RANDOM COINCIDENCE POINTS FOR MULTI-VALUED NON-LINEAR CONTRACTIONS IN PARTIALLY ORDERED METRIC SPACES

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ABSTRACT: The aim of this work is to derive the results for random coincidence points for multivalued nonlinear contractions in partially ordered metric spaces. We do it from two different approaches, the first is Δ *-symmetric property and the second is by using* g *-mixed monotone property. These results are the random versions of Hussain and Alotabi [Fixed Point Theory and Appl.,2011, 2011:82]. The present theorems extend certain results due to Ciric, Samet and Vetro.*

Key Words: Partially ordered set, Δ -symmetric property, mixed g -monotone property, Compatible maps, Couple random coincidence point.

AMS 2010 Subject Classification: Primary 47H10, Secondary 54H25

1. INTRODUCTION

Random fixed point theorems are stochastic generalization of classical fixed point theorems. Random fixed point theorems for contraction mappings on separable complete metric spaces were first proved by Spacek [23] and Hans [5,6]. The survey article by Bharucha-Ried [1] attereched the attention of several mathematicians (see Zhang and Huang [26], Hans [5,6], Huang [7], Itoh [10], Lin [14], Papageorgiou [16,17], Shahzad and Hussain [21], Shahzad and Latif [22], Tan and Yuan [24] and give wings to this theory. Itoh [10] extended Spacek and Hans's theorem to multivalued contraction mappings. The stochastic version of the well-known Schauder's fixed point theorem was proved by Sehgal and Singh [20], Ciric and Lakshmikantham [4], Zhu and Xiao [27], Hussain et all [9] and Khan et all [11] proved some coupled random fixed point and coupled random coincidence results in partially ordered complete metric spaces. Ciric et all [3] proved fixed point theorems for single-valued mappings, extended to a coincidence theorems for a pair of a random operator $f:\Omega\times X\to X$ and a multi-valued random operator $T:\Omega\times X\to CB(X)$. The aim of this article is to prove a stochastic analog of the Hussain and Alotaibi [8] coupled coincidences for multi-valued contractions in partially ordered metric spaces for a pair of random operators $g:\Omega\times X\to X$ and a multi-valued random operator $T:\Omega\times X\to CL(X)$.

2. Preliminaries

Let (X,d) be a metric space. We denote by $CB(X)$ the collection of non-empty closed bounded subsets of X.
For $A,B\in CB(X)$ and $x\in X$, suppose that $D(x,A) =\inf_{a\in A} d(x,a)$ and

$H(A,B)=\max\{\sup_{a\in A} D(a,B),\sup_{b\in B} D(b,A)\}$, such a mapping H is called a Hausdorff metric on $CB(X)$ induced by d.

Definition 2.1. [8]. An element $x\in X$ is said to be a fixed point of a multi-valued mapping $T:X\to CB(X)$ if and only if $x\in Tx$.

Definition 2.2. [2]. Let X be a nonempty set and $F:X\times X\to X$ be a given mapping. An element $(x,y)\in X\times X$ is said to be a coupled fixed point of the mapping F if $F(x,y)=x$ and $F(y,x)=y$.

Definition 2.3. [13]. Let $(x,y)\in X\times X$, $F:X\times X:\to X$ and $g:X\to X$. We say that the pair (x,y) is a coupled coincidence point of F and g if $F(x,y)=gx$ and $F(y,x)=gy$ for all $x,y\in X$.

Definition 2.4 [8]. A function $f:X\to\mathbb{R}$ is called lower semi-continuous if and only if for any sequence $\{x_n\},\{y_n\}\subset X$ and $(x,y)\in X\times X$, we have,

$$\lim_{n\to\infty}(x_n,y_n)=(x,y)$$

implies

$$f(x,y)\le\liminf_{n\to\infty} f(x_n,y_n).$$

Let $CL(X):=\{A\subset X\,|\,A\neq,\overline{A}=A\}$, where $\overline{A}$ denotes the closure of A in the metric space (X,d). Let (X,d) be a metric space endowed with a partial order and $G:X\to X$ be a given mapping. We define the set $\Delta\subset X\times X$ by $\Delta:=\{(x,y)\in X\times X\,|\,G(x)\le G(y)\}$. In [18], Samet and Vetro introduced the binary relation R on $CL(X)$ defined by $ARB\Leftrightarrow A\times B\subseteq\Delta,$ where $A,B\in CL(X)$.

Definition 2.5 [8]. Let $F:X\times X\to CL(X)$ be a given mapping. We say that F is a Δ-symmetric mapping if and only if $(x,y)\in\Delta\Rightarrow F(x,y)\ \ R\ \ F(y,x)$.

Example 2.6. [8]. Suppose that $X=[0,1]$, endowed with the usual order $\preceq$. Let $G:[0,1]\to[0,1]$ be the mapping defined by $G(x)=M$ for all $x\in[0,1]$, where M is a constant in $[0,1]$. Then $\Delta=[0,1]\times[0,1]$ and $F:X\times X\to CL(X)$ is a Δ-symmetric mapping.

Definition 2.7. [19]. Let $F:X\times X\to CL(X)$ be a given mapping. We say that $(x,y)\in X\times X$ is a coupled fixed point of F if and only if $x \in F(x,y)$ and $y \in F(y,x)$.

Definition 2.8. [8]. Let $F: X \times X \to CL(X)$ be a given mapping and let $g: X \to X$. We say that $(x,y) \in X \times X$ is a coupled coincidence point of F and g if and only if $g(x) \in F(x,y)$ and $g(y) \in F(y,x)$.

Hussain and Alotaibi [8] proved the following theorems for the existence of coupled coincidence for multi-valued nonlinear contractions using two different approaches, first is based on Δ-symmetric property recently studied in [19] and second one is based on mixed g-monotone property studied by Lakishmikantham and C/iric [13].

Theorem 2.9. [8]. Let (X,d) be a metric space endowed with a partial order $\preceq$ and $\Delta \neq \phi$. Suppose that $F: X \times X \to CL(X)$ is Δ-symmetric mapping, $g: X \to X$ is continuous, gX is complete, the function $f: g(X) \times g(X) \to [0,+\infty)$ defined by $f(gx,gy) = D(gx,F(x,y)) + D(gy,F(y,x))$ for all $x,y \in X$ is a lower semi-continuous and that there exists a functio $\varphi:[0,\infty] \to [a,1),\ 0<a<1$ satisfying $\limsup_{r \to t^+} \varphi(r) < 1$ for each $t \in [0,+\infty)$. Assume that for any $(x,y) \in \Delta$ there exist $gu \in F(x,y)$ and $gv \in F(y,x)$ satisfying

$$\sqrt{\varphi(f(gx,gy))}[d(gx,gu)+d(gy,gv)] \leq f(gx,gy)$$

Such that

$$f(gu,gv) \leq \varphi(f(gx,gy)[d(gx,gu)+d(gy,gv)].$$

Then, F and g have a coupled coincidence point, that is, there exists $gz = (gz_1, gz_2) \in X \times X$ such that $gz_1 \in F(z_1, z_2)$ and $gz_2 \in F(z_2, z_1)$.

Theorem 2.10. [8]. Let (X,d) be a metric space endowed with a partial order $\preceq$ and $\Delta \neq \phi$. Suppose that $F: X \times X \to CL(X)$ is Δ-symmetric mapping, $g: X \to X$ is continuous, gX is complete. Suppose that the function $f: g(X) \times g(X) \to [0,+\infty)$ defined in Theorem 2.9 is a lower semi-continuous and that there exists a function $\varphi:[0,\infty] \to [a,1),\ 0<a<1$ satisfying $\limsup_{r \to t^+} \varphi(r) < 1$ for each $t \in [0,+\infty)$. Assume that for any $(x,y) \in \Delta$, there exist $gu \in F(x,y)$ and $gv \in F(y,x)$ satisfying

$$\sqrt{\varphi\left(d(gx,gu)+d(gy,gv)\right)}\left[d(gx,gu)+d(gy,gv)\right]$$
$$\leq \left[D(gx,F(x,y)) + D(gy,F(y,x)\right]$$

such that

$$D\left(gu,F(u,v)\right) + D\left(gv,F(v,u\right) \leq \varphi\left(d(gx,gu)+d(gy,gv)\right)$$
$$\left[d(gx,gu)+d(gy,gv)\right]$$

.Then, F and g have a coupled coincidence point, that is, there exists $gz = (gz_1, gz_2) \in X \times X$ such that $gz_1 \in F(z_1, z_2)$ and $gz_2 \in F(z_2, z_1)$.

