GENERALIZED EXPONENTIAL CHAIN ESTIMATORS USING TWO AUXILIARY VARIABLES FOR STRATIFIED SAMPLING WITH NON-RESPONSE

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ABSTRACT: *In this paper, some generalized exponential chain ratio and chain product type estimators have been proposed for finite population mean in the presence of non-response in stratified two-phase sampling when means of the auxiliary variables are not available. The expressions for the bias and mean square error (MSE) of proposed estimators have been derived for two different situations of non-responses. Theoretical comparisons for proposed class of exponential estimators have been made with Hansen and Hurwitz (1946), stratified two-phase ratio and stratified two-phase product estimators. An empirical study has also been carried out to demonstrate the performances of the estimators.*

Keywords: Non-response, Stratified random sampling, Exponential chain estimator, Bias, Mean square error.

1. INTRODUCTION

The problem of non-response is very common in surveys and consequently the estimators may produce bias results. Hansen and Hurwitz [1], were the first to develop a procedure to elicit response from the sub-sample of non-response. They suggested the first attempt by mail questionnaire and the second attempt by a personal interview. Survey based on Hansen-Hurwitz technique costs more because of extra work of personal interviews. They envisaged an estimator for the estimation of population mean in the presence of non-response. Variance expression along with the optimum sampling fraction among non-respondents was also derived. They proposed a sampling plan that involves taking a subsample of non-respondents after the first mail attempt and then interviewing the subsample by personal interview. Cochran [2] provided some ratio and regression estimators following Hansen-Hurwitz [1] for the situation of non response. Further improvement in the estimation procedure for the population mean in the presence of non-response using auxiliary character was suggested by several authors including Rao[3], Khare and Srivastava [4,5], Tabasum and Khan [6,7], Singh and Kumar [8], Chaudhary *et al*. [9,10], Singh et al. [11] and Khare *et al*.[12], Sanaullah *et al*. [13], envisaged some generalized exponential-type ratio and product estimator for stratified two-phase sampling in the presence of non-response.

Bhal and Tuteja [14] introduced the exponential ratio and product type estimators in simple random sampling for estimating the population mean. Singh and Vishwakarma [15] suggested a generalized exponential estimator using single auxiliary variable, and Noor-ul-amin and Hanif [16] modified the exponential estimator for two auxiliary variables in two-phase sampling design. Sanaullah et al. [17] proposed some improved exponential-type estimators using information on two auxiliary variables and Sharma et al. [18] have discussed some exponential product type estimators using information on auxiliary attributes. Singh and Kumar [19] envisaged a generalized exponential estimator for two auxiliary variables in stratified sampling following Singh et al [20]. Koyuncu and Kadilar [21], and Sanaullah et al. [22] discussed some generalized exponential-type shrinkage estimator for stratified two-phase sampling. These all above said improvements are based on the fact if population mean(s) of the auxiliary variable(s) are known before the survey planned. In some situations of practical importance prior information about auxiliary variables are not available and this is the situation we are focusing in this study.

Let a finite population of size N be stratified into L homogenous strata. Let N_h be the size of h^{th} stratum $(h=1,2,.......,L)$ such that $\sum_{h=1}^{L} N_h = N$ and (y_{hi}, x_{hi}) be the observations of the study variable (y) and the auxiliary variable (x), on the i^{th} unit of h^{th} stratum, respectively. Let, c and $\bar{x}_h$ be the sample means of h^{th} stratum corresponding to the population means $\bar{Y}_h$ and $\bar{X}_h$ respectively. However in many practical conditions it is not possible to get hold the population mean of two auxiliary variables $\bar{X}_h$ and $\bar{Z}_h$, in such a situation it is usual to replace these by their sample means $\bar{x}'_h$ and $\bar{z}'_h$ respectively based on a preliminary first-phase sample of size n'_h ($n' = \sum_{h=1}^{L} n'_h$) of which n_h ($n = \sum_{h=1}^{L} n_h$) is a sub-sample i.e.($n_h \subset n'_h$). A bit in a different way we can use the knowledge of the sample mean of secondary auxiliary variable which is closely related to the primary auxiliary variable but remotely related to the study variable. That is if

$\bar{X}_h$ is replaced by $\hat{\bar{X}}_h = \bar{x}'_h \dfrac{\bar{z}'_{st}}{\bar{z}_{st}}$, where $\bar{z}_{st}$ is based on the 2nd phase sample $n\,(=\sum_{h=1}^{L} n_h)$, this would provide a better estimate of $\bar{X}_h$ than $\bar{x}'_h$.

