COMPARISON OF ABOODH TRANSFORMATION AND DIFFERENTIAL TRANSFORMATION METHOD NUMERICALLY

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ABSTRACT: *Differential Transformation gives semi analytical numerical solution. These methods are capable of reducing the calculation and works efficiently. In this paper we use Aboodh Transformation Method (ATM) and Differential Transformation Method (DTM) to get a Numerical solution of the system of linear ordinary differential equation. We compare the results to see which Transformation converges faster to true solution .we also presented the results with relative error.*

Keywords: Taylor expansion, Fourier integral, Aboodh Transform Method, Differential Transform Method

INTRODUCTION

Differential Transform method was first proposed and applied to solve linear and nonlinear initial value problems in electric circuit analysis by X.Zhou [1]. Chen and Liu have applied this method to solve two boundary value problems [2].Jan, Chen and Liu apply the two dimensional DTM to solve partial differential equation [3]. Yu and Chen apply the DTM for the optimization of the rectangular fins with variable thermal parameters [4, 5]. The high order Taylor series method which required a lot of symbolic computations. DTM is an iterative procedure for obtaining Taylor series solutions. The DTM is a numerical method based on a Taylor expansion .This method constructs an analytical solution in the form of a polynomial. This method is easy to handle and compute the work in less time even when apply to nonlinear or parameter varying systems. But, it is different from Taylor series method that requires computation of the high order derivatives. The DTM is an iterative procedure that is described by the transformed equation of original functions for solutions of differential equations. In recent year, many papers were devoted to the problem of approximate solution of system of differential equation. The implementation of the DTM [2, 4, 6, 7] amongst others has shown reliable results to solve ordinary differential equations, partial differential equations, Blasius equation, nonlinear fractional differential equations and delay differential equations. Aboodh Transform is derived from the classical Fourier integral. Based on the mathematical simplicity of the Aboodh Transform and its fundamental properties, Aboodh Transform was introduced by Khalid Aboodh in 2013, to facilitate the process of solving ordinary and partial differential equations in the time domain. This transformation has deeper connection with the Laplace and Elzaki Transform. [8, 9].

2 Aboodh Transform Method (ATM)

The Aboodh Transform Method over a set of function define as

$\mathbb{A} = \{f(t): \exists\, M, k_1, k_2 > 0, |f(t)| < Me^{-vt}\}$ By formula $A\{f(t)\} = K(v) = \frac{1}{v}\int_0^\infty e^{-vt} f(t)dt$ where M, k_1, k_2 are constants, M is finite but k_1, k_2 may or may not finite and $k_1 \le v \le k_2$

Application 1: Consider the System of Differential equation

$$\frac{dx}{dt} = 2x - 3y, \quad \frac{dy}{dt} = y - 2x$$
$$x(0) = 8, y(0) = 3$$

Solution:-

Given system of ordinary differential equation is

$$\frac{dx}{dt} = 2x - 3y, \quad \frac{dy}{dt} = y - 2x$$

By Applying Aboodh transformation method

$$A\left\{\frac{dx}{dt}\right\} = A\{2x - 3y\}$$
$$\Longrightarrow A\left\{\frac{dx}{dt}\right\} = 2A\{x\} - 3A\{y\}$$
$$v\,x(v) - \frac{x(0)}{v} = 2\,x(v) - 3\,y(v)$$
$$\Longrightarrow v\,x(v) - 2\,x(v) = \frac{8}{v} - 3\,y(v)$$
$$(v-2)x(v) = \frac{8}{v} - 3y(v)$$
$$\Rightarrow x(v) = \frac{8}{v(v-2)} - 3\frac{y(v)}{(v-2)} \quad (1)$$
$$A\left\{\frac{dy}{dt}\right\} = A\{y - 2x\}$$
$$\Longrightarrow A\left\{\frac{dy}{dt}\right\} = A\{y\} - 2A\{x\}$$
$$v\,y(v) - \frac{y(0)}{v} = y(v) - 2x(v)$$
$$\Longrightarrow v\,y(v) - y(v) = \frac{3}{v} - 2\,x(v)$$
$$(v-1)y(v) = \frac{3}{v} - 2\,x(v)$$
$$\Rightarrow y(v) = \frac{3}{v(v-1)} - 2\,\frac{x(v)}{(v-1)} \quad (2)$$