Using the concept of commuting maps and mixed g-monotone property, Lakshmikantham and C/iric [13] established the existence of coupled coincidence point results to generalize the results of Bhaskar and lakshmikantham [2]. Hussain and Alotaibi [8] proved the following results by using the mixed g-monotone property for compatible maps F and g in partially ordered metric space, where F is the multi-valued mapping.

Theorem 2.11. [8]. Let $F: X \times X \to CL(X)$, $g: X \to X$ be such that

(a) there exists $k \in (0,1)$ with

$$H(F(x,y),F(u,v) \leq \frac{k}{2} d((gx,gy),(gu,gv)$$

for all $(gx,gy) \succeq (gu,gv)$,

(b) if $g(x_1) \preceq g(x_2), g(y_1) \preceq g(y_2)$, $x_i, y_i \in X (i=1,2)$; then for all $g(u_1) \in F(x_1,y_1)$, there exists $g(u_2) \in F(x_2,y_2)$ with $g(u_1) \preceq g(u_2)$ and for all $g(v_1) \in F(y_1,x_1)$, there exists $g(v_2) \in F(y_2,x_2)$ with $g(v_2) \preceq g(v_1)$ provided, $d((gu_1,gv_1),(gu_2,gv_2)) < 1$; F has the mixed g-monotone property, provided $d((gu_1,gv_1),(gu_2,gv_2)) < 1$,

(c) there exists $x_0, y_0 \in X$ and some $gx_1 \in F(x_0,y_0), gy_1 \in F(y_0,x_0)$ with $g(x_0) \preceq g(x_1), g(y_0) \succeq g(y_1)$ such that $d((gx_0,gy_0),(gx_1,gy_1)) < 1-k$ with $k \in (0,1)$;

(d) if a non-decreasing sequence $\{x_n\} \to x$, then $x_n \leq x$ for all n and if non-increasing sequence $\{y_n\} \to y$, then $y \leq y_n$ for all n and if gX is complete, then F and g have a coupled coincidence point.

Theorem 2.12. [8]. Let $F: X \times X \to CB(X)$, $g: X \to X$ be such that condition (i)-(iii) of Theorem 2.11 hold. Let X be complete, F and g be continuous and compatible. Then F and g have a coupled coincidence point.

3. MAIN RESULTS

Let (Ω, Σ) be a measurable space with a Σ, sigma algebra of subsets of Ω and let (X,d) be metric space. We denote by 2^X the family of all subsets of X. A mapping $T: \Omega \to 2^X$ is called Σ-measurable if for any open subset

U of X, $T^{-1}(U)=\{w: T(w)\cap U\neq\phi\}\in\Sigma$. In what follows, when we speak of measurability, we will mean Σ-measurability. A mapping $f:\Omega\times X\to X$ is called a random operator if for any $x\in X$, $f(.,x)$ is measurable. A mapping $T:\Omega\times X\to CL(X)$ is called multivalued random operator if for every $x\in X$, $T(.,x)$ is measurable. A mapping $s:\Omega\to X$ is called a measurable selector of a measurable multifunction $T:\Omega\to 2^X$ if s is measurable and $s(\omega)\in T(\omega)$ for all $\omega\in\Omega$. A measurable mapping $\xi:\Omega\to X$ is called a random fixed point of a random multifunction $T:\Omega\times X\to CL(X)$ if $\xi(\omega)\in T(\omega,\xi(\omega))$ for every $\omega\in\Omega$. A measurable mapping $\xi:\Omega\to X$ is called a random coincidence of $T:\Omega\times X\to CL(X)$ and $f:\Omega\times X\to X$ if $f(\omega,\xi(\omega))\in T(\omega,\xi(\omega))$ for every $\omega\in\Omega$.

Theorem 3.1. Let $(X,\preceq,d)$ be a complete separable partially ordered metric space, (Ω,Σ) be a measurable space and let $F:\Omega\times(X\times X)\to CL(X)$ is a Δ-symmetric mapping, $g:\Omega\times X\to X$ be continuous, such that

(i) $F(.,v)$ and $g(.,x)$ are measurable for all $v\in X\times X$ and $x\in X$ respectively,

(ii) $F(\omega,.)$ is continuous for all $\omega\in\Omega$.

The function $f:\Omega\times(gX\times gX)\to[0,+\infty)$ defined by

$$f(\omega,(g(\omega,x),g(\omega,y)))=D(g(\omega,x),F(\omega,(x,y)))+D(g(\omega,y),F(\omega,(y,x)))$$

for all $x,y\in X,\omega\in\Omega$ is lower semi-continuous and there exists a function $\phi:[0,\infty)\to[a,1)$, $0<a<1$ satisfying,

$$\limsup_{r\to t^+}\phi(r)<1 \text{ for each } t\in[0,+\infty). \quad (1)$$

Assume that for any $(\omega,(x,y))\in\Omega\times\Delta$, there exist $g(\omega,u)\in F(\omega,(x,y))$ and $g(\omega,v)\in F(\omega,(y,x))$ satisfying

$$\sqrt{\phi\begin{pmatrix} f(\omega,(g(\omega,x)\\ ,g(\omega,u)))\end{pmatrix}}\begin{bmatrix} d(g(\omega,x),g(\omega,u))\\ +d(g(\omega,y),g(\omega,v))\end{bmatrix} \quad (2)$$
$$\leq f(\omega,(g(\omega,x),g(\omega,u)))$$

such that

$$f(\omega,(g(\omega,x),g(\omega,u)))$$
$$\leq\phi\begin{pmatrix} f(\omega,(g(\omega,x),\\ g(\omega,u)))\end{pmatrix}\begin{bmatrix} d(g(\omega,x),g(\omega,u))\\ +d(g(\omega,y),g(\omega,v))\end{bmatrix} \quad (3)$$

If $g(\omega\times X)=X$, then there exist measurable mappings $\zeta,\theta:\Omega\to X$ such that $g(\omega,\zeta(\omega))\in F(\omega,(\zeta(\omega),\theta(\omega)))$ and $g(\omega,\theta(\omega))\in F(\omega,(\theta(\omega),\zeta(\omega)))$ for all $\omega\in\Omega$, that is F and g have a coupled random coincidence point.

Proof. Let $\Psi=\{\zeta:\Omega\to X\}$ be a family of measurable mappings. Define a function $g:\Omega\times X\to R^+$ as follows

$$h(\omega,x)=d(\omega,F(\omega,x)) \quad (4)$$

Since $x\to F(\omega,x)$ is continuous for all $\omega\in\Omega$, we conclude that $h(\omega,.)$ is continuous for all $\omega\in\Omega$. Also, since $x\to F(\omega,x)$ is measurable for all $x\in X$, we conclude that $h(\omega,.)$ is measurable (see Wanger [25, page 868]) for all $\omega\in\Omega$. Thus $h(\omega,x)$ is the Caratheodry function. Therefore, if $\zeta:\Omega\to X$ is a measurable mapping, then $\omega\to h(\omega,\zeta(\omega))$ is also measurable (see [18]). Now, we shall construct two sequences of measurable mappings $\{\zeta_n\}$ and $\{\eta_n\}$ in Ψ, and two sequences $\{g(\omega,\zeta_n(\omega))\}$ and $\{g(\omega,\eta_n(\omega))\}$ in X as follows. Let $(\omega,(\zeta_0,\eta_0))\in\Omega\times\Delta$ be arbitrary, then the multifunction $G:\Omega\to CB(X)$ defined by $G(\omega)=F(\omega,(\zeta_0(\omega),\eta_0(\omega)))$ is measurable. From the Kuratowski and Ryll-Nardzewski [11] selector theorem, there are measurable selectors $\lambda_1,\mu_1:\Omega\to X$ such that $\lambda_1(\omega)\in F(\omega,(\zeta_0(\omega),\eta_0(\omega)))$ and $\mu_1(\omega)\in F(\omega,(\eta_0(\omega),\zeta_0(\omega)))$ for all $\omega\in\Omega$. Since $\lambda_1(\omega)\in F(\omega,(\zeta_0(\omega),\eta_0(\omega)))\subseteq X=g(\omega\times X)$ and $\mu_1(\omega)\in F(\omega,(\eta_0(\omega),\zeta_0(\omega)))$, then there are $(\omega,(\lambda_1(\omega),\mu_1(\omega)))\in\Omega\times\Delta$ such that $g(\omega,\zeta_1(\omega))=\lambda_1(\omega), g(\omega,_1\eta_1(\omega))=\mu_1(\omega)$. Thus $g(\omega,\zeta_1(\omega))\in F(\omega,(\zeta_0(\omega),\eta_0(\omega)))$ and $g(\omega,\eta_1(\omega))\in F(\omega,(\eta_0(\omega),\zeta_0(\omega)))$.

