SOME COUETTE FLOWS OF A SECOND GRADE FLUID DUE TO TANGENTIAL STRESSES

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ABSTRACT: *In this paper we study the Couette flows of a second grade fluid, between two parallel plates, produced by the motion of a plate that applies a tangential stress on the fluid. Exact solutions for velocity are determined by means of the Laplace transform. Two particular cases corresponding to constant and sinusoidal tangential stresses on the plate, are studied. Expressions for the velocity field corresponding to the motion of a Newtonian fluid as limiting cases are extracted from general solutions. Some relevant properties of the velocity and the influences of the pertinent parameters on the fluid motion are presented using graphical illustrations.*

Keywords-: Couttee Flow Second grade Fluid, Velocity field, Shear stress, Exact solutions.

1. INTRODUCTION

Due to the nonlinear nature of Navier-Stocks equations, it is very complicated to find the exact solutions except a few particular cases available in literature. For the equations of motion, the exact solutions have some physical meaning and these solutions can be used as a check against complicated numerical codes that have been developed for much more complex flows. Taylor [1] investigated that the non-linear convective term vanished and found an exact solution by representing doubly infinite array of vertices by taking vorticity to be proportional to the stream functions perturbed by a uniform stream. later, Kovasnay [2] found an exact solution which represents the motion behind a two dimensional grid and showed that the non-linarites in the Navier-Stokes equations are self-canceling. Wang (1996) was also able to linearize the non-linear part of Navier-Stocks equations and showed the results of Taylor [1] and Kovasznay (1948) as special case in his work. Similar results were obtained by Lin and Tobak [3] and Hui [4] in which the non-linear inertial part canceled automatically.

The non-linearity in non-Newtonian fluids, namely fluids of second grade, has caused much difficulty in solving these problems. Rajagopal [5] investigated that, in the equations of motion of a second grade fluids, the nonlinear convective term vanish for the specific problems and Rajagopal [6] obtained solutions for unsteady flows. Rajagopal and Gupta [7] also found a class of exact solutions to the equations of motion of second grade fluids in which the non-linearities are self-canceling and showed a subclass of the solutions obtained by Wang [8] for the Navier-Stokes equations and Rajagopal [9] in (1995) study the boundary conditions for the differential type fluids .

The motion of a fluid can be obtained as a result of several effects such as various types of motion of the boundaries, application of a body force, wall that applies a tangential stress on the fluid or application of a pressure gradient. Exact solutions have been established, by Rajagopal [10] for unsteady unidirectional flows, Rajagopal and Gupta [11-12] and Rajagopal [13] for the flow between infinite parallel plates. The effects of side walls on steady and unsteady flows have been studied in [14-16]. The unsteady Couette flow problem has been considered in several works containing various effects. The effects of fluid slippage at the boundary for Couette flow are considered in the paper of Marques et al. [17] under steady state conditions and only for gases. Khaled and Vafai [18] have studied the effect of slip condition on Couette flows due to an oscillating wall. Other interesting results regarding flows of Newtonian or non-Newtonian fluids can be found in the references [19-24].

This paper deals with the Couette flows of a second grade fluid caused by the bottom plate which applies a tangential stress $\tau_W(t)=\tau_0 f(t)$ on the fluid. Exact expressions for velocity are determined by means of a Laplace transform. Two particular cases, namely constant tension on the bottom plate and sinusoidal oscillations of the wall tension, are studied. Some relevant properties of the velocity are presented using graphical illustrations generated by the software Mathcad.

2. BASIC EQUATIONS

The Cauchy stress **T** for an incompressible fluid of second grade, is related to the fluid motion by the constitutive equation [25-27]

$$\mathbf{T}=-p\mathbf{I}+\mathbf{S},\mathbf{S}=\mu\mathbf{A}_1+\alpha_1\mathbf{A}_2+\alpha_2\mathbf{A}_1^2, \qquad (1)$$

where p is the hydrostatic pressure, $\mathbf{I}$ is the unit tensor, **S** is the extra-stress tensor, $\mathbf{A}_1=\mathbf{L}+\mathbf{L}^T$ is the first Rivlin-Ericksen tensor, $\mathbf{L}$ is the velocity gradient, the superscript T indicates the transpose operation, $\mathbf{A}_2$ is the kinematic tensor defined by $\mathbf{A}_2=d\mathbf{A}_1/dt+\mathbf{A}_1\mathbf{L}+\mathbf{L}^T\mathbf{A}_1$, d/dt denotes the material time derivative, μ is the dynamic viscosity of the fluid and α_1, α_2 are the normal stress moduli that meet the following restrictions