Using (2) in (1) we have

$$x(v) = \frac{8v - 17}{v(v+1)(v-4)} = 5\left(\frac{1}{v+v^2}\right) + 3\left(\frac{1}{v^2 - 4\,v}\right)$$

And after applying Inverse Aboodh Transform, we have

$$x(t) = 5\,e^{-t} + 3\,e^{4t}$$

Also

$$y(v) = 5\left(\frac{1}{v+v^2}\right) - 2\left(\frac{1}{v^2 - 4\,v}\right)$$

And after applying Inverse Aboodh Transform, we have

$$y(t) = 5\,e^{-t} - 2e^{4t}$$

Table. 1
Values of x (t) and y (t) using Aboodh transformation

t	x(t)	y(t)
0.1	8.999661183103608	1.540537694897257
0.2	10.770276550867314	-0.357428091595026
0.3	13.664441871618232	-2.936142742064507
0.4	18.210697503363541	-6.554464618612033
0.5	25.199821595355118	-11.745458899298134
0.6	35.813587322394937	-19.302294580813069
0.7	51.816866832248216	-30.406367023237063
0.8	75.844235411914170	-46.818415573632599
0.9	111.827551629736960	-71.163620588652975
1	165.633847305289920	-107.356902860431260

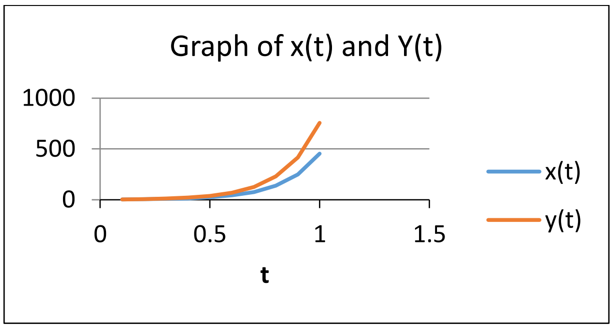

Figure 1

Application 2:

Consider the system of differential equations

$$\frac{dx}{dt} = x + 3y \quad , \quad \frac{dy}{dt} = 5x + 3y$$

$$x(0) = 2 \quad , \quad y(0) = 1$$

Solution : Given system of ordinary differential equations

$$\frac{dx}{dt} = x + 3y \quad , \quad \frac{dy}{dt} = 5x + 3y$$

By applying Aboodh Transformation method

$$A\left\{\frac{dx}{dt}\right\} = A\{x + 3y\} \Longrightarrow A\left\{\frac{dx}{dt}\right\} = A\{x\} + 3A\{y\}$$

$$v\,x(v) - \frac{x(0)}{v} = x(v) + 3\,y(v)$$

$$\Longrightarrow v\,x(v) - x(v) = \frac{2}{v} + 3\,y(v)$$

$$(v-1)x(v) = \frac{2}{v} + 3y(v) \Rightarrow$$

$$x(v) = \frac{2}{v(v-1)} + 3\frac{y(v)}{(v-1)} \qquad (3)$$

$$A\left\{\frac{dy}{dt}\right\} = A\{3y + 5x\} \Longrightarrow A\left\{\frac{dy}{dt}\right\} = 3A\{y\} + 5A\{x\}$$

$$v\,y(v) - \frac{y(0)}{v} = 3y(v) + 5x(v)$$

$$\Longrightarrow v\,y(v) - 3y(v) = \frac{1}{v} + 5\,x(v)$$

$$(v-3)y(v) = \frac{1}{v} + 5\,x(v)$$

$$\Rightarrow y(v) = \frac{1}{v(v-3)} + 5\,\frac{x(v)}{(v-3)} \qquad (4)$$

Using (4) in (3) we have

$$x(v) = \frac{2v-3}{v(v-6)(v+2)} = \frac{9}{8}\left(\frac{1}{v^2-6v}\right) + \frac{7}{8}\left(\frac{1}{v^2+2\,v}\right)$$