Now by (2) and (3), we have

$$\sqrt{\phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_0(\omega)),\\ g(\omega,\eta_0(\omega))))\end{pmatrix}}\begin{bmatrix} d(g(\omega,\zeta_0(\omega)),g(\omega,\zeta_1(\omega)))\\ +d(g(\omega,\eta_0(\omega)),g(\omega,\eta_1(\omega)))\end{bmatrix}$$
$$\leq f(\omega,(g(\omega,\zeta_0(\omega)),g(\omega,\eta_0(\omega)))) \quad (5)$$

and

$$f(\omega,(g(\omega,\zeta_1(\omega)),g(\omega,\eta_1(\omega))))$$
$$\leq\phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_0(\omega)),\\ g(\omega,\eta_0(\omega))))\end{pmatrix}\begin{bmatrix} d(g(\omega,\zeta_0(\omega)),g(\omega,\zeta_1(\omega)))\\ +d(g(\omega,\eta_0(\omega)),g(\omega,\eta_1(\omega)))\end{bmatrix} \quad (6)$$

Since by definition of ϕ, we have $\phi(f(\omega,(x,y)))<1$, for each $(\omega,(x,y))\in\Omega\times(X\times X)$, it follows from (5) and (6),

$$f(\omega,(g(\omega,\zeta_1(\omega)),g(\omega,\eta_1(\omega))))$$

$$\leq \phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_0(\omega)), \\ g(\omega,\eta_0(\omega)))) \end{pmatrix}\begin{bmatrix} d(g(\omega,\zeta_0(\omega)),g(\omega,\zeta_1(\omega))) \\ +d(g(\omega,\eta_0(\omega)),g(\omega,\eta_1(\omega))) \end{bmatrix}$$

$$= \sqrt{\phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_0(\omega)), \\ g(\omega,\eta_0(\omega)))) \end{pmatrix}}\sqrt{\phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_0(\omega)), \\ g(\omega,\eta_0(\omega)))) \end{pmatrix}}$$

$$\begin{bmatrix} d(g(\omega,\zeta_0(\omega)),g(\omega,\zeta_1(\omega))) \\ +d(g(\omega,\eta_0(\omega)),g(\omega,\eta_1(\omega))) \end{bmatrix}$$

$$\leq \sqrt{\phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_0(\omega)), \\ g(\omega,\eta_0(\omega)))) \end{pmatrix}}\begin{pmatrix} f(\omega,(g(\omega,\zeta_0(\omega)), \\ g(\omega,\eta_0(\omega)))) \end{pmatrix}. \qquad (7)$$

Since F is a Δ-symmetric mapping and $(\omega,(\zeta_0,\eta_0))\in\Omega\times\Delta$, we have, $F(\omega,(\zeta_0(\omega),\eta_0(\omega)))RF(\omega,(\eta_0(\omega),\zeta_0(\omega)))$ implies that $(\omega,(\zeta_1(\omega),\eta_1(\omega)))\in\Omega\times\Delta$. Similarly, as $g(\omega,\zeta_1(\omega))\in F(\omega,(\zeta_0(\omega),\eta_0(\omega)))$ and $g(\omega,\eta_1(\omega))\in F(\omega,(\eta_0(\omega),\zeta_0(\omega)))$ there are measurable selectors $g(\omega,\zeta_2(\omega)),g(\omega,\eta_2(\omega))$ of $F(\omega,(\zeta_1(\omega),\eta_1(\omega)))$ and $F(\omega,(\eta_1(\omega),\zeta_1(\omega)))$ respectively, we have again from (2) and (3),

$$\sqrt{\phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_1(\omega)), \\ g(\omega,\eta_1(\omega)))) \end{pmatrix}}\begin{bmatrix} d(g(\omega,\zeta_1(\omega)),g(\omega,\zeta_2(\omega))) \\ +d(g(\omega,\eta_1(\omega)),g(\omega,\eta_2(\omega))) \end{bmatrix}$$

$$\leq f(\omega,(g(\omega,\zeta_1(\omega)),g(\omega,\eta_1(\omega))))$$

and

$$f(\omega,(g(\omega,\zeta_2(\omega)),g(\omega,\eta_2(\omega))))$$

$$\leq \phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_1(\omega)), \\ g(\omega,\eta_1(\omega)))) \end{pmatrix}\begin{bmatrix} d(g(\omega,\zeta_1(\omega)),g(\omega,\zeta_2(\omega))) \\ +d(g(\omega,\eta_1(\omega)),g(\omega,\eta_2(\omega))) \end{bmatrix}.$$

Hence, we get

$$f(\omega,(g(\omega,\zeta_2(\omega)),g(\omega,\eta_2(\omega))))$$

$$\leq \sqrt{\phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_1(\omega)), \\ g(\omega,\eta_1(\omega)))) \end{pmatrix}}\left[f(\omega,g(\omega,\zeta_1(\omega)),g(\omega,\eta_1(\omega)))\right]$$

with $(\omega,(\zeta_2(\omega),\eta_2(\omega)))\in\Omega\times\Delta$. Continuing this process, such that for all $n\in\mathbb{N}$, we have $(\omega,(\zeta_n(\omega),\eta_n(\omega)))\in\Omega\times\Delta$,

$$g(\omega,\zeta_{n+1}(\omega))\in F(\omega,(\zeta_n(\omega),\eta_n(\omega)))$$

and

$$g(\omega,\eta_{n+1}(\omega))\in F(\omega,(\eta_n(\omega),\zeta_n(\omega))).$$

Again from (2) and (3), we have

$$\sqrt{\phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_n(\omega)), \\ g(\omega,\eta_n(\omega)))) \end{pmatrix}}\begin{bmatrix} d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega))) \\ +d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega))) \end{bmatrix}$$

$$\leq f(\omega,(g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))) \qquad (8)$$

and

$$f(\omega,(g(\omega,\zeta_{n+1}(\omega)),g(\omega,\eta_{n+1}(\omega))))$$

$$\leq \sqrt{\phi\begin{pmatrix} f(\omega,(g(\omega,\zeta_n(\omega)), \\ g(\omega,\eta_n(\omega)))) \end{pmatrix}}\left[f(\omega,g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))\right]. \quad (9)$$

Now, we shall show that $f(\omega,g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))\to 0$ as $n\to\infty$. We shall assume that $f(\omega,g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))>0$ for all $n\in\mathbb{N}$, since if $f(\omega,g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))=0$, then we get $D(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega)))=0$, which implies that

$$g(\omega,\zeta_n(\omega))\in\overline{F(\omega,(\zeta_n(\omega),\eta_n(\omega))}=F(\omega,(\zeta_n(\omega),\eta_n(\omega))$$

and

$$D(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega)))=0,$$

implies that

$$g(\omega,\eta_n(\omega))\in\overline{F(\omega,(\eta_n(\omega),\zeta_n(\omega))}=F(\omega,(\eta_n(\omega),\zeta_n(\omega)).$$

.In this case, $(\zeta_n(\omega),\eta_n(\omega))$ is a random couple-d coincidence point of F and g and the assertion of the theorem is proved. From (9) and $\phi(t)<1$, we deduced that $\{f(\omega,(g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega))))\}$ is strictly decreasing sequence of positive real numbers. Therefore, there is some $\delta\geq 0$ such that

$$\lim_{n\to\infty} f(\omega,g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))=\delta. \qquad (10)$$

Now, we will show that $\delta=0$. Suppose that this is not the case, by (1) and taking the limit on both sides of (9), we have

$$\delta\leq \lim_{f(\omega,g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))\to\delta^+}\sup\sqrt{\phi\begin{pmatrix} f(\omega,g(\omega,\zeta_n(\omega)), \\ g(\omega,\eta_n(\omega))) \end{pmatrix}}\delta<\delta,$$

a contradiction. Thus, δ =0, that is

$$\lim_{n\to\infty} f(\omega,g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))=0. \qquad (11)$$

Now, we have to show that $\{g(\omega,\zeta_n(\omega))\}$ and $\{g(\omega,\eta_n(\omega))\}$ are Cauchy sequences in $g(\omega\times X)=X$. Suppose that

$$\alpha=\lim_{f(\omega,g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))\to\delta^+}\sup\sqrt{\phi\begin{pmatrix} f(\omega,g(\omega,\zeta_n(\omega)), \\ g(\omega,\eta_n(\omega))) \end{pmatrix}}. \quad (12)$$

Then, by (1), we have $\alpha<1$. Let $\alpha<q<1$, then there is some $n_0\in\mathbb{N}$ such that

$$\sqrt{\phi(f(\omega,g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega))))}<q. \qquad (13)$$

for each $n\geq n_0$.

