 1.6. The fifth GA index for graph G is defined as following

$$GA_5(G) = \sum_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}.$$

In this paper, we compute formulas of General Randić index ($R_\alpha(G)$) for different values of α, First Zagreb index, atom-bond connectivity (ABC) index, GA index, ABC_4 index, GA_5 index for Cayley graph $Cay(Z_n \oplus Z_m)$ for n≥3 and m≥2, and generalized antiprism graph A_n^m for n≥3 and m≥1.

2. Results for Cayley graph $Cay(Z_n \oplus Z_m)$

Let G be a semigroup, and let S be a nonempty subset of G. The Cayley graph Cay(G,S) of G relative to S is defined as the graph with vertex set G and edge set E(S) consisting of those ordered pairs (x,y) such that sx = y for some s∈S. Cayley graphs of groups are significant both in group theory and in constructions of interesting graphs with nice properties. The Cayley graph Cay(G,S) of a group G is symmetric or undirected if and only if $S = S^{-1}$.

The Cayley graph $Cay(Z_n \oplus Z_m)$, n≥3, m≥2, is a graph which can be obtained as the cartesian product $P_m \square C_n$ of a path on m vertices with a cycle on n vertices. The vertex set and edge set of $Cay(Z_n \oplus Z_m)$ defined as: $V(Cay(Z_n \oplus Z_m))= \{(x_i,y_j): 1\le i\le n, 1\le j\le m\}$ and $E(Cay(Z_n \oplus Z_m)) =\{(x_i,y_j)(x_{i+1},y_j): 1\le i\le n, 1\le j\le m\} \cup \{(x_i,y_j)(x_i,y_{j+1}): 1\le i\le n, 1\le j\le m-1\}$. We make the convention that $(x_{n+1},y_j) = (x_1, y_j)$ for 1≤j≤m. We have $|V(Cay(Z_n \oplus Z_m))|=mn$, $|E(Cay(Z_n \oplus Z_m))| = (2m-1)n$, where $|V(Cay(Z_n \oplus Z_m))|$, $|E(Cay(Z_n \oplus Z_m))|$ denote the number of vertices, edges of the Cayley graph $Cay(Z_n \oplus Z_m)$ respectively.

Theorem 2.1. Consider the Cayley graph $Cay(Z_n \oplus Z_m)$, then its general Randić index is equal to

$$R_\alpha(Cay(Z_n \oplus Z_m)) = \begin{cases} 32mn-38n & ,\alpha = 1; \\ 8mn+(4\sqrt{3}-14)\,n & ,\alpha=\frac{1}{2}; \\ \frac{1}{8}mn+\frac{11}{144}n & ,\alpha=-1; \\ \frac{1}{2}mn+\frac{4\sqrt{3}-7}{12}\,n & ,\alpha=-\frac{1}{2}. \end{cases}$$

Proof. Consider the Cayley graph $Cay(Z_n \oplus Z_m)$. The number of vertices and edges in $Cay(Z_n \oplus Z_m)$ are mn and (2m-1)n, respectively. There are three types of edges in $Cay(Z_n \oplus Z_m)$ based on degrees of end vertices of each edge. Table 1 shows such partition of $Cay(Z_n \oplus Z_m)$.

(d_u,d_v)	Number of edges
(3,4)	2n
(4,4)	2mn-5n
(3,3)	2n

Table 1: The edge partition of $Cay(Z_n \oplus Z_m)$ based on degrees of end vertices of each edge

Now for α= 1, we apply the formula of $R_\alpha(Cay(Z_n \oplus Z_m))$.

$$R_1(Cay(Zn \oplus Zm)) = \sum_{uv\in E(Cay(Zn\oplus Zm))} (d_u d_v)$$

By using edge partition given in Table 1, we obtain

$R_1(Cay(Z_n \oplus Z_m))=(2n)(3 \times 4)+(2mn-5n)(4 \times 4)+(2n)(3\times3)=32mn-38n$.

For $\alpha=\frac{1}{2}$, we apply the formula of $R_\alpha(Cay(Z_n \oplus Z_m))$.

$$R_{\frac{1}{2}}(Cay(Zn \oplus Zm)) = \sum_{uv\in E(Cay(Zn\oplus Zm))} \sqrt{d_u d_v}$$

By using edge partition given in Table 1, we obtain

$R_{\frac{1}{2}}(Cay(Z_n \oplus Z_m))=(2n)(\sqrt{3\times4})+(2mn-5n)(\sqrt{4\times4})+(2n)(\sqrt{3\times3})=8mn+(4\sqrt{3}-14)n$.