And after applying Inverse Aboodh Transform, we have

$$x(t) = \frac{9}{8}\,e^{6t} + \frac{7}{8}\,e^{-2t}$$

Also

$$y(v) = \frac{15}{8}\left(\frac{1}{v^2-6v}\right) - \frac{7}{8}\left(\frac{1}{v^2+2v}\right)$$

And after applying Inverse Aboodh Transform, we have

$$y(t) = \frac{15}{8}\,e^{6t} - \frac{7}{8}e^{-2t}$$

Table 2.
Values of x (t) and y (t) using Aboodh transformation

t	x(t)	y(t)
0.1	2.766273059382557	2.700083341788971
0.2	4.321661578359802	5.638689189849844
0.3	7.286063579046839	10.862878814192005
0.4	12.794236271824374	20.275292870100440
0.5	22.918123549611138	37.338487219951865
0.6	41.436558684560900	68.358144646473022
0.7	75.237894764489695	124.821098358285770
0.8	136.875879161822160	227.655373394382740
0.9	249.226854756904460	414.992393855657040
1	453.975811052159030	756.310569426046300

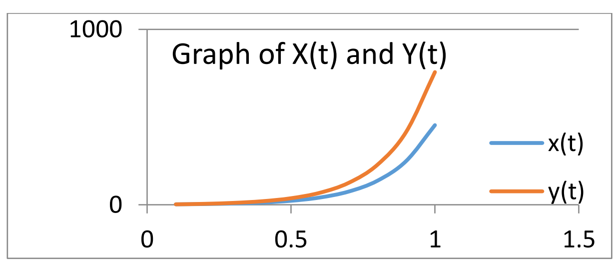

Figure 2.

Application of DTM

Application 1a

Consider the system of differential equation

$$\frac{dx}{dt} = 2x - 3y, \frac{dy}{dt} = y - 2x$$

$$x(0) = 8, y(0) = 3$$

Solution:-

Given system of ordinary differential equation is

$$\frac{dx}{dt} = 2x - 3y, \quad \frac{dy}{dt} = y - 2x$$

By applying differential transformation method,

$$\left.\begin{aligned} (k+1)X(k+1) &= 2X(k) - 3Y(k) \\ (k+1)Y(k+1) &= Y(k) - 2X(k) \end{aligned}\right\} \quad (5)$$

By applying differential transformation on the initial conditions, $X(0) = 8$, $Y(0) = 3$

Now (5) implies that

$$\left.\begin{aligned} X(k+1) &= \frac{1}{k+1}[2X(k) - 3Y(k)] \\ Y(k+1) &= \frac{1}{k+1}[Y(k) - 2X(k)] \end{aligned}\right\} \; ; \; k = 0,1,2,3, \dots$$

For $k = 0$ $X_1 = 7$, $Y_1 = -13$

For $k = 1$ $X_2 = \frac{53}{7}$, $Y_2 = -\frac{27}{2}$

For $k = 2$ $X_3 = \frac{187}{6}$, $Y_3 = -\frac{133}{6}$

For $k = 3$ $X_4 = \frac{773}{24}$, $Y_4 = -\frac{169}{8}$

For $k = 4$ $X_5 = \frac{3067}{120}$, $Y_5 = -\frac{2053}{120}$

For $k = 5$ $X_6 = \frac{12293}{720}$, $Y_6 = -\frac{2729}{240}$

For $k = 6$ $X_7 = \frac{7021}{720}$, $Y_7 = -\frac{32773}{5040}$

For $k = 7$ $X_8 = \frac{196613}{40320}$, $Y_8 = -\frac{14563}{4480}$

For $k = 8$ $X_9 = \frac{786427}{362880}$, $Y_9 = -\frac{74899}{51840}$

For $k = 9$ $X_{10} = \frac{3145733}{3628800}$, , $Y_{10} = -\frac{699049}{1209600}$