$$\mu>0,\ \alpha_1\geq 0,\ \alpha_1+\alpha_2=0, \qquad (2)$$

The unidirectional flows to be considered here have the velocity field [25]

$$\mathbf{V}=\mathbf{V}(y,t)=u(y,t)\mathbf{i}, \qquad (3)$$

here $\mathbf{i}$ is the unit vector along the x-direction of the cartesian co-ordinate system $x, y and z$. For such flows the constraint of incompressibility is automatically satisfied. In the case of a conservative body force field and in the absence of a pressure gradient in the $x-$ direction the governing equation corresponding to Eqs. (1) and (3), as it was deduced in [25] from the balance of linear momentum, is

$$\tau(y,t)=\left(\mu+\alpha_1\frac{\partial}{\partial t}\right)\frac{\partial u(y,t)}{\partial y}, \quad (4)$$

$$\frac{\partial u(y,t)}{\partial t}=\left(\nu+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 u(y,t)}{\partial y^2},\,(y,t)\in(0,h)\times(0,\infty), \quad (5)$$

where $\tau(y,t)$ is the tangential shear stress, $\alpha=\alpha_1/\rho$, $\nu=\mu/\rho$ is the kinematic viscosity and ρ being the constant density of the fluid.

3. PROBLEM FORMULATION AND SOLUTION

Let us consider an incompressible second grade fluid between two infinite rigid plates which are at rest initially. The plates are situated in the planes $y=0$ and $y=h$ of a Cartesian coordinate system $Oxyz$ with the positive y-axis in the upward direction, Fig. 1. After $t=0$ the fluid is set in motion by the lowwer plate that applies a tangential stress $\tau_W(t)=\tau(y,t)|_{y=0}=\tau_0 f(t)$ to the fluid. Here, $f(\cdot)$ is a piecewise continuous function defined on $[0,\infty)$ and $f(0)=0$. Also, we suppose that the Laplace transform of function $f(\cdot)$ exists.

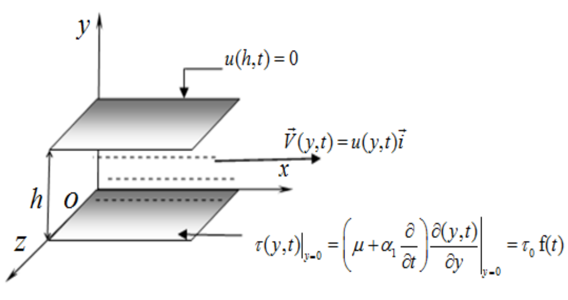


Figure 1: Geometry of the problem

Owing to the shear the fluid between the plates is gradually moved. Its velocity is of the form (3) and the governing equation are (4) and (5). The associate boundary and initial conditions are

$$\tau(y,t)|_{y=0}=\left(\mu+\alpha_1\frac{\partial}{\partial t}\right)\frac{\partial u(y,t)}{\partial y}|_{y=0}=\tau_0 f(t),\;\; u(h,t)=0,\;\; t\geq 0, \quad (6)$$

$$u(y,0)=0,\;\; y\in[0,h]. \quad (7)$$

By using the following dimensionless variables and functions

$$t^*=\frac{\nu t}{h^2},\; y^*=\frac{y}{h},\; \tau^*=\frac{\tau}{\tau_0},$$

$$u^*=\frac{u}{\left(\frac{h\tau_0}{\mu}\right)},\; g(t^*)=f\left(\frac{h^2t^*}{\nu}\right), \quad (8)$$

we obtain the nondimensionalized initial-boundary value problem (dropping the ``*'' notation)

$$\tau(y,t)=\left(1+\beta\frac{\partial}{\partial t}\right)\frac{\partial u(y,t)}{\partial y}, \quad (9)$$

$$\frac{\partial u(y,t)}{\partial t}=\left(1+\beta\frac{\partial}{\partial t}\right)\frac{\partial^2 u(y,t)}{\partial y^2},\, y,t>0 \quad (10)$$

$$\tau(y,t)|_{y=0}=\left(1+\beta\frac{\partial}{\partial t}\right)\frac{\partial u(y,t)}{\partial y}|_{y=0}=g(t),$$
$$u(1,t)=0,\; t\geq 0 \quad (11)$$