Thus, from (9) and (13), we have

$$f(\omega,g(\omega,\zeta_{n+1}(\omega)),g(\omega,\eta_{n+1}(\omega)))$$

$$\leq qf(\omega,g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega))) \qquad (14)$$

for each $n \geq n_0$. Hence, by induction

(ii)
$$f(\omega, g(\omega, \zeta_{n+1}(\omega)), g(\omega, \eta_{n+1}(\omega))) \leq q^{n+1-n_0}(f(\omega, g(\omega, \zeta_0(\omega)), g(\omega, \eta_0(\omega)))) \quad (15)$$

for each $n \geq n_0$. Since $\phi(t) \geq \alpha > 0$ for all $t \geq 0$, from (8) and (15), we have

$$\begin{bmatrix} d(g(\omega, \zeta_n(\omega)), g(\omega, \zeta_{n+1}(\omega))) \\ +d(g(\omega, \eta_n(\omega)), g(\omega, \eta_{n+1}(\omega))) \end{bmatrix} \leq \frac{1}{\sqrt{\alpha}} q^{n-n_0}(f(\omega, g(\omega, \zeta_0(\omega)), g(\omega, \eta_0(\omega)))). \quad (16)$$

From (16) and since $q < 1$, we conclude that $\{g(\omega, \zeta_n(\omega))\}$ and $\{g(\omega, \eta_n(\omega))\}$ are Cauchy sequences in $g(\omega \times X) = X$. Now, since gX is complete, there exist $\zeta_0(\omega), \eta_0(\omega) \in \Psi$ such that

$$\lim_{n\to\infty} g(\omega, \zeta_n(\omega)) = g(\omega, \zeta_0(\omega))$$

and

$$\lim_{n\to\infty} g(\omega, \eta_n(\omega)) = g(\omega, \eta_0(\omega)).$$

As $g(\omega, \zeta_0(\omega))$ and $g(\omega, \eta_0(\omega))$ are measurable, then the functions $\zeta(\omega), \theta(\omega)$ defined by, $\zeta(\omega) = g(\omega, \zeta_0(\omega))$ and $\theta(\omega) = g(\omega, \eta_0(\omega))$ are measurable.

Since by assumption f is lower semi-continuous, so from (11), we get

$$\begin{aligned} 0 &\leq f(\omega, (g(\omega, \zeta(\omega)), g(\omega, \eta_0(\omega)))) \\ &= D(g(\omega, \zeta(\omega), F(\omega, (\zeta(\omega), \theta(\omega)))) \\ &\quad + D(g(\omega, \theta(\omega), F(\omega, (\theta(\omega), \zeta(\omega)))) \\ &\leq \liminf_{n\to\infty} f(\omega, g(\omega, \zeta_n(\omega)), g(\omega, \eta_n(\omega)) \\ &= 0. \end{aligned}$$

Hence,

$$\begin{aligned} &D(g(\omega, \zeta(\omega), F(\omega, (\zeta(\omega), \theta(\omega)))) \\ &= D(g(\omega, \theta(\omega), F(\omega, (\theta(\omega), \zeta(\omega)))). \\ &= 0, \end{aligned}$$

which implies that,

$$g(\omega, \zeta(\omega)) \in F(\omega, (\zeta(\omega), \theta(\omega)))$$

and

$$g(\omega, \theta(\omega) \in F(\omega, (\theta(\omega), \zeta(\omega))).$$

This proves that F and g have a coupled random coincidence points. □

Theorem 3.2: Let $(X, \preceq, d)$ be a complete separable partially ordered metric space, (Ω, Σ) be a measurable space and let $F : \Omega \times (X \times X) \to CL(X)$ is a Δ-symmetric mapping, $g : \Omega \times X \to X$ be continuous, such that

(i) $F(., v)$ and $g(., x)$ are measurable for all $v \in X \times X$ and $x \in X$ respectively,

$F(\omega, .)$ is continuous for all $\omega \in \Omega$.

The function $f : \Omega \times (gX \times gX) \to [0, +\infty)$ defined by

$$f(\omega, (g(\omega, x), g(\omega, y))) = D(g(\omega, x), F(\omega, (x, y))) + D(g(\omega, y), F(\omega, (y, x))) \quad (17)$$

for all $x, y \in X, \omega \in \Omega$ is lower semi-continuous and there exists a function $\phi : [0, \infty) \to [a, 1), 0 < a < 1$ satisfying

$$\limsup_{r\to t^+} \phi(r) < 1 \quad (18)$$

for each $t \in [0, \infty)$.

Assume that for any $(\omega, (x, y)) \in \Omega \times \Delta$, there exist $g(\omega, u) \in F(\omega, (x, y)))$ and $g(\omega, v) \in F(\omega, (y, x)))$ satisfying

$$\sqrt{\phi(d(g(\omega, x), g(\omega, u)) + d(g(\omega, y), g(\omega, v)))} [d(g(\omega, x), g(\omega, u)) + d(g(\omega, y), g(\omega, v))] \leq D(g(\omega, x), F(\omega, (x, y))) + D(g(\omega, y), F(\omega, (y, x))) \quad (19)$$

such that

$$D(g(\omega, u), F(\omega, (u, v))) + D(g(\omega, v), F(\omega, (v, u))) \leq \phi(d(g(\omega, x), g(\omega, u)) + d(g(\omega, y), g(\omega, v))) [d(g(\omega, x), g(\omega, u)) + d(g(\omega, y), g(\omega, v))]. \quad (20)$$

If $g(\omega \times X) = X$, then there exist measurable mappings $\zeta, \theta : \Omega \to X$ such that, $g(\omega, \zeta(\omega)) \in F(\omega, (\zeta(\omega), \theta(\omega)))$ and $g(\omega, \theta(\omega)) \in F(\omega, (\theta(\omega), \zeta(\omega)))$ for all $\omega \in \Omega$, that is, F and g have a coupled random coincidence point.

Proof: Replacing $\phi(f(\omega, (g(\omega, x), g(\omega, y))))$ with $\phi(d(g(\omega, x), g(\omega, u)) + d(g(\omega, y), g(\omega, v)))$ and as in the proof of Theorem 3.1, we can construct sequences $\{g(\omega, \zeta_n(\omega))\}$ and $\{g(\omega, \eta_n(\omega))\}$ in $g(\omega \times X) = X$ such that for all $n \in$ □, we have $(\omega, (\zeta_n(\omega), \eta_n(\omega)) \in \Omega \times \Delta$, $g(\omega, \zeta_{n+1}(\omega)) \in F(\omega, (\zeta_n(\omega), \eta_n(\omega)))$ and $g(\omega, \eta_{n+1}(\omega)) \in F(\omega, (\eta_n(\omega), \zeta_n(\omega)))$. Now by (19)

$$\sqrt{\begin{array}{l}\phi(d(g(\omega, \zeta_n(\omega)), g(\omega, \zeta_{n+1}(\omega))) \\ +d(g(\omega, \eta_n(\omega)), g(\omega, \eta_{n+1}(\omega))))\end{array}} \begin{array}{l}[d(g(\omega, \zeta_n(\omega)), g(\omega, \zeta_{n+1}(\omega))) \\ +d(g(\omega, \eta_n(\omega)), g(\omega, \eta_{n+1}(\omega))))]\end{array} \quad (21)$$
$$\leq D(g(\omega, \zeta_n(\omega)), F(\omega, (\zeta_n(\omega), \eta_n(\omega)))) + D(g(\omega, \eta_n(\omega)), F(\omega, (\eta_n(\omega), \zeta_n(\omega))))$$

and

$$\begin{aligned}&[D(g(\omega,\zeta_{n+1}(\omega)),F(\omega,(\zeta_{n+1}(\omega),\eta_{n+1}(\omega))))\\&+D(g(\omega,\eta_{n+1}(\omega)),F(\omega,(\eta_{n+1}(\omega),\zeta_{n+1}(\omega))))]\\&\le\sqrt{\begin{matrix}\phi(d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega))))\end{matrix}}\\&\quad[D(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))\\&\quad+D(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))]\end{aligned}\tag{22}$$

for all $n\ge 0$. Again following the lines of the proof of Theorem 3.1, we conclude that,

$$\begin{aligned}&\{D(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))\\&+D(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))\}\end{aligned}$$

is a strictly decreasing sequence of positive real numbers. Therefore, there is some $\delta\ge 0$ such that

$$\begin{aligned}&\lim_{n\to\infty}\{D(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))\\&+D(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))\}=\delta.\end{aligned}\tag{23}$$