For $k = 10$ $X_{11} = \frac{12582907}{39916800}$, $Y_{11} = -\frac{8388613}{39916800}$

For $k = 11$ $X_{12} = \frac{50331653}{479001600}$, $Y_{12} = -\frac{11184809}{159667200}$

For $k = 12$ $X_{13} = \frac{2614631}{80870400}$, $Y_{13} = -\frac{10324441}{479001600}$

For $k = 13$ $X_{14} = \frac{805306373}{87178291200}$, $Y_{14} = -\frac{59652323}{9686476800}$

For $k = 14$ $X_{15} = \frac{3221225467}{1307674368000}$, $Y_{15} = -\frac{43826197}{26687232000}$

For $k = 15$ $X_{16} = \frac{12884901893}{20922789888000}$,

$Y_{16} = -\frac{220254733}{536481792000}$ Up to so on

Now by applying the inverse differential transform $x(t) = \sum_{k=0}^{31} X(k)\, t^k$

$$x(t) = 8 + 7t + \frac{53}{2}t^2 + \frac{187}{6}t^3 + \frac{773}{24}t^4 + \frac{3067}{120}t^5 + \frac{12293}{720}t^6 + \frac{7021}{720}t^7 + \frac{196613}{40320}t^8 + \frac{786427}{362880}t^9 + \frac{3145733}{3628800}t^{10} + \frac{12582907}{39916800}t^{11} + \frac{50331653}{479001600}t^{12} + \frac{2614631}{80870400}t^{13} + \frac{805306373}{87178291200}t^{14} + \frac{3221225467}{130767436800}t^{15} + \frac{12884901893}{20922789888000}t^{16} + \frac{51539607547}{355687428096000}t^{17} + \frac{206158430213}{6402373705728000}t^{18} + \frac{6929695133}{1022227734528000}t^{19} + \frac{3298534883333}{2432902008176640000}t^{20} + \frac{13194139533307}{51090942171709440000}t^{21} + \frac{52776558133253}{1124000727777607680000}t^{22} + \frac{1128910334401}{138246078817566720000}t^{23} + \frac{4443417375367}{326151790385915494400000}t^{24} + \frac{482528531503981}{221588714904728371200000}t^{25} + \frac{13510798882111493}{403291461126605635584000000}t^{26} + \frac{3179011501673291}{640251732377550127104000000}t^{27} + \frac{216172782113783813}{30488834461171386050150400000}t^{28} + \frac{864691128455135227}{8841761993739701954543616000000}t^{29} + \frac{3458764513820540933}{265252859812191058636308480000000}t^{30} + \frac{116260992061194653}{69099484488890107711979520000000s}t^{31} + \cdots \quad (6)$$

By using (6) and giving values of t, we obtain

Table 3.

For $x(t)$ by Differential Transformation in application 1a

t	$x(t)$ by DTM
0.1	8.999661183103610
0.2	10.770276550867300
0.3	13.664441871618200
0.4	18.210697503363500
0.5	25.199821595355100
0.6	35.813587322395000
0.7	51.816866832248200
0.8	75.844235411914300
0.9	111.827551629737000
1	165.633847305290000

Table 3a.:

