AN ANALYSIS OF THERMAL EFFECTS ON ELECTRICALLY CONDUCTING CASSON FLUID FLOW PAST A PERMEABLE SHRINKING SURFACE IN A POROUS MEDIUM

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ABSTRACT: *This research paper examines the thermal effects in magnetohydrodynamic (MHD) stagnation point flow of Casson fluids over a flate shrinking sheet. The model of governing equations has been transformed in ordinary differential form by employing similarity transformation. The resulting equations are then solved numerically. The results for physical quantities like velocity and temperature have been computed and presented graphically for sufficient values of the parameters of interest namely Casson Parameter β, stretching/shrinking parameter ε, heat source parameter λ, radiative heat parameter R_n, Eckert number E_c, Prandtl number P_r, magnetic field parameter M, and porosity parameter K_p.*

Key words: Casson fluids, Magnetohydrodynamic flow, Heat source, Prandtl number, Porous Medium.

1. INTRODUCTION

Several fluids being used in industries possesses non-Newtonian behaviour. Non-Newtonian fluids are more useful than Newtonian fluids due to their various applications in industries like certain separation processes, petroleum drilling, food manufacturing, polymer engineering etc. There is no single constitutive equation for non-Newtonian fluids because for these fluids, the relationship between stress and the rate of strain is not linear. Many researchers of this research area [1-9] have proposed, different models of these fluids depending on various physical characters. One such type of non-Newtonian fluids is Casson fluid which behaves like an elastic solid. Examples of such fluids include tomato puree, chocolate, foams, molten cosmetics, yogurt, nail polish etc. Casson[10] was first who introduced this model for flow behaviour of the suspensions pigment oil for printing ink type. Mustafa *et al.* [11] investigated the unsteady boundary layer flow and heat transfer of a Casson fluid over a moving flat plate with a parallel free stream. Bhattacharyya et al. [12-13] studied the exact solution for boundary layer flow of Casson fluid due to porous stretching/shrinking sheet. Nakamura et al and Bird et al [14-15] respectively worked on the Casson fluid and concluded that infinite viscosity at zero rate of shear and a yield stress below which no flow occurs and a zero velocity at an infinite shear rate. In the references [16-18] an excellent collection of articles can be found. Crane [19] considered the study of boundary layer flow due to a stretching sheet. Gupta [20] considered the flow of incompressible fluid and mass transfer with suction or blowing towards a linearly stretching/shrinking sheet. Santosh *et al.* [21] investigated the unsteady 2-dimensional, laminar flow of a viscous fluid over a shrinking surface. Singh *et al.* [22] investigated for two-dimensional flow of a viscous fluid for a stretching or shrinking surface in porous medium. Jena [23] examined a steady, laminar, MHD flow of a viscous fluid over a shrinking sheet. Reddy [24] discussed the effects of buoyancy and radiation for heat and mass transfer flow past a moving vertical porous plate. Babu *et al.* [25] studied numerically the effects of radiation and heat source/sink on MHD boundary layer flow of heat and mass transfer past a shrinking sheet with wall mass suction. Kishore *et al*. [26] considered the hydromagnetic flow in a porous medium with radiation and viscous dissipation. Poornima *et al.* [27] analyzed the boundary-layer flow of nanofluid to a non-linear boundary moving sheet. Sharma *et al.* [28] considered the boundary layer flow due to exponentially shrinking sheet in the existence of the thermal radiation. Khan et al [29] discussed the thermo diffusion flow of the nanofluid past a stretching surface.

In this work, we considered an analysis of thermal effects in electrically conducting Casson fluid flow past a permeable shrinking surface. We extend the work of Lok *et.al.* [30], the investigation of 2-dimensional MHD stagnation point flow of viscous fluid over a porous shrinking sheet.