Since in our assumption there appears

$$\phi\begin{pmatrix}d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\end{pmatrix},$$

we need to prove that

$$\begin{Bmatrix}d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\end{Bmatrix}$$

admits a subsequence converging to a certain τ^+ for some $\tau\ge 0$. Since $\phi(t)\ge a>0$ for all $t\ge 0$, from (21), we have

$$\begin{aligned}&d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\\&\le\frac{1}{\sqrt{a}}\begin{bmatrix}D(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))\\+D(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))\end{bmatrix}.\end{aligned}\tag{24}$$

From (23) and (24), we conclude that the sequence

$$\begin{Bmatrix}d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\end{Bmatrix}$$

is bounded, therefore, there is some $\beta\ge 0$ such that

$$\liminf_{n\to+\infty}\begin{Bmatrix}d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\end{Bmatrix}=\beta.\tag{25}$$

Since $g(\omega,\zeta_{n+1}(\omega))\in F(\omega,(\zeta_n(\omega),\eta_n(\omega)))$ and $g(\omega,\eta_{n+1}(\omega))\in F(\omega,(\eta_n(\omega),\zeta_n(\omega)))$, it follows that

$$\begin{aligned}&\begin{bmatrix}d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\end{bmatrix}\\&\ge\begin{bmatrix}D(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))\\+D(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))\end{bmatrix}\end{aligned}$$

for each $n\ge 0$. This implies that $\beta\ge\delta$. Now, we shall show that $\beta=\delta$. If we assume that $\delta=0$, then from (23) and (24), we have

$$\lim_{n\to+\infty}\begin{Bmatrix}d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\end{Bmatrix}=0.$$

Thus, if $\delta=0$, then $\beta=\delta$. Suppose now that $\delta>0$.

Assume to contrary that $\beta>\delta$. Then, $\beta-\delta>0$ and so from (23) and (25) there is a positive integer n_0 such that

$$\begin{aligned}&\begin{Bmatrix}D(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))\\+D(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))\end{Bmatrix}\\&<\delta+\frac{\beta-\delta}{4}\end{aligned}\tag{26}$$

and

$$\beta-\frac{\beta-\delta}{4}<\begin{Bmatrix}d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\end{Bmatrix}\tag{27}$$

for all $n\ge n_0$. Then, combining (21), (26) and (27), we get

$$\begin{aligned}&\sqrt{\begin{matrix}\phi(d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega))))\end{matrix}}\left(\beta-\frac{\beta-\delta}{4}\right)\\&<\sqrt{\begin{matrix}\phi(d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega))))\end{matrix}}\begin{bmatrix}d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\end{bmatrix}\\&\le\begin{bmatrix}D(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))\\+D(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))\end{bmatrix}\\&<\delta+\frac{\beta-\delta}{4}\end{aligned}$$

for all $n\ge n_0$. Hence, we get

$$\begin{aligned}&\sqrt{\phi\begin{pmatrix}d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\\+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\end{pmatrix}}\\&\le\frac{\beta+3\delta}{3\beta+\delta}\end{aligned}\tag{28}$$

for all $n\ge n_0$. Put $h=\dfrac{\beta+3\delta}{3\beta+\delta}<1.$ Now, form (22) and (28), it follows that

$$\begin{aligned}&\begin{bmatrix}D(g(\omega,\zeta_{n+1}(\omega)),F(\omega,(\zeta_{n+1}(\omega),\eta_{n+1}(\omega))))\\+D(g(\omega,\eta_{n+1}(\omega)),F(\omega,(\eta_{n+1}(\omega),\zeta_{n+1}(\omega))))\end{bmatrix}\\&\le h\begin{bmatrix}D(g(\omega,\zeta_n(\omega)),F(\omega,(\zeta_n(\omega),\eta_n(\omega))))\\+D(g(\omega,\eta_n(\omega)),F(\omega,(\eta_n(\omega),\zeta_n(\omega))))\end{bmatrix}\end{aligned}$$

for all $n\ge n_0$. Since, we assume that $\delta>0$ and $h<1$, proceeding by induction and combining the above inequalities, it follows that

$$\begin{aligned}\delta&\le\begin{bmatrix}D(g(\omega,\zeta_{n_0+k_0}(\omega)),F(\omega,(\zeta_{n_0+k_0}(\omega),\eta_{n_0+k_0}(\omega))))\\+D(g(\omega,\eta_{n_0+k_0}(\omega)),F(\omega,(\eta_{n_0+k_0}(\omega),\zeta_{n_0+k_0}(\omega))))\end{bmatrix}\\&\le h^{k_0}\begin{bmatrix}D(g(\omega,\zeta_{n_0}(\omega)),F(\omega,(\zeta_{n_0}(\omega),\eta_{n_0}(\omega))))\\+D(g(\omega,\eta_{n_0}(\omega)),F(\omega,(\eta_{n_0}(\omega),\zeta_{n_0}(\omega))))\end{bmatrix}\\&<\delta\end{aligned}$$

for a positive integer k_0, which is a contradiction to the assumption that $\beta>\delta$ and so we must have $\beta=\delta$. Now, we shall show that $\beta=0$. Since

$\beta = \delta$

$$\leq \begin{bmatrix} D(g(\omega,\zeta_n(\omega)), F(\omega,(\zeta_n(\omega),\eta_n(\omega)))) \\ +D(g(\omega,\eta_n(\omega)), F(\omega,(\eta_n(\omega),\zeta_n(\omega)))) \end{bmatrix}$$

$$\leq \begin{bmatrix} d(g(\omega,\zeta_n(\omega)), g(\omega,\zeta_{n+1}(\omega))) \\ +d(g(\omega,\eta_n(\omega)), g(\omega,\eta_{n+1}(\omega)))) \end{bmatrix}.$$

S0, we can read (25) as

$$\liminf_{n\to\infty} \begin{Bmatrix} d(g(\omega,\zeta_n(\omega)), g(\omega,\zeta_{n+1}(\omega))) \\ +d(g(\omega,\eta_n(\omega)), g(\omega,\eta_{n+1}(\omega)))) \end{Bmatrix} = \beta^+.$$

Thus, there exists a subsequence

$$\begin{Bmatrix} d(g(\omega,\zeta_{n_k}(\omega)), g(\omega,\zeta_{n_k+1}(\omega))) \\ +d(g(\omega,\eta_{n_k}(\omega)), g(\omega,\eta_{n_k+1}(\omega)))) \end{Bmatrix} = \beta_{n_k},$$

such that

$$\lim_{n\to\infty} \beta_{n_k} = \beta^+.$$

Now, by (180, we have

$$\limsup_{\beta_{n_k}\to\beta^+} \sqrt{\phi(\beta_{n_k})} < 1. \qquad (29)$$

From (22),

$$\begin{bmatrix} D(g(\omega,\zeta_{n_k+1}(\omega)), F(\omega,(\zeta_{n_k+1}(\omega),\eta_{n_k+1}(\omega)))) \\ +D(g(\omega,\eta_{n_k+1}(\omega)), F(\omega,(\eta_{n_k+1}(\omega),\zeta_{n_k+1}(\omega)))) \end{bmatrix}$$

$$\leq \sqrt{\phi(\beta_{n_k})} \begin{bmatrix} D(g(\omega,\zeta_{n_k}(\omega)), F(\omega,(\zeta_{n_k}(\omega),\eta_{n_k}(\omega)))) \\ +D(g(\omega,\eta_{n_k}(\omega)), F(\omega,(\eta_{n_k}(\omega),\zeta_{n_k}(\omega)))) \end{bmatrix}.$$