Comparison Table for $x(t)$ in application 1a , 1

COMPARISON TABLE FOR $x(t)$			
t	DTM	Aboodh	Error
0.1	8.99966118310 3610	8.99966118310 3608	0.0000000000 00000
0.2	10.7702765508 67300	10.7702765508 67314	0.0000000000 00000
0.3	13.6644418716 18200	13.6644418716 18232	0.0000000000 00000
0.4	18.2106975033 63500	18.2106975033 63541	0.0000000000 00000
0.5	25.1998215953 55100	25.1998215953 55118	0.0000000000 00000
0.6	35.8135873223 95000	35.8135873223 94937	0.0000000000 00099
0.7	51.8168668322 48200	51.8168668322 48216	0.0000000000 00000
0.8	75.8442354119 14300	75.8442354119 14170	0.0000000000 00199
0.9	111.827551629 737000	111.827551629 736960	0.0000000000 00995
1	165.63384730 5290000	165.633847305 289920	0.0000000000 00995

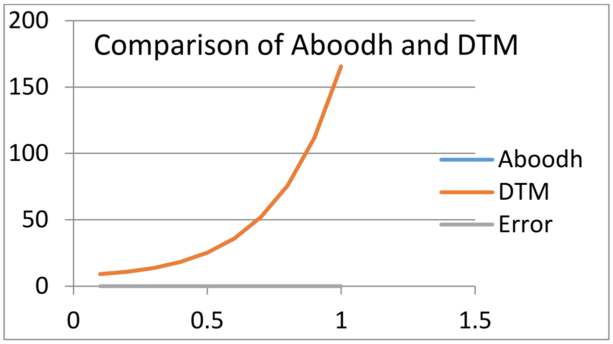

Figure 3.

Also by applying inverse differential transform

$$y(t) = \sum_{k=0}^{32} Y(k)\, t^k$$

$$\mathrm{y}\,(\mathrm{t}) = 3 - 13\mathrm{t} - \frac{27}{2}t^2 - \frac{133}{6}t^3 - \frac{169}{8}t^4 - \frac{2053}{120}t^5 - \frac{2729}{240}t^6 - \frac{32773}{5040}t^7 - \frac{14563}{4480}t^8 - \frac{74899}{51840}t^9 - \frac{699049}{1209600}t^{10} - \frac{8388613}{39916800}t^{11} - \frac{11184809}{159667200}t^{12} - \frac{10324441}{479001600}t^{13} - \frac{59652323}{9686476800}t^{14} - \frac{43826197}{26687232000}t^{15} - \frac{220254733}{536481792000}t^{16} - \frac{34359738373}{355687428096000}t^{17} - \frac{45812984489}{2134124568576000}t^{18} - \frac{3844446251}{850665037824000}t^{19} - \frac{81445305761}{90107481784320000}t^{20} - \frac{66136037761}{384142422343680000}t^{21} - \frac{902163386893}{28820531481477120000}t^{22} - \frac{140737488355333}{25852016738884976640000}t^{23} - \frac{187649984473769}{206816133911079813120000}t^{24} - \frac{173215370283481}{1193170003333152768000000}t^{25} - \frac{1000799917193443}{44810162347400626176000000}t^{26} - \frac{5146971002709139}{1555552778631193165824000000}t^{27} - \frac{3695261232714253}{7817649861838816935936000000}t^{28} - \frac{52405522936674863}{803796544885427450413056000000}t^{29} - \frac{768614336404564649}{884176199373970195454361600000000}t^{30} - \frac{709490156681136601}{632526050321378678286581700000000}t^{31} \quad (7)$$

By using (7) and giving values of t we obtain

Table 4.
For $y(t)$ by Differential Transformation in Applcation 1a

t	$y(t)$ by DTM
0.1	1.540537694897260
0.2	-0.357428091595026
0.3	-2.936142742064510
0.4	-6.554464618612040
0.5	-11.745458899298100
0.6	-19.302294580813100
0.7	-30.406367023237100
0.8	-46.818415573632600
0.9	-71.163620588653100
1	-107.356902860432000

Table 4a.