$$u(y,0)=0,\;\; y\in[0,1]. \quad (12)$$

Here $\beta=\alpha h^2$

By applying the temporal Laplace transform [28] to Eqs. (10) and (11) and employing the initial condition (12) we obtain the problem

$$\frac{\partial^2\bar{u}(y,q)}{\partial y^2}-\frac{q}{1+\beta q}\bar{u}(y,q)=0, \quad (13)$$

$$(1+\beta q)\frac{\partial\bar{u}(0,q)}{\partial y}=G(q),\; \bar{u}(1,q)=0, \quad (14)$$

where the image function $\bar{u}(y,q)=\mathrm{L}\{u(y,t)\}=\int_0^\infty e^{-qt}u(y,t)dt$ is the Laplace transforms of functions u(y,t).

Solving ordinary differential equation (13) with respect to boundary conditions (14) we get

$$\bar{u}(y,q)=G(q)G_1(y,q), \quad (15)$$

where

$$G_1(y,q)=\frac{sh[(y-1)\sqrt{\frac{q}{1+\beta q}}]}{(1+\beta q)\sqrt{\frac{q}{1+\beta q}}ch(\sqrt{\frac{q}{1+\beta q}})}. \quad (16)$$

The singular points of the function $G_1(y,q)$ are simple poles located at

$$q_k = -\frac{\alpha_k^2}{1+\beta\alpha_k^2},\ \alpha_k = \frac{(2k+1)\pi}{2},\ k = 0,1,2,... \quad (17)$$

Inverting Eq (16), by using the residue theorem to evaluate the Laplace inversion integral [28], we obtain

$$g_1(y,t) = L^{-1}\{G_1(y,q)\} = \sum_{k=0}^{\infty} Res\left[G_1(y,q)e^{qt};q_k\right]$$

$$= -2\sum_{k=0}^{\infty}\frac{\cos(\alpha_k y)}{1+\beta\alpha_k^2}\exp\left(-\frac{\alpha_k^2 t}{1+\beta\alpha_k^2}\right). \quad (18)$$

4. CONSTANT TENSION ON THE LOWER PLATE

In this section we consider $g(t) = H(t)$, where H(t) is the Heaviside step unit function. Using Eqs. (15), (18) and the convolution theorem we obtain the exact (y,t)-domain solution of the set of equations (10)-(12) given by

$$u(y,t) = (g * g_1)(t) = \int_0^t g(t-s)g_1(y,s)ds, \quad (19)$$

and, using the identity

$$-2\sum_{k=0}^{\infty}\frac{\cos\alpha_k y}{\alpha_k^2} = (y-1), y\in[0,1], \quad (20)$$

we obtain the velocity field

$$u(y,t) = H(t)\int_0^t g_1(y,s)ds=$$

$$H(t)\left[(y-1)+2\sum_{k=0}^{\infty}\frac{1}{\alpha_k{}^2}\cos(\alpha_k y)\exp\left(-\frac{\alpha_k^2 t}{1+\beta\alpha_k^2}\right)\right], \quad (21)$$

The velocity given by Eq. (19) has the following temporal limits

$$\lim_{t\to 0^+} u(y,t) = 0,\ \lim_{t\to\infty} u(y,t) = y-1, \quad (22)$$

In the case $\beta = 0$, we recover the velocity field

$$u_N(y,t) = H(t)\left[(y-1)+2\sum_{k=0}^{\infty}\frac{1}{\alpha_k{}^2}\cos(\alpha_k y)\exp\left(-\alpha_k^2 t\right)\right], \quad (23)$$

corresponding to Newtonian fluid given in [29, Eq. (21)] The same limits can also be obtained using the known relations $\lim_{t\to 0^+} u(y,t) = \lim_{q\to\infty} q\bar{u}(y,q)$ and $\lim_{t\to\infty} u(y,t) = \lim_{q\to 0} q\bar{u}(y,q)$.

As a result from (22), we have that the velocity u(y,t) does not exhibit a jump of discontinuity at $t = 0$ and, for $t\to\infty$ it reduces to the ``stationary solution" $u_s = y-1$.