For α=-1, we apply the formula of $R_\alpha(Cay(Z_n \oplus Z_m))$.

$$R_{-1}(Cay(Zn \oplus Zm)) = \sum_{uv\in E(Cay(Zn\oplus Zm))} \left(\frac{1}{d_u d_v}\right)$$

By using edge partition given in Table 1, we obtain

$R_{-1}(Cay(Zn \oplus Zm))=(2n)(\frac{1}{3\times4})+(2mn-5n)(\frac{1}{4\times4})+(2n)(\frac{1}{3\times3}) = \frac{1}{8}mn+\frac{11}{144}n$.

For $\alpha= -\frac{1}{2}$, we apply the formula of $R_\alpha(Cay(Z_n \oplus Z_m))$.

$$R_{-\frac{1}{2}}(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u d_v}}$$

By using edge partition given in Table 1, we obtain

$R_{-\frac{1}{2}}(Cay(Zn \oplus Zm)) =(2n)(\frac{1}{\sqrt{3\times4}})+(2mn-5n)(\frac{1}{\sqrt{4\times4}})+(2n)(\frac{1}{\sqrt{3\times3}})=\frac{1}{2}mn+\frac{4\sqrt{3}-7}{12}n$.

In the following theorem, we determine the first Zagreb index for Cayley graph $Cay(Zn \oplus Zm)$.

Theorem 2.2. For Cayley graph $Cay(Zn \oplus Zm)$, the first Zagreb index is equal to $M_1(Cay(Zn \oplus Zm))=2n(8m-7)$.

Proof. Consider the Cayley graph $Cay(Zn \oplus Zm)$. By using the edge partition given in Table 1, the result follows. We know

$$M_1(Cay(Zn \oplus Zm)) = \sum_{uv\in E(Cay(Zn\oplus Zm))} (d_u + d_v)$$

Substituting the values, we get

$M_1(Cay(Zn \oplus Zm))=(2n)(3+4)+(2mn-5n)(4+4)+(2n)(3+3) =2n(8m-7)$.

Now we compute ABC index for Cayley graph $Cay(Zn \oplus Zm)$.

Theorem 2.3. For Cayley graph $Cay(Zn \oplus Zm)$, the ABC index is equal to

$ABC(Cay(Zn \oplus Zm)) = \sqrt{\frac{3}{2}}mn + \left(\sqrt{\frac{5}{3}} - \frac{5}{2}\sqrt{\frac{3}{2}} + \frac{4}{3}\right)n.$

Proof. Consider the Cayley graph $Cay(Zn \oplus Zm)$. The proof is just calculation based. By using edge partition given in Table 1, we get the result. We know

$$ABC\big(Cay(Zn \oplus Zm)\big) = \sum_{uv\in E(Cay(Zn\oplus Zm))} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

$ABC(Cay(Zn \oplus Zm))=(2n)(\sqrt{\frac{3+4-2}{3\times4}})+(2mn-5n)(\sqrt{\frac{4+4-2}{4\times4}})+(2n)(\sqrt{\frac{3+3-2}{3\times3}})$

By doing some calculation, we get

$ABC(Cay(Zn \oplus Zm)) = \sqrt{\frac{3}{2}}mn + \left(\sqrt{\frac{5}{3}} - \frac{5}{2}\sqrt{\frac{3}{2}} + \frac{4}{3}\right)n.$

In the next theorem, we compute the GA index for Cayley graph $Cay(Zn \oplus Zm)$.

Theorem 2.4. Consider the Cayley graph Cay(Zn $\oplus$ Zm), then its GA index is equal to GA(Cay(Zn $\oplus$ Zm))=2mn+ $(\frac{8\sqrt{3}}{7}-3)$ n.

Proof. Consider the Cayley graph Cay(Zn $\oplus$ Zm). The proof is just calculation based. By using edge partition given in Table 1, we get the result. We know

$$GA(Cay(Zn \oplus Zm)) = \sum_{uv\in E(Cay(Zn\oplus Zm))} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$$

GA(Cay(Zn $\oplus$ Zm))=(2n)($\frac{2\sqrt{3\times 4}}{3+4}$)+(2mn-5n)($\frac{2\sqrt{4\times 4}}{4+4}$)+(2n) ($\frac{2\sqrt{3\times 3}}{3+3}$)

By doing some calculation, we get

GA(Cay(Zn $\oplus$ Zm))=2mn+$(\frac{8\sqrt{3}}{7}-3)$ n.

In the following theorem, we compute the fourth version of ABC index for Cayley graph Cay(Zn $\oplus$ Zm).