COMPARISON TABLE FOR $y(t)$ in Application 1a, 1			
t	DTM	Aboodh	Error
0.1	1.540537694897260	1.540537694897250	0.000000000000010
0.2	-0.357428091595026	-0.357428091595026	0.000000000000000
0.3	-2.936142742064510	-2.936142742064500	0.000000000000010
0.4	-6.554464618612040	-6.554464618612030	0.000000000000010
0.5	-11.745458899298100	-11.745458899298100	0.000000000000000
0.6	-19.302294580813100	-19.302294580813000	0.000000000000099
0.7	-30.406367023237100	-30.406367023237000	0.000000000000099
0.8	-46.818415573632600	-46.818415573632500	0.000000000000099
0.9	-71.163620588653100	-71.163620588652900	0.000000000000199
1	-107.356902860432000	-107.356902860431000	0.000000000000995

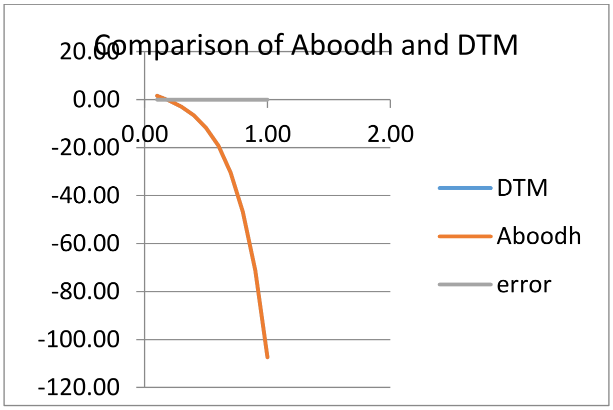

Figure 4.

Application No. 2a

Consider the system of differential equations

$$\frac{dx}{dt} = x + 3y \quad , \quad \frac{dy}{dt} = 5x + 3y$$

$$x(0) = 2 \quad , \quad y(0) = 1$$

Solution

Given system of ordinary differential equations is

$$\frac{dx}{dt} = x + 3y \quad , \quad \frac{dy}{dt} = 5x + 3y$$

By applying differential transformation method

$$\left.\begin{aligned}(k+1)X(k+1) - X(k) - 3Y(k) &= 0 \\ (k+1)Y(k+1) - 5X(k) - 3Y(k) &= 0\end{aligned}\right\} \quad (8)$$

By applying differential transformation on the initial conditions,

X (0) = 2 , Y (0) = 1 Now (8) implies that

$$\left.\begin{aligned}X(k+1) &= \frac{1}{k+1}[X(k) + 3Y(k)] \\ Y(k+1) &= \frac{1}{k+1}[5X(k) + 3Y(k)]\end{aligned}\right\} \quad ; k = 0,1,2, \ldots$$

For $k = 0$ $X_1 = 5$, $Y_1 = 13$

For $k = 1$ $X_2 = 22$, $Y_2 = 32$

For $k = 2$ $X_3 = \frac{118}{3}$, $Y_3 = \frac{206}{3}$ For

$k = 3$ $X_4 = \frac{184}{3}$, $Y_4 = \frac{302}{3}$

For $k = 4$ $X_5 = \frac{218}{3}$, $Y_5 = \frac{1826}{15}$ For

$k = 5$ $X_6 = \frac{3284}{45}$, $Y_6 = \frac{5464}{45}$

For $k = 6$ $X_7 = \frac{19676}{315}$, $Y_7 = \frac{32812}{315}$

For $k = 7$ $X_8 = \frac{14764}{315}$, $Y_8 = \frac{24602}{315}$

For $k = 8$ $X_9 = \frac{17714}{567}$, $Y_9 = \frac{147626}{2835}$

For $k = 9$ $X_{10} = \frac{265724}{14175}$, $Y_{10} = \frac{442864}{14175}$

For $k = 10$ $X_{11} = \frac{1594316}{155925}$, $Y_{11} = \frac{2657212}{155925}$ Up to so on