In Fig. 2. we plotted the velocity field $u(y,t)$ given by Eq. (21), versus y for $t = \{0.3, 0.5, 2.2\}$ and versus t for $y = \{0, 0.5, 0.8\}$. For a given value of t, the velocity is an increasing function with respect to y. It is clear that for a given value of y, the velocity $u(y,t)$ decreases as function

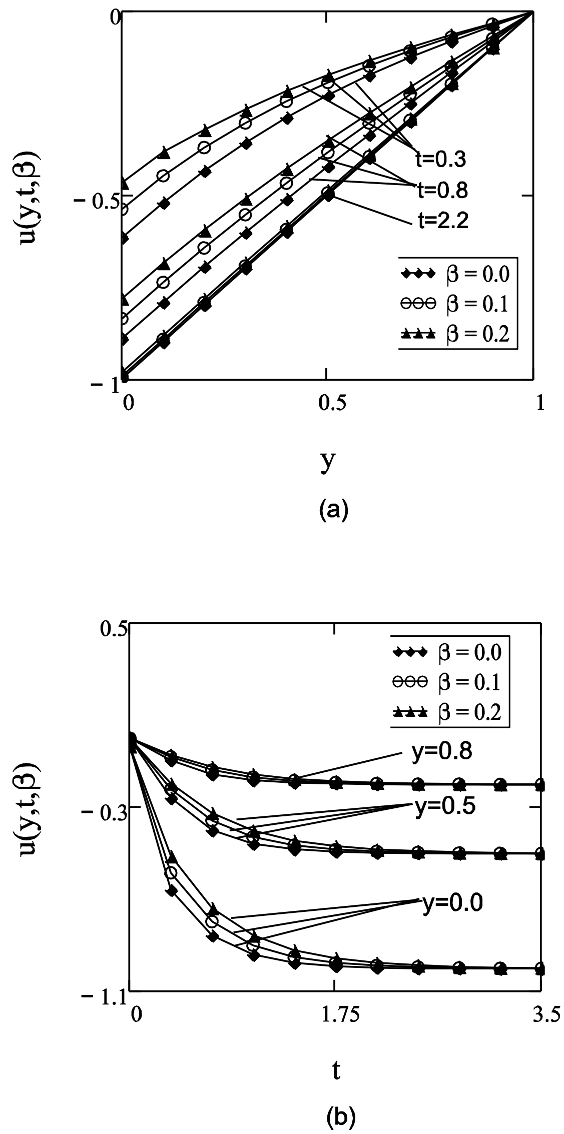


Figure 2: Profile of the velocity field versus (Fig.2a) and versus (Fig.2b) corresponding to the Eq. (21) for constant tension on the wall

of t and tends to the ``stationary velocity" $u_s = y-1$ for increasing t. The influence of the parameter β on the fluid motion is also shown in Figs. 2 (a) and (b). As expected, Figs. 2 show that velocity is a decreasing function with respect to β in absolute value, One can also see from these figures that second grade fluid is slower than Newtonian fluid, in absolute value. It is observed that non-Newtonian effects vanish in time, after some time both fluids show same behavior.

Finally, we determine the volume flux

$$Q(t) = \int_0^1 u(y,t)dy$$
$$= H(t)\left\{-\frac{1}{2} + 2\sum_{k=0}^{\infty} \frac{(-1)^k}{\alpha_k^{\ 3}} \exp\left(-\frac{\alpha_k^2 t}{1+\beta\alpha_k^2}\right)\right\}. \quad (24)$$

corresponding to second grade fluid, respectively and volume flux

$$Q_N(t) = H(t)\left\{-\frac{1}{2} + 2\sum_{k=0}^{\infty} \frac{(-1)^k}{\alpha_k^{\ 3}} \exp\left(-\alpha_k^2 t\right)\right\}. \quad (25)$$

corresponding to constant stress on the lower plate obtained by Nazish *et al.* [29 (Eq.(24))] for Newtonian fluids.

5 SINUSOIDAL TENSION ON THE BOTTOM PLATE

Let us now consider $f(t) = \sin(\omega t)$, $\omega > 0$ being the frequency of oscillations of the tension on the lower plate. Using Eq. (8) we obtain, after dropping the ``$*$" notation,

$$g(t) = \sin(\Omega t), \ \Omega = \frac{h^2\omega}{\nu}, \quad (26)$$

Using Eqs. (15), (18) and the convolution theorem we obtain the exact (y,t)-domain solution of the set of equations (10)-(12) given by