2. MATHEMATICAL MODEL:

A steady, two-dimensional flow of electrically conducting Casson fluid past a moving boundary surface in a porous medium. The strength of the uniform magnetic field applied normally to the surface is B_0. $T_w(x) = T_\infty + bx^3$ is the wall temperature $U = u_w$ and T_∞ are respectively free stream velocity and temperature functions. The magnetic Reynolds number is negligible.

Under the above assumptions, the equations governing the problems are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \left(1+\frac{1}{\beta}\right)\upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^{\,2}}{\rho}(U-u) + \left(1+\frac{1}{\beta}\right)\frac{\upsilon}{K}u + 1 \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0(T-T_\infty)}{\rho C_p} +$$

$$\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{16\alpha}{3\beta\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho C_p}u^2 \quad (3)$$

Where ρ is density, μ is dynamic viscosity coefficient, σ is the electrical conductivity, k is the thermal conductivity, c_p is the specific heat capacity at constant pressure, υ is kinematic viscosity, Q_0 is the volumetric rate of heat generation, β is Casson fluid parameter.

The boundary conditions are:

$$v = v_w(x),\ u = u_w(x),\ T = T_w \text{ at } y=0$$
$$u(x) \to U(x) \quad T \to T_\infty \text{ as } y \to \infty \quad (4)$$

Where $u_w(x) = -cx$ is wall shrinking velocity and $v_w(x)$ is mass flux velocity.where *a, b, c* and *n* are constant with $a > 0, b > 0$ and $c \geq 0$

Using similarity transformations:

The velocity components are described in terms of the stream function ψ (x, y):

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \qquad \psi(x,y) = x\sqrt{a\upsilon}\, f(\eta),$$

$$\eta = y\sqrt{\frac{a}{\upsilon}},$$

$$u = xaf',\ v = -\sqrt{\upsilon a}\, f,\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (5)$$

$$v_W(x) = -(a\upsilon)^{\frac{1}{2}} s,$$

Where $s = f(0)$ is the constant mass flux with $s > 0$ for the suction $s < 0$

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equations (2), (3) and (4), we get

$$(1+\frac{1}{\beta}) f''' + ff'' - f'^2 + M(1 - f') + (1+\frac{1}{\beta})\frac{1}{K'} f' + 1 = 0 \quad (6)$$

$$(4 + 3Rn)\theta'' + 3Rn\Pr(f\theta' + \lambda\theta + Ecf''^2 + ME_C f'^2) = 0 \quad (7)$$

and the boundary conditions are

$$f'(0) = -\varepsilon,\ f(0) = S,\ \theta(0) = 1,$$
$$f'(\infty) = 1,\ \theta(\infty) = 0, \quad (8)$$

Where $P_r = \frac{\upsilon}{\alpha}$ is the Prandtl number, $E_C = \frac{U_w^2}{C_p(T_W - T_\infty)}$ is Eckert number, $\lambda = \frac{Q_0}{\rho C_p c}$ is the heat source (λ<0) or sink (λ>0), $M = \frac{\sigma B_0^2}{\rho c}$ is the magnetic parameter, $K_p = \frac{aK'}{\upsilon}$ is the porosity parameter, and $\varepsilon = \frac{c}{a}$ is stretching/shrinking velocity parameter.

3. RESULTS AND DISCUSSION:

The flow of the problem, finally governed by the system of highly non-linear differential equations (6) to (8) cannot be solved easily solved analytically. Hence numerical solution of the problem is sought out. The higher order of the system is reduced by setting $f' = p, p' = q, \theta' = w$. Then the first order differential equations with appropriate boundary conditions are solved by using straight forward and efficient coding in Computational software Mathematica version 10. A rigorous computational work has been carried out to observe the effects of physical parameters involved in the study on flow and thermal characteristics.

The results have been presented in the form of plots for non-dimensional velocity and temperature function.