Now by applying the inverse differential transform

$$x(t) = \sum_{k=o}^{39} X(k)t^k$$

$$x(t)=2+5t+22t^2 + \frac{118}{3}t^3 + \frac{184}{3}t^4 + \frac{218}{3}t^5 + \frac{3284}{45}t^6 + \frac{19676}{315}t^7 + \frac{14764}{315}t^8 + \frac{17714}{567}t^9 + \frac{265724}{14175}t^{10} + \frac{1594316}{155925}t^{11} + \frac{217408}{42525}t^{12} + \frac{573956}{243243}t^{13} + \frac{43046728}{42567525}t^{14} + \frac{258280312}{638512875}t^{15} + \frac{96855124}{638512875}t^{16} + \frac{116226146}{2170943775}t^{17} + \frac{1743392204}{97692469875}t^{18} + \frac{10460353196}{1856156927625}t^{19} + \frac{15690529808}{9280784638125}t^{20} + \frac{18828635764}{38979295480125}t^{21} + \frac{25675412408}{194896477400625}t^{22} + \frac{1694577218872}{49308808782358125}t^{23} + \frac{1270932914168}{147926426347074375}t^{24} + \frac{89712911588}{43507772455021875}t^{25} + \frac{994643150216}{2090264720121703125}t^{26} + \frac{137260754729752}{1298054391195577640625}t^{27} + \frac{205891132094656}{9086380738369043484375}t^{28} + \frac{247069358513576}{52701008282540452209375}t^{29} + \frac{3706040377703696}{3952575621190533915703125}t^{30} + \frac{2223624226622064}{1225298442569065513867968 75}t^{31} + \cdots \quad (9)$$

By using (9) and giving values of t, we obtain

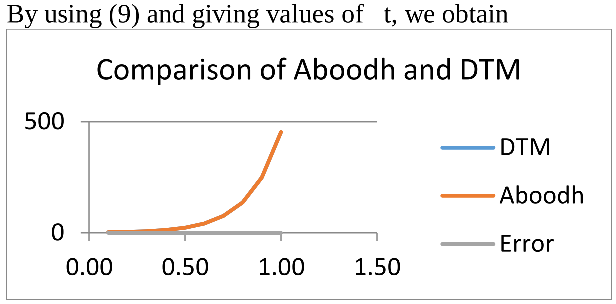

Figure 5.

Now by applying the inverse differential transform

$$y(t) = \sum_{k=0}^{39} Y(k)t^k$$

$$y(t)=1+13t+32t^2 + \frac{206}{3}t^3 + \frac{302}{3}t^4 + \frac{1826}{15}t^5 + \frac{5464}{45}t^6 + \frac{32812}{315}t^7 + \frac{24602}{315}t^8 + \frac{147626}{2835}t^9 + \frac{442864}{14175}t^{10} + \frac{2657212}{155925}t^{11} + \frac{3985804}{467775}t^{12} + \frac{1839604}{467775}t^{13} + \frac{71744528}{42567525}t^{14} + \frac{39133384}{58046625}t^{15} + \frac{161425202}{638512875}t^{16} + \frac{968551226}{10854718875}t^{17} + \frac{2905653664}{97692469875}t^{18} + \frac{1341070924}{142781302125}t^{19} + \frac{26150883004}{9280784638125}t^{20} + \frac{156905298052}{194896477400625}t^{21} + \frac{470715894128}{2143861251406875}t^{22} + \frac{2824295364824}{49308808782358125}t^{23} + \frac{2118221523604}{147926426347074375}t^{24} + \frac{88876427564}{25861263347390625}t^{25} + \frac{38127987424928}{48076088562799171875}t^{26} + \frac{228767924549624}{1298054391195577640625}t^{27} + \frac{343151886824408}{9086380738369043484375}t^{28} + \frac{12111243064391 2}{155002965536883682968 75}t^{29} + \frac{6176733962839456}{3952575621190533915703125}t^{30} + \frac{2850800290541296}{94253726351466577989843 75}t^{31} \quad (10)$$

using (10) and giving values of t, we obtain in table 6.