$$u(y,t) = (g * g_1)(t) = \int_0^t g(t-s) g_1(y,s) ds, \quad (27)$$

we obtain the velocity field

$$u(y,t) = \int_0^t \sin\left(\Omega t - \Omega s\right) g_1(y,s) ds$$
$$= -2\sum_{k=0}^{\infty} \frac{\cos(\alpha_k y)}{1+\beta\alpha_k^2} \int_0^t \sin\left(\Omega t - \Omega s\right) \exp\left(-\frac{\alpha_k^2 t}{1+\beta\alpha_k^2}\right) ds$$
$$= -2\sin(\Omega t)\sum_{k=0}^{\infty} \frac{\cos(\alpha_k y)\alpha_k^2}{\alpha_k^4 + \Omega^2\left(1+\beta\alpha_k^2\right)^2}$$
$$+ 2\Omega\cos(\Omega t)\sum_{k=0}^{\infty} \frac{\cos(\alpha_k y)\left(1+\beta\alpha_k^2\right)}{\alpha_k^4 + \Omega^2\left(1+\beta\alpha_k^2\right)^2}$$
$$- 2\Omega\sum_{k=0}^{\infty} \frac{\cos(\alpha_k y)\left(1+\beta\alpha_k^2\right)}{\alpha_k^4 + \Omega^2\left(1+\beta\alpha_k^2\right)^2} \exp\left(-\frac{\alpha_k^2 t}{1+\beta\alpha_k^2}\right) \quad (28)$$

The temporal limits of the velocity $u(y,t)$ given by Eq. (28) are

$$\lim_{t\to 0^+} u(y,t) = 0,$$
$$\lim_{t\to\infty} u(y,t) = u_p(y,t) = -2\sin(\Omega t)\sum_{k=0}^{\infty} \frac{\cos(\alpha_k y)\alpha_k^2}{\alpha_k^4 + \Omega^2\left(1+\beta\alpha_k^2\right)^2}$$
$$+ 2\Omega\cos(\Omega t)\sum_{k=0}^{\infty} \frac{\cos(\alpha_k y)\left(1+\beta\alpha_k^2\right)}{\alpha_k^4 + \Omega^2\left(1+\beta\alpha_k^2\right)^2} \quad (29)$$

For large values of the time t, the velocity $u(y,t)$ given by Eq. (28) reduces to the ``permanent solution" $(29)_2$.

The corresponding solution for Newtonian fluids are In addition let us give another form if the permanent velocity given by Eq. (29). For this we rewrite the Eq. (15) in the form

$$\bar{u}(y,q) = \frac{\Omega}{q^2+\Omega^2} G_1(y,q).$$

The poles of function $\bar{u}(y,q)$ are $\pm\Omega i$ and q_k given by (17). Using the residue theorem, after lengthy but straightforward computations, the exact (y,t)-domain solution is

$$u(y,t) = L^{-1}\left\{\frac{\Omega}{q^2+\Omega^2} G_1(y,q)\right\} = \frac{1}{\left(a^2+b^2\right)\left(1+\beta^2\Omega^2\right)} \times$$
$$\times \frac{A_1(y)\cos(\Omega t) + A_2(y)\sin(\Omega t)}{\left[sh^2(a) + \cos^2(b)\right]}, \quad (30)$$

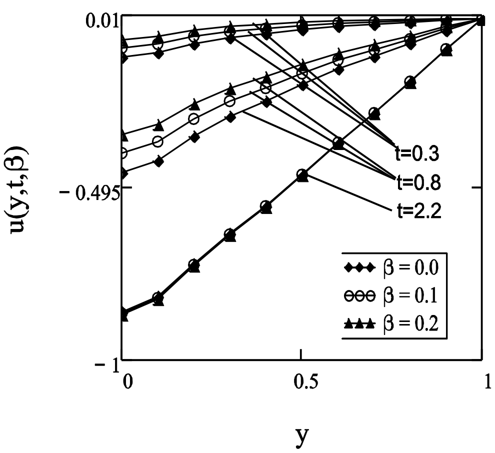
where

$$A_1(y) = A(y)(a - b\beta\Omega) - B(y)(b + a\beta\Omega),$$
$$A_2(y) = B(y)(a - b\beta\Omega) + A(y)(b + a\beta\Omega), \quad (31)$$