Theorem 2.5. Consider the Cayley graph Cay(Zn $\oplus$ Zm), then its fourth ABC index is equal to

ABC_4(Cay(Zn $\oplus$ Zm))=$\frac{\sqrt{30}}{8}$mn+$(\frac{3\sqrt{2}}{5}+\frac{2}{5}\sqrt{\frac{23}{6}}+\frac{4\sqrt{7}}{15}+\frac{1}{2}\sqrt{\frac{29}{15}}-\frac{9\sqrt{30}}{16})$n.

Proof. Consider the Cayley graph Cay(Zn $\oplus$ Zm). We find the edge partition of Cay(Zn $\oplus$ Zm) based on the degree sum of vertices lying at unit distance from end vertices of each edge. Table 2 explains such partition for Cay(Zn $\oplus$ Zm).

(S_u, S_v)	Number of edges
(10,10)	2n
(10,15)	2n
(15,15)	2n
(15,16)	2n
(16,16)	2mn-9n

Table 2: The edge partition of Cay(Zn $\oplus$ Zm) based on degree sum of vertices lying at unit distance from end vertices of each edge.

Now we can apply the formula of ABC_4 index for Cay(Zn $\oplus$ Zm). Since

$$ABC_4(Cay(Zn \oplus Zm)) = \sum_{uv\in E(Cay(Zn\oplus Zm))} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$$

Using Table 2, we have

ABC_4(Cay(Zn $\oplus$ Zm))=(2n)($\sqrt{\frac{10+10-2}{10\times 10}}$)+(2n) ($\sqrt{\frac{10+15-2}{10\times 15}}$)+(2n)($\sqrt{\frac{15+15-2}{15\times 15}}$)+(2n)($\sqrt{\frac{15+16-2}{15\times 16}}$)+(2mn-9n) ($\sqrt{\frac{16+16-2}{16\times 16}}$)

After an easy simplification, we get

ABC_4(Cay(Zn $\oplus$ Zm))=$\frac{\sqrt{30}}{8}$mn+$(\frac{3\sqrt{2}}{5}+\frac{2}{5}\sqrt{\frac{23}{6}}+\frac{4\sqrt{7}}{15}+\frac{1}{2}\sqrt{\frac{29}{15}}-\frac{9\sqrt{30}}{16})$n.

In the following theorem, we determine the fifth version of ABC index for Cayley graph Cay(Zn $\oplus$ Zm).

Theorem 2.6. Consider the Cayley graph Cay(Zn $\oplus$ Zm), then its fifth GA index is equal to

GA_5(Cay(Zn $\oplus$ Zm))=2mn+$(\frac{4\sqrt{6}}{5}+\frac{16\sqrt{15}}{31}-5)$n.

Proof. Consider the Cayley graph Cay(Zn $\oplus$ Zm). We apply the formula of GA_5 index of Cay(Zn $\oplus$ Zm) which is given as

$$GA_5(Cay(Zn \oplus Zm)) = \sum_{uv\in E(Cay(Zn\oplus Zm))} \frac{2\sqrt{S_u S_v}}{S_u + S_v}$$

Using values from Table 2, we easily get the result

GA_5(Cay(Zn $\oplus$ Zm))=(2n)($\frac{2\sqrt{10\times 10}}{10+10}$)+(2n)($\frac{2\sqrt{10\times 15}}{10+15}$)+(2n) ($\frac{2\sqrt{15\times 15}}{15+15}$)+(2n)($\frac{2\sqrt{15\times 16}}{15+16}$)+(2mn-9n)($\frac{2\sqrt{16\times 16}}{16+16}$)

After an easy calculation, we get

GA_5(Cay(Zn $\oplus$ Zm))=2mn+$(\frac{4\sqrt{6}}{5}+\frac{16\sqrt{15}}{31}-5)$n.

3. Results for generalized antiprism graph A_n^m

For n≥3 and m≥1, we denote by A_n^m the plane graph of a convex polytope, which is obtained as a combination of m antiprisms A_n defined in [4].

Let us denote the vertex set and edge set of A_n^m by V(A_n^m) and E(A_n^m) respectively. We have V (A_n^m)= {(x_i,y_j) : 1≤i≤n, 1≤ j≤ m+1} and E(A_n^m) ={(x_i,y_j)(x_{i+1},y_j): 1≤i≤n, 1≤j≤m+1} ∪{(x_i,y_j)(x_i,y_{j+1}): 1≤i≤n, 1≤j≤ m}∪{(x_{i+1},y_j)(x_i,y_{j+1}): 1≤i≤n, 1≤j≤ m, j odd}∪{(x_i,y_j) (x_{i+1},y_{j+1}): 1≤i≤n, 1≤j≤ m, j even}. We make the convention that (x_{n+1},y_j) = (x_1,y_j) for 1≤j≤m+1.