Table 5.
For $x(t)$ by differential Transformation in Application 2a

t	x(t) by DTM
0.1	2.766273059382560
0.2	4.321661578359800
0.3	7.286063579046840
0.4	12.794236271824400
0.5	22.918123549611200
0.6	41.436558684561000
0.7	75.237894764489800
0.8	136.875879161822000
0.9	249.226854756905000
1	453.975811052161000

Table 5a.

COMPARISON TABLE FOR $x(t)$ in Application 2a , 2			
t	DTM	Aboodh	Error
0.1	2.766273059382560	2.766273059382557	0.000000000000010
0.2	4.321661578359800	4.321661578359802	0.000000000000000
0.3	7.286063579046840	7.286063579046839	0.000000000000010
0.4	12.794236271824400	12.794236271824374	0.000000000000099
0.5	22.918123549611200	22.918123549611138	0.000000000000099
0.6	41.436558684561000	41.436558684560900	0.000000000000099
0.7	75.237894764489800	75.237894764489695	0.000000000000199
0.8	136.875879161822000	136.875879161822160	0.000000000000000
0.9	249.226854756905000	249.226854756904460	0.000000000000995
1	453.975811052161000	453.975811052159030	0.000000000002046

Table 6.
For $y(t)$ by Differential Transformation in application 2a

t	$y(t)$by DTM
0.1	2.700083341788970
0.2	5.638689189849850
0.3	10.862878814192000
0.4	20.275292870100500
0.5	37.338487219951900
0.6	68.358144646473200
0.7	124.821098358286000
0.8	227.655373394384000
0.9	414.992393855659000
1	756.310569426050000

Table 6a.

COMPARISON TABLE FOR $y(t)$ in Application 2a , 2			
t	DTM	Aboodh	Error
0.1	2.700083341788970	2.700083341788971	0.000000000000000
0.2	5.638689189849850	5.638689189849844	0.000000000000010
0.3	10.862878814192000	10.862878814192005	0.000000000000000
0.4	20.275292870100500	20.275292870100440	0.000000000000099
0.5	37.338487219951900	37.338487219951865	0.000000000000099

0.6	68.358144646473200	68.358144646473022	0.000000000000213
0.7	124.821098358286000	124.821098358285770	0.000000000000995
0.8	227.655373394384000	227.655373394382740	0.000000000001990
0.9	414.992393855659000	414.992393855657040	0.000000000001990
1	756.310569426050000	756.310569426046300	0.000000000004093

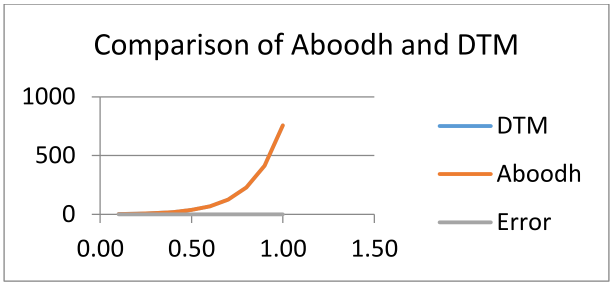

Figure 6.

CONCLUSION

In this paper, solved system of differential equations by two different transformations, one is Aboodh Integral Transformations and the other one is Differential transformation; and solutions obtained by these two transformations are compared. Solution obtained by DTM is series solutions; on the other hand Aboodh integral Transformation gives exact solution. In this paper, it is found that solutions by DTM converge rapidly. Results are compared in tabular form as well as with graph. It is found that results are approximately near to the exact solution. Hence DTM is a reliable, effective tool for the solution of system of differential equations.

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