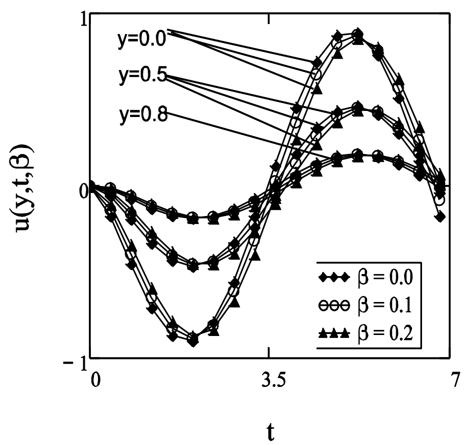
$$A(y) = ch(a)\cos(b)ch\left[a(y-1)\right]\sin\left[b(y-1)\right]$$
$$- sh(a)\sin(b)sh\left[a(y-1)\right]\cos\left[b(y-1)\right],$$
$$B(y) = ch(a)\cos(b)sh\left[a(y-1)\right]\cos\left[b(y-1)\right] \quad (32)$$
$$+ sh(a)\sin(b)ch\left[a(y-1)\right]\sin\left[b(y-1)\right],$$

and

$$a,b = \sqrt{\left(\frac{\Omega}{1+\beta\Omega}\right)\left(\frac{\pm\beta\Omega + \sqrt{\beta^2\Omega^2+1}}{2}\right)}.$$



(a)



(b)

Figure 3: Profile of the velocity field versus (Fig.3a) and versus (Fig.3b) corresponding to the Eq. (28) for sinusoidal tension on the wall

Integrating the Equation (26) with respect to $y \in [0,1]$, we obtain the volume flux

$$Q(t) = \quad = -2\sin(\Omega t)\sum_{k=0}^{\infty}\frac{(-1)^k \alpha_k}{\alpha_k^4+\Omega^2\left(1+\beta\alpha_k^2\right)^2} + 2\Omega\cos(\Omega t)\sum_{k=0}^{\infty}\frac{(-1)^k\left(1+\beta\alpha_k^2\right)}{\alpha_k[\alpha_k^4+\Omega^2\left(1+\beta\alpha_k^2\right)^2]}$$
$$-2\Omega\sum_{k=0}^{\infty}\frac{(-1)^k\left(1+\beta\alpha_k^2\right)}{\alpha_k[\alpha_k^4+\Omega^2\left(1+\beta\alpha_k^2\right)^2]}\exp\left(-\frac{\alpha_k^2 t}{1+\beta\alpha_k^2}\right) \tag{33}$$

which, for large values of time t, reduces to the "permanent flux"

$$Q(t) = -2\sin(\Omega t)\sum_{k=0}^{\infty}\frac{(-1)^k \alpha_k}{\alpha_k^4+\Omega^2\left(1+\beta\alpha_k^2\right)^2} + 2\Omega\cos(\Omega t)\sum_{k=0}^{\infty}\frac{(-1)^k\left(1+\beta\alpha_k^2\right)}{\alpha_k[\alpha_k^4+\Omega^2\left(1+\beta\alpha_k^2\right)^2]} \tag{34}$$

Some properties of the flow are revealed in Fig. 3. This figure contains diagrams of velocity, $u(y,t)$ given by Eq. (28) versus y and $t \in \{0.3, 0.8, 2.2,\}$ respectively, the diagrams of velocity $u(y,t)$ versus t for $y \in \{0.0, 0.5, 0.8\}$. The influence of the parameter β on the fluid motion is also shown in Figs. 3 (a) and (b). As expected, Figs. 3 show that velocity is an increasing function with respect to β in absolute value. From these figures, it is evident that, second grade fluid is slower than Newtonian fluid, in absolute value. It is observed that for sinusoidal motion the non-Newtonian effects also vanish in time like constant shear stress case disused in Fig. 2.

6 CONCLUSIONS

Couette flows of second grade fluids have been analyzed in the assumption that the lower plate, situated in the plane $y = 0$, applies a tangential stress to the fluid. Two particular cases, corresponding to constant and sinusoidal shear stresses on the wall, were considered. Exact expressions for the velocity $u(y,t)$ have been determined by means of the Laplace transform. Some properties of the velocity $u(y,t)$ were presented. In the case of a constant tangential tension on the bottom plate, the velocity $u(y,t)$ is an increasing function on y. For large values of the time t the velocity tends to the stationary velocity" $u_S = y-1$ and non-Newtonian effects vanish in time (See Fig. 2).

If the plate applies a sinusoidal shear stress on the fluid, the velocity $u(y,t)$ is written as a sum between the ``permanent solution" $u_p(y,t)$ and the transient solution $u_t(y,t) = u(y,t) - u_p(y,t)$. For large values of the time t, the transient velocity can be neglected and the fluid flows according to the ``permanent solution" $u_p(y,t)$. In both cases the volume flux was determined. we extracted the expressions for the velocity field corresponding to the motion of a Newtonian fluid as limiting cases of general solutions. Moreover, we compared second grade and Newtonian fluid graphically and observed that Newtonian fluid is faster than second grade fluid for both cases

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