UNSTEADY MHD FLOW AND HEAT TRANSFER FOR NEWTONIAN FLUIDS OVER AN EXPONENTIALLY STRETCHING SHEET

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ABSTRACT: *Analysis is made for MHD flow and heat transfer for viscous fluids over a stretching sheet in the presence of a uniform magnetic field that is applied perpendicular to the sheet. Similarity functions of a new exponential form have been used to convert the highly non-linear partial differential equations of motion to their ordinary differential form. The resulting equations have been solved numerically. Numerical results have been computed and presented for different values of the parameters of interest involved in present study namely stretching velocity ratio parameter, Hartmann number, Grashof number, heat source parameter and Prandtl number. The results have been presented in graphical form to elaborate the flow pattern*.

AMS Subject Classification: 76M20.
Key Words: MHD flow**,** Hartmann number, Grashof number, Prandtl number, stretching sheet

1. INTRODUCTION

Studies of fluids flow due to stretching phenomena and heat transfer have relevance to a score of important applications that occur in industry such as extrusion process and production of plastic sheets. The quality of final product can be improved with proper adjustment of cooling and heat transfer. Hence, in order to slow the rate of solidification, the application of magnetic field is useful because of its easy implementation. After the preliminary work of Sakiadas [1, 2] several important investigations have been carried out for flow referred to stretching phenomenon. Crane [3] obtained a similarity solution in closed analytical form for steady two dimensional incompressible boundary layer flow caused by the stretching of a sheet. Cortell [4, 5] examined the flow and heat transfer on a nonlinear stretching sheet for two different types of thermal boundary conditions on the sheet, namely, constant surface temperature and prescribed surface temperature. Baag et al. [6] analyzed MHD flow on a stretching sheet embedded in a porous medium. Ganji et al. [7] reported the analytical solution of the magnetohydrodynamic flow over a nonlinearly stretching sheet. Peker et al. [8] and Azimi et al. [9] have studied the flow of a conducting viscous fluid over a stretching sheet with a constant rate of stretching and the flow is subjected to variable magnetic field. Ishak et al. [10], Prasad et al. [11], Abbas and Hayat [12] and Raftari et al. [13] are, among others, who studied similar problems. Ahmad et al. [14] obtained closed form solution for a viscous, incompressible, MHD flow over a porous stretching sheet. Ali [15] studied a power law driven sheet problem.

Most of the previous work related to this study is for continuously stretching sheet problems but sometimes this phenomenon becomes insufficient to practical applications and industry needs. Magyari and Keller [16] examined the steady heat and mass transfer of two dimensional boundary layers on an exponentially stretching surface. Liu et al. [17] analyzed flow and heat transfer due to a horizontal surface stretched exponentially in two lateral directions. Al-Odat et al. [18] and Sajid and Hayat [19] studied magnetic field effect on hydromagnetic fluid and the thermal radiation effects on viscous fluids respectively, over an exponentially stretching sheet. Nadeem and Lee [20] employed the homotopy analysis method to the boundary layer flow and heat and mass transfer problem of nano-fluids over an exponentially stretching surface.

Most of the studies above deal with steady-state condition but in certain cases, the flow becomes time dependent, and consequently, it is necessary to consider the unsteady flow condition. Unsteady flow over a stretching sheet was investigated by Pop and Na [21]. Bhattacharyya et al. [22] considered unsteady MHD boundary layer flow with diffusion and first-order chemical reaction over a permeable stretching sheet with suction or blowing. Later, the heat transfer past an unsteady stretching sheet was analyzed under different physical conditions by Sharidan et al. [23], Elbashbeshy and Bazid [24], Tsai et al. [25], El-Aziz [26], Mukhopadhyay [27]. Surma Devi et al. [28] demonstrated the heat and species transport in an unsteady, three-dimensional flow caused by stretching of a flat surface. Adhikary et al. [29] discussed the unsteady two dimensional hydromagnetic flow and heat transfer of a fluid. Ali et al. [30] obtained numerical solution of MHD flow of fluid and heat transfer over porous stretching sheet. Hussain et al. [31] examined MHD stagnation point flow of micropolar fluids towards a stretching sheet.