The number of vertices and edges of A_n^m are (m+1)n and 3mn+n respectively.

Theorem 3.1. Consider the generalized antiprism graph A_n^m , then its general Randić index is equal to

$$R_\alpha(A_n^m)=\begin{cases} 108mn-52n & ,\alpha = 1; \\ 18mn+(8\sqrt{6}-22)n & ,\alpha=\frac{1}{2}; \\ \frac{1}{12}mn+\frac{11}{72}n & ,\alpha=-1; \\ \frac{1}{2}mn+\frac{\sqrt{6}-1}{3}n & ,\alpha=-\frac{1}{2}. \end{cases}$$

Proof. There are three types of edges in A_n^m based on degrees of end vertices of each edge. Table 3 shows such partition.

(d_u, d_v)	Number of edges
(4,4)	2n
(4,6)	4n
(6,6)	3mn-5n

Table 3: The edge partition of A_n^m based on degrees of end vertices of each edge

Now for α= 1, we apply the formula of $R_\alpha(A_n^m)$.

$$R_1(A_n^m) = \sum_{uv\in E(A_n^m)} (d_u d_v)$$

By using edge partition given in Table 3, we obtain

$R_1(A_n^m)$=(2n)(4×4)+(4n)(4×6)+(3mn-5n)(6×6)=108mn-52n.

For α=$\frac{1}{2}$, we apply the formula of $R_\alpha(A_n^m)$.

$$R_{\frac{1}{2}}(A_n^m) = \sum_{uv\in E(A_n^m)} \sqrt{d_u d_v}$$

By using edge partition given in Table 3, we obtain

$R_{\frac{1}{2}}$ (A_n^m)=(2n)($\sqrt{4\times 4}$)+(4n)($\sqrt{4\times 6}$)+(3mn-5n)($\sqrt{6\times 6}$)= 18mn+(8$\sqrt{6}$-22)n.

For α=-1, we apply the formula of $R_\alpha(A_n^m)$.

$$R_{-1}(A_n^m) = \sum_{uv\in E(A_n^m)} (\frac{1}{d_u d_v})$$

By using edge partition given in Table 3, we obtain

$R_{-1}(A_n^m)$=(2n)($\frac{1}{4\times 4}$)+(4n)($\frac{1}{4\times 6}$)+(3mn-5n)($\frac{1}{6\times 6}$)= $\frac{1}{12}$mn+$\frac{11}{72}$n.

For α= -$\frac{1}{2}$, we apply the formula of $R_\alpha(A_n^m)$.

$$R_{-\frac{1}{2}}(A_n^m) = \sum_{uv\in E(A_n^m)} \frac{1}{\sqrt{d_u d_v}}$$

By using edge partition given in Table 3, we obtain

$R_{-\frac{1}{2}}(A_n^m)$ =(2n)($\frac{1}{\sqrt{4\times 4}}$)+(4n)($\frac{1}{\sqrt{4\times 6}}$)+(3mn-5n)($\frac{1}{\sqrt{6\times 6}}$)= $\frac{1}{2}$mn+$\frac{\sqrt{6}-1}{3}$n.

Theorem 3.2. For a generalized antiprism graph A_n^m, the first Zagreb index is equal to $M_1(A_n^m) = 4n(9m-1)$.

Proof. Consider the generalized antiprism graph A_n^m. By using the edge partition given in Table 3, the result follows. We know

$$M_1(A_n^m) = \sum_{uv\in E(A_n^m)} (d_u + d_v)$$

Substituting the values, we get

$M_1(A_n^m)=(2n)(4+4)+(4n)(4+6)+(3mn-5n)(6+6)=4n(9m-1)$.

Theorem 3.3. For generalized prism graph A_n^m, the ABC index is equal to

$ABC(A_n^m) = \sqrt{\frac{5}{2}}mn + \left(\sqrt{\frac{3}{2}} + \frac{4}{\sqrt{3}} - \frac{5}{3}\sqrt{\frac{5}{2}}\right)n.$

Proof. Consider the generalized antiprism graph A_n^m. The proof is just calculation based. By using edge partition given in Table 3, we get the result. We know

$$ABC(A_n^m) = \sum_{uv\in E(A_n^m)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

$ABC(A_n^m)=(2n)(\sqrt{\frac{4+4-2}{4\times4}})+(4n)(\sqrt{\frac{4+6-2}{4\times6}})+(3mn-5n)(\sqrt{\frac{6+6-2}{6\times6}})$

By doing some calculation, we get

$ABC(A_n^m)=\sqrt{\frac{5}{2}}mn + \left(\sqrt{\frac{3}{2}} + \frac{4}{\sqrt{3}} - \frac{5}{3}\sqrt{\frac{5}{2}}\right)n.$

Theorem 3.4. Consider the generalized antiprism graph A_n^m, then its GA index is equal to $GA(A_n^m)=3mn+(\frac{8\sqrt{6}-15}{5})n$.