Taking the limit as $k \to +\infty$ and using (23), we get

$$\delta = \limsup_{k\to+\infty} \begin{bmatrix} D(g(\omega,\zeta_{n_k+1}(\omega)), F(\omega,(\zeta_{n_k+1}(\omega),\eta_{n_k+1}(\omega)))) \\ +D(g(\omega,\eta_{n_k+1}(\omega)), F(\omega,(\eta_{n_k+1}(\omega),\zeta_{n_k+1}(\omega)))) \end{bmatrix}$$

$$\leq (\limsup_{k\to+\infty} \sqrt{\phi(\beta_{n_k})}) \begin{bmatrix} D(g(\omega,\zeta_{n_k}(\omega)), F(\omega,(\zeta_{n_k}(\omega),\eta_{n_k}(\omega)))) \\ +D(g(\omega,\eta_{n_k}(\omega)), F(\omega,(\eta_{n_k}(\omega),\zeta_{n_k}(\omega))))] \end{bmatrix}$$

$$= (\limsup_{\beta_{n_k}\to\beta^+} \sqrt{\phi(\beta_{n_k})})\delta.$$

From the last inequality, if we suppose that $\delta > 0$, we get

$$1 \leq (\limsup_{\beta_{n_k}\to\beta^+} \sqrt{\phi(\beta_{n_k})}),$$

a contradiction with (29). Thus, $\delta = 0$. Then from (23) and (24) we have

$$\alpha = \limsup_{\beta_{n_k}\to\beta^+} \sqrt{\phi(\beta_{n_k})})$$

$$< 1.$$

Once again, proceeding as in the proof of Theorem 3.1, one can prove that $\{g(\omega,\zeta_n(\omega))\}$ and $\{g(\omega,\eta_n(\omega))\}$ in $g(\omega \times X) = X$ and that F and g have a coupled random coincidence point. □

4. Random coupled coincidence point by mixed g-monotone property

Definition 4.1: [13] Let $(X,\preceq)$ be a partially ordered set and $F : X \times X \to X$ and $g : X \to X$. We say that F has the mixed g-monotone property if F is monotone g-non-decreasing in its first argument and is monotone g-non-increasing in its second argument, that is, for any $x, y \in X$, $x_1, x_2 \in X$, $g(x_1) \preceq g(x_2)$ implies $F(x_1, y) \preceq F(x_2, y)$ and $y_1, y_2 \in X$, $g(y_1) \preceq g(y_2)$ implies $F(x, y_1) \succeq F(x, y_2)$.

Definition 4.2: [13] Let $(X,\preceq)$ be a partially ordered set, $F : X \times X \to CL(X)$ and $g : X \to X$ be mappings. We say that the mapping F has the mixed g-monotone property, if for all $x_1, x_2, y_1, y_2 \in X$ with $g(x_1) \preceq g(x_2)$ and $g(y_1) \succeq g(y_2)$, we get for all $g(u_1) \in F(x_1, y_1)$, there exists $g(u_2) \in F(x_2, y_2)$ such that $g(u_1) \preceq g(u_2)$ and for all $g(v_1) \in F(y_1, x_1)$, there exists $g(v_2) \in F(y_2, x_2)$ such that $g(v_1) \succeq g(v_2)$.

Lemma 4.3: [15] If $A, B \in CB(X)$ with $H(A, B) < \varepsilon$, then for each $a \in A$ there exists an element $b \in B$ such that $d(a,b) < \varepsilon$.

Lemma 4.4: [15] Let $\{A_n\}$ be a sequence in $CB(X)$ and $\lim_{n\to\infty} H(A_n, A) = 0$ for $A \in CB(X)$. If $x_n \in A_n$ and $\lim_{n\to\infty} d(x_n, x) = 0$, then $x \in A$.

Let $(X,\preceq)$ be a partially ordered set and d be a metric on X such that (X,d) is a complete metric space. We define the partial order on the product space $X \times X$ as:

for $(u,v),(x,y) \in X \times X$, $(u,v) \preceq (x,y)$ if and only if $u \preceq x, v \succeq y$. The product metric on $X \times X$ is defined as:

$$d((x_1, y_1),(x_2, y_2)) := d(x_1, x_2) + d(y_1, y_2)$$

for all $x_i, y_i \in X$ $(i = 1,2)$.

For notational convenience, we use the symbol d for the product metric as well as for the metric on X.

Now, we prove the following result that provide the existence of a coupled random coincidence point for compatible maps F and g in partially ordered metric spaces, where F is the multi-valued mapping.

Theorem 4.5: Let $(X,\preceq,d)$ be a complete separable partially ordered metric space, $(\Omega,\sum)$ be a measurable space and $F : \Omega \times (X \times X) \to CB(X)$ and $g : \Omega \times X \to X$ be measurable mappings. If $g : \Omega \times X \to X$ be continuous and there exists $k \in (0,1)$ such that

$$H(F(\omega,(x, y)), F(\omega,(u,v)))$$

$$\leq \frac{k}{2} d((g(\omega, x), g(\omega, y)),(g(\omega,u), g(\omega,v))) \qquad (30)$$

for all $x, y,u,v \in X$, $\omega \in \Omega$ for each

$(g(\omega,x),g(\omega,y))\succeq(g(\omega,u),g(\omega,v))$. Suppose that If $g(\omega,x_1)\preceq g(\omega,x_2)$, $g(\omega,y_2)\preceq g(\omega,y_1)$, *for all* $x_i,y_i\in X$ $(i=1,2)$, then for all $g(\omega,u_1)\in F(\omega,(x_1,y_1))$, there exists $g(\omega,u_2)\in F(\omega,(x_2,y_2))$ with $g(\omega,u_1)\preceq g(\omega,u_2)$ and for all $g(\omega,v_1)\in F(\omega,(y_1,x_1))$, there exists $g(\omega,v_2)\in F(\omega,(y_2,x_2))$ with $g(\omega,v_2)\preceq g(\omega,v_1)$ provided

$d((g(\omega,u_1),g(\omega,v_1)),(g(\omega,u_2),g(\omega,v_2)))<1,$

(i) There exist $x_0,y_0\in X$, $\omega\in\Omega$ and some $g(\omega,x_1)\in F(\omega,(x_0,y_0))$, $g(\omega,y_1)\in F(\omega,(y_0,x_0))$ with $g(\omega,x_0)\preceq g(\omega,x_1)$, $g(\omega,y_0)\succeq g(\omega,y_1)$ such that $d((g(\omega,x_0),g(\omega,y_0)),(g(\omega,x_1),g(\omega,y_2)))<1-k$ $k\in(0,1)$;

(ii) If a non-decreasing sequence $\{x_n\}\to x$, then $x_n\le x$ for all n and if a non-increasing sequence $\{y_n\}\to y$, then $y\le y_n$ for all n. $g(\omega\times X)=X$ is complete, then there exist measurable mappings $\zeta,\theta:\Omega\to X$ such that

$$g(\omega,\zeta(\omega))\in F(\omega,(\zeta(\omega),\theta(\omega)))$$

and

$$g(\omega,\theta(\omega))\in F(\omega,(\theta(\omega),\zeta(\omega)))$$

for all $\omega\in\Omega$, that is F and g have a coupled random coincidence point.

Proof: Let $\Psi=\{\zeta:\Omega\to X\}$ be a family of measurable mappings. Define a function $g:\Omega\times X\to R^+$ as follows

$$h(\omega,x)=d(\omega,F(\omega,x))$$

Since $x\to F(\omega,x)$ is continuous for all $\omega\in\Omega$, we conclude that $h(\omega,.)$ is continuous for all $\omega\in\Omega$. Also, since $x\to F(\omega,x)$ is measurable for all $x\in X$, we conclude that $h(\omega,.)$ is measurable (see Wanger [25, page 868]) for all $\omega\in\Omega$. Thus $h(\omega,x)$ is the Caratheodry function. Therefore, if $\zeta:\Omega\to X$ is a measurable mapping, then $\omega\to h(\omega,\zeta(\omega))$ is also measurable (see [18]). Now, we shall construct two sequences of measurable mappings $\{\zeta_n\}$ and $\{\eta_n\}$ in Ψ, and two sequences $\{g(\omega,\zeta_n(\omega))\}$ and $\{g(\omega,\eta_n(\omega))\}$ in X as follows.