This paper looks in to the unsteady MHD flow and heat transfer for Newtonian fluids over an exponentially stretching sheet. Effects of the parameters namely Hartmann number M, Grashof number G_r, heat source parameter N and Prandtl number P_r have been observed on velocity and temperature distributions. Section 2, presents mathematical formulation of the problem, section 3, provides results and discussion. The results for non dimensional velocity and temperature are presented in graphical form for two values of stretching velocity ratio parameter $\frac{a}{c}$.

2. FORMULATION OF THE PROBLEM

The fluid flow is considered to be unsteady, incompressible and electrically conducting. Magnetic field of strength B_0 is applied perpendicular to the stretching sheet. At time t, the fluid velocity is $\underline{V} = V(u,v)$ and temperature is T. Cartesian coordinates are used. The governing equations of fluid motion with boundary layer approximation and the energy equation take the form as follows:

$$\partial u/\partial x + \partial v/\partial y = 0, \quad (1)$$

$$\partial u/\partial t + \frac{1}{\rho} dP/dx = \upsilon \partial^2 u/\partial y^2 - \sigma B_0^2 u/\rho + g\beta(T - T_0), \quad (2)$$

$$\partial T/\partial t = \frac{K}{\rho C_p} \partial^2 T/\partial y^2 + \frac{Q'}{\rho C_p}(T - T_0). \quad (3)$$

The associated boundary conditions are:

$$u = cxe^{-\lambda^2 t},\ v = 0,$$
$$T = T_0 = e^{-\lambda^2 t} \text{ as } y = 0 \quad (4)$$
$$\text{and } u = axe^{-\lambda^2 t},\ T \to T_\infty \text{ as } y \to \infty,$$

where a and c are constant.

Using similarity transformations

$$u = cxf'(\eta)e^{-\lambda^2 t},\ v = -\sqrt{c\upsilon}\, f(\eta)e^{-\lambda^2 t}, \quad (5)$$

$$(T - T_\infty) = (T_0 - T_\infty)\theta(\eta)e^{-\lambda^2 t},\ \eta = \sqrt{\frac{c}{\upsilon}}y. \quad (6)$$

The equation of mass conservation (1) is readily satisfied and the equations (2) and (3) are respectively transformed to ordinary differential form, to yield:

$$f''' - (M^2 - \frac{\lambda^2}{c})f' + G_r\theta - h = 0, \quad (7)$$

$$\theta'' + (N + P_r\lambda^2\upsilon)\theta = 0, \quad (8)$$

where $G_r = g\beta(T_0 - T_\infty)/c^2 x$ is Grashof number, $h = \frac{dP}{dx}/c^2 x\rho e^{-\lambda^2 t}$ is pressure gradient, $M = \sqrt{\frac{\sigma B_0^2}{\rho c}}$ is Hartmann number, $N = \frac{Q'\upsilon}{Kc}$ is heat source parameter and $P_r = \rho c_p / Kc$ is the Prandtl number.

Taking $R = N + P_r\lambda^2\upsilon$ then equation (8) becomes:

$$\theta'' + R\theta = 0. \quad (9)$$

The boundary conditions in equations (5) and (6) take the form as:

$$f'(0) = 1,\ f(0) = 0,\ \theta(0) = e \text{ as } \eta \to 0, \quad (10)$$

$$f'(\infty) = \frac{a}{c},\ \theta(\infty) = 0 \text{ as } \eta \to 1. \quad (11)$$

3. RESULTS AND DISCUSSION

The ordinary differential equations (7) and (9) with boundary conditions (10) and (11) have been solved numerically using Mathematica software version 6.0. The effects of the parameters M, N, G_r and P_r are presented for velocity and temperature distributions. However, for computational process, fixed values of the parameters are chosen arbitrarily as $M = 1,\ N = 1,$ $G_r = 0.5,\ h = 0.5,\ .\ t = 1,\ \lambda = 1,\ P_r = 1,\ \upsilon = 0.5$