Fig.1 display the velocity f' under the effect of the magnetic field strength parameter *M*. the magnitude of the velocity decreases because of the opposing effect of Lorentz force. But increase in the value of the Casson parameter β causes increase in the magnitude of velocity f' as shown in fig.2. The stretching parameter ε also increases the flow velocity f' as presented in fig.3.

Similarly the fig.4 and fig.5 demonstrates the increasing effect of porosity parameter K_p and suction parameter *S* respectively on velocity f'.

Fig.6, fig.7, fig.8 respectively presents the effects of suction parameter, shrinking velocity parameter $(\varepsilon > 0)$, and Prandtl number P_r on non-dimensional temperature $\theta(\eta)$. It is noticed that the temperature distribution decreases with increases in the values of these parameters. But the temperature function increases in the value of the radiation parameter R_n, heat source/sink parameter λ, Eckert number E_c and magnetic parameter *M* as depicted respectively in fig 9 to 12.

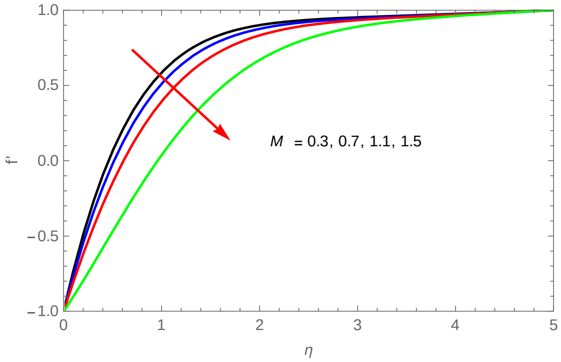

Fig.1: The plot for curves of f' under the effect of magnetic parameter M when λ=0.1 S=2, P_r=0.7, E_c=0.1, $\varepsilon=1$ and R_n=0.1

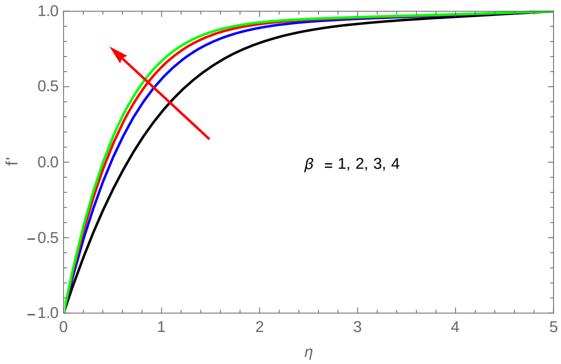

Fig.2: The plot for curves of f' under the effect of various values of β λ=0.1, P_r=0.7, E_c=0.1, $\varepsilon=1$ and R_n=0.1

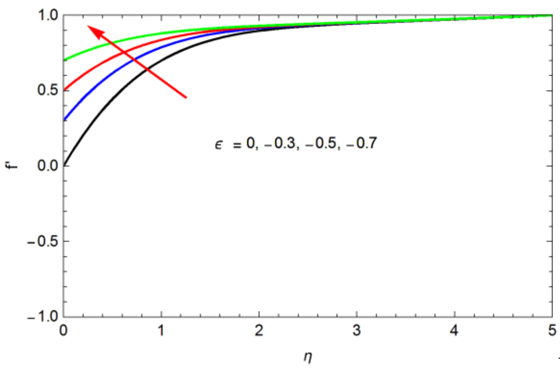

Fig.3: The plot for curves of f' under the effect of velocity ratio parameter ε when λ=0.1,M=2, S=2,P_r=0.7, E_c=0.1, and R_n=0.1.