Let $\zeta_0,\eta_0\in\Psi$ such that by (ii), there exist $g(\omega,\zeta_1(\omega))\in F(\omega,(\zeta_0(\omega),\eta_0(\omega)))$, $g(\omega,\eta_1(\omega))\in F(\omega,(\eta_0(\omega),\zeta_0(\omega)))$ with $g(\omega,\zeta_0(\omega))\preceq g(\omega,\zeta_1(\omega))$ and $g(\omega,\eta_0(\omega))\succeq g(\omega,\eta_1(\omega))$ such that

$$d\begin{pmatrix}(g(\omega,\zeta_0(\omega)),g(\omega,\eta_0(\omega))),\\(g(\omega,\zeta_1(\omega)),g(\omega,\eta_1(\omega)))\end{pmatrix}<1-k. \quad (31)$$

Since

$$(g(\omega,\zeta_0(\omega)),g(\omega,\eta_0(\omega)))\preceq(g(\omega,\zeta_1(\omega)),g(\omega,\eta_1(\omega)))$$

using (30) and (31), we have

$$H\begin{pmatrix}F(\omega,(\zeta_0(\omega),\eta_0(\omega)))\\,F(\omega,(\zeta_1(\omega),\eta_1(\omega)))\end{pmatrix}$$

$$\le\frac{k}{2}d\begin{pmatrix}(g(\omega,\zeta_0(\omega)),g(\omega,\eta_0(\omega))),\\(g(\omega,\zeta_1(\omega)),g(\omega,\eta_1(\omega))))\end{pmatrix} \quad (32)$$

$$<\frac{k}{2}(1-k)$$

and similarly,

$$H\begin{pmatrix}F(\omega,(\eta_0(\omega),\zeta_0(\omega))),\\F(\omega,(\eta_1(\omega),\zeta_1(\omega)))\end{pmatrix} \quad (33)$$

$$<\frac{k}{2}(1-k).$$

Using (i) and Lemma (4.3), we have the existence of $g(\omega,\zeta_2(\omega))\in F(\omega,(\zeta_1(\omega),\eta_1(\omega)))$ and $g(\omega,\eta_2(\omega))\in F(\omega,(\eta_1(\omega),\zeta_1(\omega)))$ with $g(\omega,\zeta_1(\omega))\preceq g(\omega,\zeta_2(\omega))$ and $g(\omega,\eta_1(\omega))\succeq g(\omega,\eta_2(\omega))$ such that

$$d(g(\omega,\zeta_1(\omega)),g(\omega,\zeta_2(\omega)))\le\frac{k}{2}(1-k) \quad (34)$$

and

$$d(g(\omega,\eta_1(\omega)),g(\omega,\eta_2(\omega)))\le\frac{k}{2}(1-k). \quad (35)$$

From (34) and (35), we have

$$d\begin{pmatrix}(g(\omega,\zeta_1(\omega)),g(\omega,\eta_1(\omega))),\\(g(\omega,\zeta_2(\omega)),g(\omega,\eta_2(\omega)))\end{pmatrix}<1-k. \quad (36)$$

Now, by (30) and (36), we have

$$H(F(\omega,(\zeta_1(\omega),\eta_1(\omega))),F(\omega,(\zeta_2(\omega),\eta_2(\omega))))$$

$$\le\frac{k^2}{2}(1-k)$$

and

$$H(F(\omega,(\eta_1(\omega),\zeta_1(\omega))),F(\omega,(\eta_2(\omega),\zeta_2(\omega))))$$

$$\le\frac{k^2}{2}(1-k).$$

From Lemma (4.3) and (i), we have the existence of $g(\omega,\zeta_3(\omega))\in F(\omega,(\zeta_2(\omega),\eta_2(\omega)))$ and $g(\omega,\eta_3(\omega))\in F(\omega,(\eta_2(\omega),\zeta_2(\omega)))$ with $g(\omega,\zeta_2(\omega))\preceq g(\omega,\zeta_3(\omega))$ and

$g(\omega,\eta_2(\omega))\succeq g(\omega,\eta_3(\omega))$ such that

$$d(g(\omega,\zeta_2(\omega)),g(\omega,\zeta_3(\omega)))\le\frac{k^2}{2}(1-k)$$

and

$$d(g(\omega,\eta_2(\omega)),g(\omega,\eta_3(\omega)))\le\frac{k^2}{2}(1-k).$$

It follows that

$$d\begin{pmatrix}(g(\omega,\zeta_2(\omega)),g(\omega,\eta_2(\omega))),\\(g(\omega,\zeta_3(\omega)),g(\omega,\eta_3(\omega)))\end{pmatrix}\le k^2 1-k.$$

Continuing in this way, we obtain

$g(\omega,\zeta_{n+1}(\omega))\in F(\omega,(\zeta_n(\omega),\eta_n(\omega)))$ and
$g(\omega,\eta_{n+1}(\omega))\in F(\omega,(\eta_n(\omega),\zeta_n(\omega)))$ with
$g(\omega,\zeta_n(\omega))\preceq g(\omega,\zeta_{n+1}(\omega))$ and
$g(\omega,\eta_n(\omega))\succeq g(\omega,\eta_{n+1}(\omega))$ such that

$$d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega)))\le\frac{k^n}{2}(1-k)$$

and

$$d(g(\omega,\eta_n(\omega)),g(\omega,\eta_{n+1}(\omega)))\le\frac{k^n}{2}(1-k).$$

Thus,

$$d\begin{pmatrix}(g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega))),\\(g(\omega,\zeta_{n+1}(\omega)),g(\omega,\eta_{n+1}(\omega)))\end{pmatrix}\le k^2 1-k. \quad (37)$$

Now, we prove that, for each $\omega\in\Omega$, $\{g(\omega,\zeta_n(\omega))\}$ and $\{g(\omega,\eta_n(\omega))\}$ are Cauchy sequences. Then,

$$\begin{aligned}&d((g(\omega,\zeta_n(\omega)),g(\omega,\zeta_m(\omega))))\\&\le d((g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega))))\\&\quad+d((g(\omega,\zeta_{n+1}(\omega)),g(\omega,\zeta_{n+2}(\omega))))\\&\quad+....+d((g(\omega,\zeta_{m-1}(\omega)),g(\omega,\zeta_m(\omega))))\\&\le[k^n+k^{n+1}+k^{n+2}+....+k^{m-1}](\frac{1-k}{2})\\&=k^n[1+k+k^2+...+k^{m-n-1}](\frac{1-k}{2})\\&=k^n[\frac{1-k^{m-n}}{1-k}](\frac{1-k}{2})\\&=\frac{k^n}{2}(1-k^{m-n})<\frac{k^n}{2}.\end{aligned}$$

Since $k\in(0,1)$, $1-k^{m-n}<1$. Therefore, $d((g(\omega,\zeta_n(\omega)),g(\omega,\zeta_m(\omega))))\to 0$ as $n\to\infty$ implies that $\{g(\omega,\zeta_n(\omega))\}$ is a Cauchy sequence. Similarly, we can show that $\{g(\omega,\eta_n(\omega))\}$ is a Cauchy sequence in X. Since X is complete and $g(\omega\times X)=X$, therefore $\zeta,\theta\in\Psi$ such that $g(\omega,\zeta_n(\omega))\to g(\omega,\zeta(\omega))$ and $g(\omega,\eta_n(\omega))\to g(\omega,\theta(\omega))$. Since $\{g(\omega,\zeta_n(\omega))\}$ is a non-decreasing sequence and $\{g(\omega,\eta_n(\omega))\}$ is a non-increasing sequence and $g(\omega,\zeta_n(\omega))\to g(\omega,\zeta(\omega))$ and $g(\omega,\eta_n(\omega))\to g(\omega,\theta(\omega))$, then from (iii), we have for all $n\ge 0$,

$$\begin{Bmatrix}g(\omega,\zeta_n(\omega))\preceq g(\omega,\zeta(\omega)),\\g(\omega,\eta_n(\omega))\succeq g(\omega,\theta(\omega))\end{Bmatrix}. \quad (38)$$

As $n\to\infty$, from (30), we have

$$H\begin{pmatrix}F(\omega,(\zeta_n(\omega),\eta_n(\omega))),\\F(\omega,(\zeta(\omega),\theta(\omega)))\end{pmatrix}\le\frac{k}{2}d\begin{pmatrix}(g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega))),\\(g(\omega,\zeta(\omega)),g(\omega,\theta(\omega)))\end{pmatrix}\to 0.$$