Fig. 1(a) to fig. 1(d) demonstrate the effects of M, N, P_r and G_r on velocity component f' when $\frac{a}{c} = 5 > 1$. It is noticed that velocity decreases against M but increases with increasing values of N, P_r and G_r. Similar effects have been observed in fig. 2(a) to fig. 2(b) when $\frac{a}{c} = 0.1 < 1$. But the flow pattern is different in two cases, also, the impact of the parameters is stronger (when $\frac{a}{c} = 0.1$). Fig.3(a) shows the plots of temperature function $\theta(\eta)$ versus heat source parameter N. The fluid temperature rises rapidly with increase in the values of N. Fig. 3(b) also presents the pattern of temperature distribution with variation of P_r. Temperature increases with P_r.

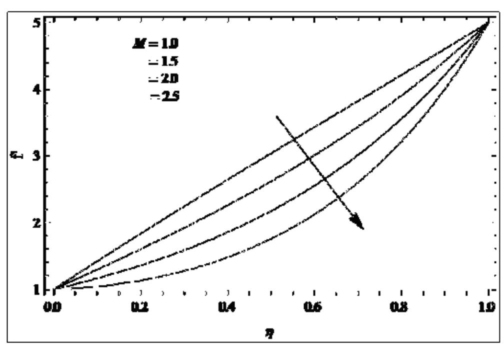

Fig. 1(a): Graph of f' for different values of M.

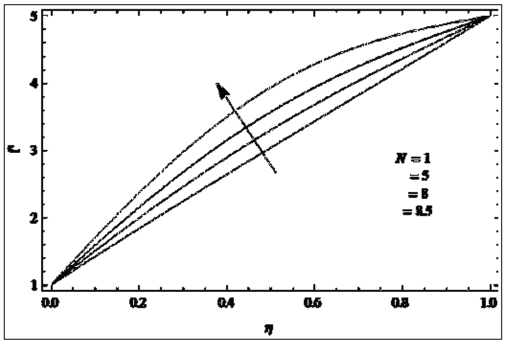

Fig. 1(b): Graph of $f^{❖}$ **for different values of** N **.**

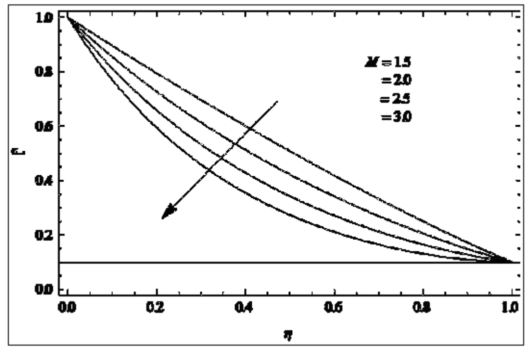

Fig. 2(a) Graph of $f^{❖}$ **for different values of** M **.**

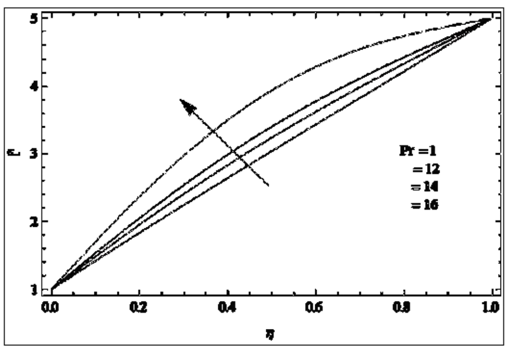

Fig. 1(c):Graph of $f^{❖}$ **for different values of** P_r **.**

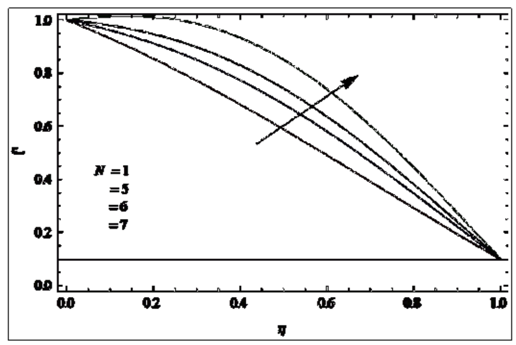

Fig. 2(b) Graph of $f^{❖}$ **for different values of** N **when** $\frac{a}{c}$ ■0.1 **.**