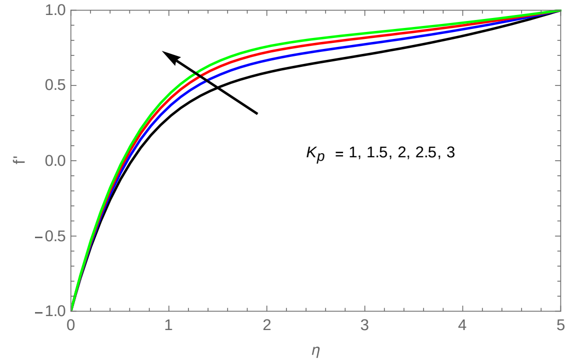

Fig.4 The plot for curves of f' under the effect porosity parameter K_p, λ=0.1,M=2, S=2, P_r=0.7, E_c=0.1, $\varepsilon=1$ and R_n=0.1.

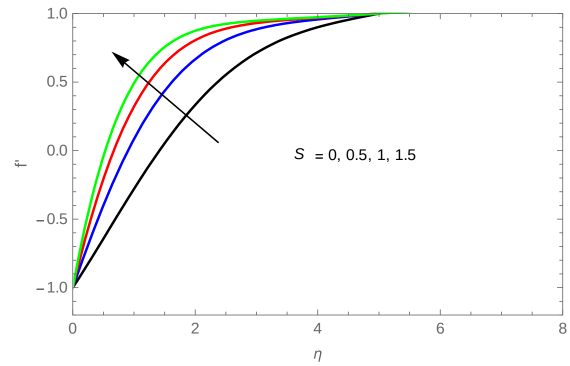

Fig.5: The plot for curves of f' effect of Suction parameter S when λ=0.1,M=2, S=2,P_r=0.7, E_c=0.1 , $\varepsilon=1$ and R_n=0.1.

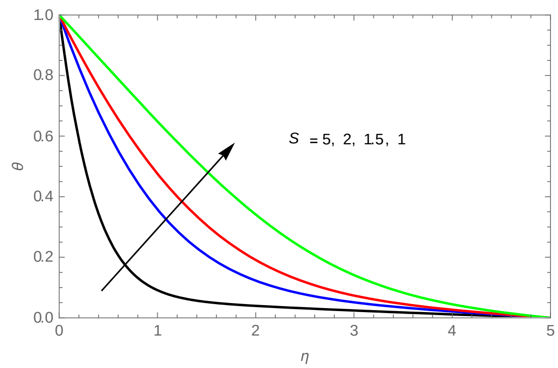

Fig.6: The plot for curves of θ under the effect of Suction parameter S when λ=0, H=0.1,A=1, E_c=0.1, $\varepsilon=1$ and P_r=0.1.

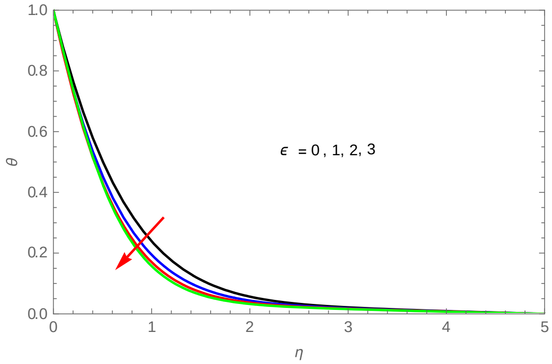

Fig.7: The plot for curves of θ under the effect of velocity ratio parameter ε when λ=0.1,M=2, S=2,P_r=0.7, E_c=0.1 and R_n=0.1.

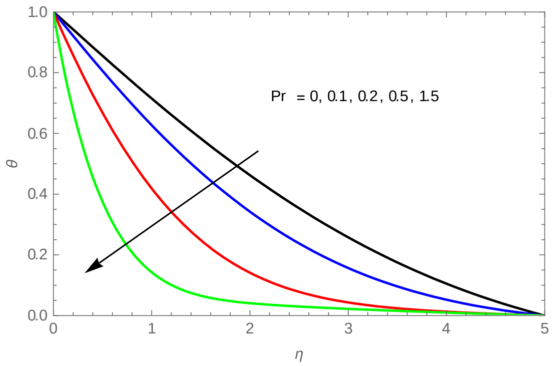

Fig.8: The plot for curves of θ under the effect of Prandtl number P_r when λ=0.1,M=2, S=2, $\varepsilon = 1$, P_r=0.7, E_c=0.1 and R_n=0.1.