Proof. The proof is just calculation based. By using edge partition given in Table 3, we get the result. We know

$$GA(A_n^m) = \sum_{uv\in E(A_n^m)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$$

$GA(A_n^m)=(2n)(\frac{2\sqrt{4\times4}}{4+4})+(4n)(\frac{2\sqrt{4\times6}}{4+6})+(3mn-5n)(\frac{2\sqrt{6\times6}}{6+6})$

By doing some calculation, we get

$GA(A_n^m)=3mn+(\frac{8\sqrt{6}-15}{5})n.$

Theorem 3.5. Consider the generalized antiprism graph A_n^m, then its fourth ABC index is equal to $ABC_4(A_n^m)=\frac{1}{6}\sqrt{\frac{35}{2}}mn+(\frac{1}{5}\sqrt{\frac{19}{2}}+\frac{\sqrt{5}}{2}+\frac{1}{8}\sqrt{\frac{31}{2}}+\frac{1}{2}\sqrt{\frac{11}{3}}-\frac{11}{18}\sqrt{\frac{35}{2}})n.$

Proof. We find the edge partition of A_n^m based on the degree sum of vertices lying at unit distance from end vertices of each edge. Table 4 explains such partition for A_n^m.

(S_u,S_v)	Number of edges
(20,20)	2n
(20,32)	4n
(32,32)	2n
(32,36)	4n
(36,36)	3mn-11n

Table 4: The edge partition of A_n^m based on degree sum of vertices lying at unit distance from end vertices of each edge

Now we can apply the formula of ABC_4 index for A_n^m. Since

$$ABC_4(A_n^m) = \sum_{uv\in E(A_n^m)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$$

Using Table 4, we have

$ABC_4(A_n^m)=(2n)(\sqrt{\frac{20+20-2}{20\times20}})+(4n)(\sqrt{\frac{20+32-2}{20\times32}})+(2n)(\sqrt{\frac{32+32-2}{32\times32}})+(4n)(\sqrt{\frac{32+36-2}{32\times36}})+(3mn-11n)(\sqrt{\frac{36+36-2}{36\times36}})$

After an easy simplification, we get

$ABC_4(A_n^m)=\frac{1}{6}\sqrt{\frac{35}{2}}mn+(\frac{1}{5}\sqrt{\frac{19}{2}}+\frac{\sqrt{5}}{2}+\frac{1}{8}\sqrt{\frac{31}{2}}+\frac{1}{2}\sqrt{\frac{11}{3}}-\frac{11}{18}\sqrt{\frac{35}{2}})n.$

Theorem 3.6. Consider the generalized antiprism graph A_n^m, then its fifth GA index is equal to $GA_5(A_n^m)=3mn+(\frac{16\sqrt{10}}{13}+\frac{48\sqrt{2}}{17}-7)n.$

Proof. We apply the formula of GA_5 index of A_n^m which is given as

$$GA_5(A_n^m) = \sum_{uv\in E(A_n^m)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}$$

Using values from Table 4, we easily get the result

$GA_5(A_n^m)=(2n)(\frac{2\sqrt{20\times20}}{20+20})+(4n)(\frac{2\sqrt{20\times32}}{20+32})+(2n)(\frac{2\sqrt{32\times32}}{32+32})+(4n)(\frac{2\sqrt{32\times36}}{32+36})+(3mn-11n)(\frac{2\sqrt{36\times36}}{36+36})$

After an easy calculation, we get

$GA_5(A_n^m)=3mn+(\frac{16\sqrt{10}}{13}+\frac{48\sqrt{2}}{17}-7)n.$

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5. CONCLUSIONs

We have computed the formulas of General Randić index, First zagreb index, ABC index, GA index, ABC_4 index, GA_5 index for Cayley graph Cay(Zn ⊕ Zm) for n≥3 and m≥2, and generalized antiprism graph A_n^m for n≥3 and m≥1. This work will give new direction for considering and computing topological indices of several other families of graphs.

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