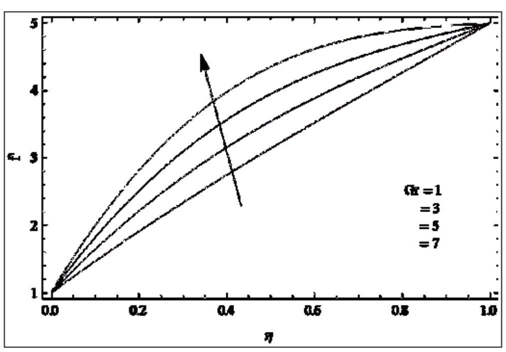

Fig. 1(d): Graph of $f^{❖}$ **for different values of** G_r **.**

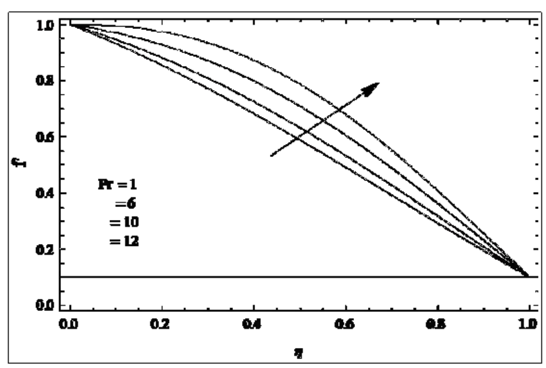

Fig. 2(c) Graph of f' **for different values of** P_r **when** $\frac{a}{c} = 5$**.**

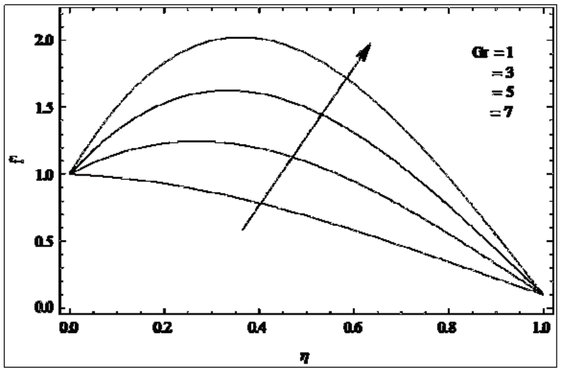

Fig. 2(d) Graph of f' for different values of G_r when $\frac{a}{c} = 0.1$.

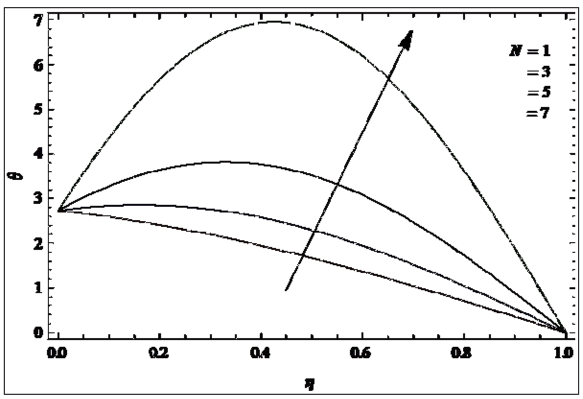

Fig. 3(a) Graph of θ for different values of N .

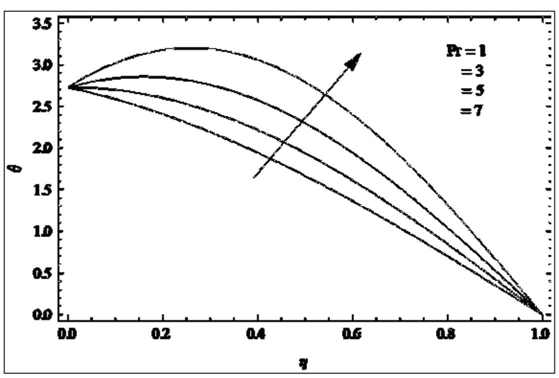

Fig. 3(b) Graph of θ for different values of P_r .

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