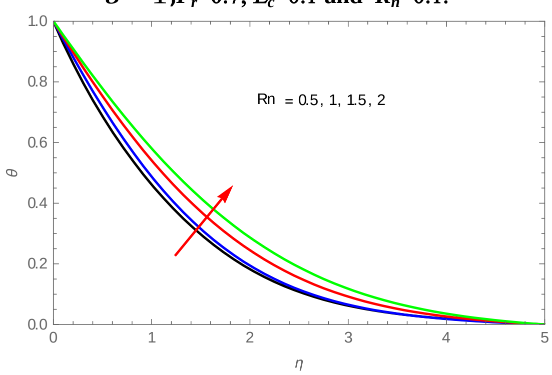

Fig.9: The plot for curves of θ under the effect of radiation parameter R_n when λ=0.1,M=2, S=2,P_r=0.7, E_c=0.1, $\varepsilon = 1$ and R_n=0.1.

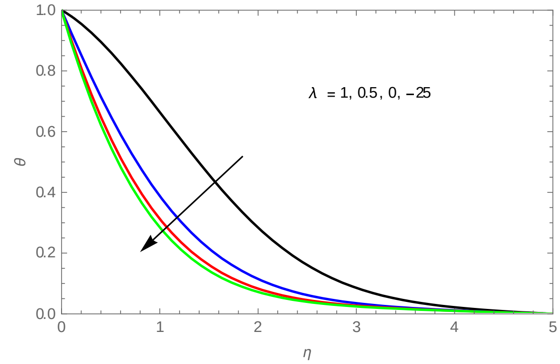

Fig.10: The plot for curves of θ under the effect of heat source parameter λ when M=2, S=2, $\varepsilon = 1$, P_r=0.7, E_c=0.1 and R_n=0.1.

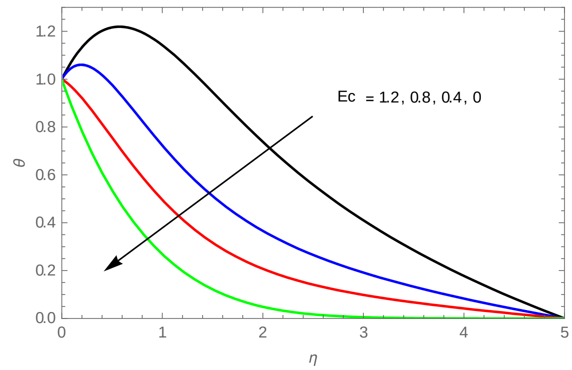

Fig.11: The plot for curves of θ under the effect of Eckert number E_c when λ=0.1,M=2, S=2,P_r=0.7, E_c=0.1 , $\varepsilon = 1$ and R_n=0.1.

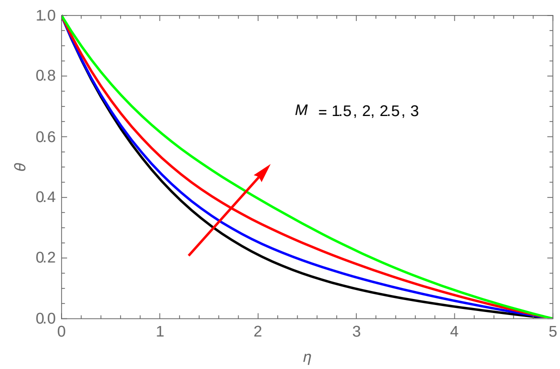

Fig.12: The plot for curves of θ under the effect of magnetic parameter M when λ=0.1, S=2, $\varepsilon = 1$, P_r=0.7, E_c=0.1 and R_n=0.1.

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