Since $g(\omega,\zeta_{n+1}(\omega))\in F(\omega,(\zeta_n(\omega),\eta_n(\omega)))$ and

$$\lim_{n\to\infty}d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta(\omega)))=0,$$

then from Lemma (4.4), we have that $g(\omega,\zeta(\omega))\in F(\omega,(\zeta(\omega),\theta(\omega)))$. Again from (30) and as, $n\to\infty$, we have

$$H\begin{pmatrix}F(\omega,(\eta_n(\omega),\zeta_n(\omega))),\\F(\omega,(\theta(\omega),\zeta(\omega)))\end{pmatrix}\le\frac{k}{2}d\begin{pmatrix}(g(\omega,\eta_n(\omega)),g(\omega,\zeta_n(\omega))),\\(g(\omega,\theta(\omega)),g(\omega,\zeta(\omega)))\end{pmatrix}\to 0.$$

Since $g(\omega,\eta_{n+1}(\omega))\in F(\omega,(\eta_n(\omega),\zeta_n(\omega)))$ and $\lim_{n\to\infty}d(g(\omega,\eta_n(\omega)),g(\omega,\theta(\omega)))=0$, then from Lemma (4.4), we have that $g(\omega,\theta(\omega))\in F(\omega,(\theta(\omega),\zeta(\omega)))$. That is, F and g have a coupled random coincidence point. □

Definition 4.6: Let (X,d) be a separable metric space, (Ω,Σ) be a measurable space and $F:\Omega\times(X\times X)\to CB(X)$ and $g:\Omega\times X\to X$ be mappings. We say that F and g are compatible if

$$\lim_{n\to\infty}H\begin{pmatrix}g(\omega,F(\omega,(x_n,y_n))),\\F(\omega,(g(\omega,x_n),g(\omega,y_n)))\end{pmatrix}=0$$

and

$$\lim_{n\to\infty}H\begin{pmatrix}g(\omega,F(\omega,(y_n,x_n))),\\F(\omega,(g(\omega,y_n),g(\omega,x_n)))\end{pmatrix}=0,$$

whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X, such that $\lim_{n\to\infty}F(\omega,(x_n,y_n))=\lim_{n\to\infty}g(\omega,x_n)=x$ and

$\lim_{n\to\infty} F(\omega,(y_n, x_n)) = \lim_{n\to\infty} g(\omega, y_n) = y$ for all $\omega \in \Omega$ and $x, y \in X$.

Theorem 4.7: Let $(X, \preceq, d)$ be a complete separable partially ordered metric space, $(\Omega, \sum)$ be a measurable space and $F : \Omega \times (X \times X) \to CB(X)$ and $g : \Omega \times X \to X$ be measurable mappings. If F and g be continuo-us and compatible mappings, there exist $k \in (0,1)$ such that

$$H\begin{pmatrix} F(\omega,(x,y)), \\ F(\omega,(u,v)) \end{pmatrix} \le \frac{k}{2} d\begin{pmatrix} (g(\omega,x), g(\omega,y)), \\ (g(\omega,u), g(\omega,v)) \end{pmatrix}$$

for all $x, y, u, v \in X$, $\omega \in \Omega$ for which

$$(g(\omega,x), g(\omega,y)) \succeq (g(\omega,u), g(\omega,v)).$$

Suppose that

(i) If $g(\omega,x_1) \preceq g(\omega,x_2)$, $g(\omega,y_2) \preceq g(\omega,y_1)$, *for all* $x_i, y_i \in X\ (i=1,2)$, then for all $g(\omega,u_1) \in F(\omega,(x_1,y_1))$, there exists $g(\omega,u_2) \in F(\omega,(x_2,y_2))$ with $g(\omega,u_1) \preceq g(\omega,u_2)$ and for all $g(\omega,v_1) \in F(\omega,(y_1,x_1))$, there exists $g(\omega,v_2) \in F(\omega,(y_2,x_2))$ with $g(\omega,v_2) \preceq g(\omega,v_1)$ provided

$$d((g(\omega,u_1), g(\omega,v_1)),(g(\omega,u_2), g(\omega,v_2))) < 1,$$

(ii) There exist $x_0, y_0 \in X$, $\omega \in \Omega$ and some $g(\omega,x_1) \in F(\omega,(x_0,y_0))$, $g(\omega,y_1) \in F(\omega,(y_0,x_0))$ with $g(\omega,x_0) \preceq g(\omega,x_1)$ and $g(\omega,y_0) \succeq g(\omega,y_1)$ such that

$$d((g(\omega,x_0), g(\omega,y_0)),(g(\omega,x_1), g(\omega,y_2))) < 1-k,$$

where $k \in (0,1)$;

(iii) If a non-decreasing sequence $\{x_n\} \to x$, then $x_n \le x$ for all n and if a non-increasing sequence $\{y_n\} \to y$, then $y \le y_n$ for all n .

If $g(\omega \times X) = X$ is complete, then there exist measurable mappings $\zeta, \theta : \Omega \to X$ such that $g(\omega,\zeta(\omega)) \in F(\omega,(\zeta(\omega),\theta(\omega)))$ and $g(\omega,\theta(\omega)) \in F(\omega,(\theta(\omega),\zeta(\omega)))$ for all $\omega \in \Omega$, that is F and g have a coupled random coincidence point.

Proof: As in the proof of Theorem 4.5, we obtain the Cauchy sequences $\{g(\omega,\zeta_n(\omega))\}$ and $\{g(\omega,\eta_n(\omega))\}$. Since X is complete and $g(\omega \times X) = X$, therefore $\zeta, \theta \in \Psi$ such that $g(\omega,\zeta_n(\omega)) \to g(\omega,\zeta(\omega))$ and $g(\omega,\eta_n(\omega)) \to g(\omega,\theta(\omega))$. Since $F : \Omega \times (X \times X) \to CB(X)$ and $g : \Omega \times X \to X$ are compatible maps, we have

$$\lim_{n\to\infty} H\begin{pmatrix} g(\omega, F(\omega,(\zeta_n(\omega),\eta_n(\omega)))), \\ F(\omega,(g(\omega,\zeta_n(\omega)), g(\omega,\eta_n(\omega)))) \end{pmatrix} = 0$$

and

$$\lim_{n\to\infty} H\begin{pmatrix} g(\omega, F(\omega,(\eta_n(\omega),\zeta_n(\omega)))), \\ F(\omega,(g(\omega,\eta_n(\omega)), g(\omega,\zeta_n(\omega)))) \end{pmatrix} = 0 .$$

Since $\{g(\omega,\zeta_n(\omega))\}$ and $\{g(\omega,\eta_n(\omega))\}$ are sequen-ces in X , such that

$$\zeta(\omega) = \lim_{n\to\infty} g(\omega,\zeta_{n+1}(\omega)) \in \lim_{n\to\infty} F(\omega,(g(\omega,\zeta_n(\omega)), g(\omega,\eta_n(\omega)))) \quad (39)$$

and

$$\theta(\omega) = \lim_{n\to\infty} g(\omega,\eta_{n+1}(\omega)) \in \lim_{n\to\infty} F(\omega,(g(\omega,\eta_n(\omega)), g(\omega,\zeta_n(\omega)))) \quad (40)$$

are satisfied. For all $n \ge 0$, we have

$$\begin{aligned} &D(g(\omega,\zeta(\omega)), F(\omega,(g(\omega,\zeta_n(\omega)), g(\omega,\eta_n(\omega))))) \\ &\le D(g(\omega,\zeta_n(\omega)), g(\omega, F(\omega,(\zeta_n(\omega),\eta_n(\omega))))) \\ &+ H\begin{pmatrix} g(\omega, F(\omega,(\zeta_n(\omega),\eta_n(\omega)))), \\ F(\omega,(g(\omega,\zeta_n(\omega)), g(\omega,\eta_n(\omega)))) \end{pmatrix}. \end{aligned}$$

Taking the limit as $n \to \infty$ and using the fact that F and g are continuous. We get from (39) that

$$D(g(\omega,\zeta(\omega)), F(\omega,(g(\omega,\zeta(\omega)), g(\omega,\theta(\omega))))) \to 0$$

,which implies that, $g(\omega,\zeta(\omega)) \in F(\omega,(g(\omega,\zeta(\omega)), g(\omega,\theta(\omega))))$.

Similarly from (40), we can prove that

$$D(g(\omega,\theta(\omega)), F(\omega,(g(\omega,\theta(\omega)), g(\omega,\zeta(\omega))))) \to 0$$

,which implies that $g(\omega,\theta(\omega)) \in F(\omega,(g(\omega,\theta(\omega)), g(\omega,\zeta(\omega))))$. Thus F and g have a coupled random coincidence points.

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