It is assumed to be that at the first phase all the n'_h units provide information on auxiliary characteristics. At the second phase from the sample n_h, let $n_{h(1)}$ units provide the response for the survey questions and $n_{h(2)}$ units do not. Following Hansen-Hurwitz [1] sampling plan, a sub-sample of size $r_h\,(=\frac{n_{h(2)}}{k_h})$ from $n_{h(2)}$ non-respondents is selected at random and is re-contacted for their direct interview, where (k_h>1) is the inverse sampling rate. Here it is assumed that all the r_h units provide their response to the survey. Hansen-Hurwitz [1] reported an unbiased estimator $\bar{y}^*$ for population mean $\bar{Y}$ in simple random sampling, which may be modified under stratified random sampling as,

$$\bar{y}^*_{st} = \sum_{h=1}^{L} P_h \left[\left(n_{h(1)}\bar{y}_{h(1)} + n_{h(2)}\bar{y}^*_{rh(2)} \right) \left(n_h \right)^{-1} \right] \qquad (1.1)$$

where $y_{h(1)}$ and $y^*_{rh(2)}$ are responses recorded from the sample of respondents of first attempt and second attempt (re-contacted for personal interview) respectively and $\bar{y}_{h(1)}$ and $\bar{y}^*_{rh(2)}$ are the sample means. The variance of this estimator may be given as,

$$Var\,(\bar{y}^*_{st}) = \sum_{h=1}^{L} P_h^2 \begin{bmatrix} \left(N_h - n_h\right)\left(N_h n_h\right)^{-1} S^2_{yh} \\ +\left(k_h - 1\right)\left(n_h\right)^{-1} W_{h(2)} S^2_{yh(2)} \end{bmatrix} \qquad (1.2)$$

where $S^2_{yh} = S^2_{yh} = \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h \right)^2 \left(N_h - 1 \right)^{-1}$, and $S^{*2}_{yh(2)} = \sum_{i=1}^{N_{h(2)}} \left(y^*_i - \bar{Y}^*_{h(2)} \right)^2 \left(N_{h(2)} - 1 \right)^{-1}$ are the population variances with population means

$\bar{Y}_h = \sum_{i=1}^{N_h} \left(N_h \right)^{-1} y_i$ and $\bar{Y}^*_h = \sum_{i=1}^{N_{h(2)}} \left(N_{h(2)} \right)^{-1} y^*_{ih(2)}$ for respondents and non-respondents groups respectively, and $W_{h(2)} = N_{h(2)}(N)^{-1}$.

The formulas reported in (1.1) and (1.2) can be defined same for the two auxiliary variables *X* and *Z*.

When there occurs non-response on study variable as well as on the auxiliary variable, the usual two-phase ratio and product estimators for population mean are defined in stratified sampling respectively as,

$$\bar{Y}^*_{Rd} = \bar{y}^*_{st}\bar{x}'_{st} \big/ \bar{x}^*_{st} \text{, (Ratio estimator)} \qquad (1.3)$$

$$\bar{Y}^*_{Pd} = \bar{y}^*_{st}\bar{x}^*_{st} \big/ \bar{x}'_{st} \text{, (Product estimator)} \qquad (1.4)$$

where $\bar{y}^*_{st}$ and $\bar{x}^*_{st}$ are Hansen-Hurwitz estimators modified to the stratified sampling for population means $\bar{X}$ and $\bar{Y}$ respectively, and ($\bar{y}_{h(1)}, \bar{x}_{h(1)}$), and ($\bar{y}_{h(2)r}, \bar{x}_{h(2)r}$) are the sample means for h^{th} stratum based on $n_{h(1)}$ and $n_{h(2)r}$ units respectively, and $\bar{x}'_{st} = \sum_{h=1}^{L} P_h \bar{x}'_h$ is the sample mean based on $n'_h = \sum_{h=1}^{L} n_h$. It is to be pointed out that $\bar{Y}^*_{Rd}$ is modified form of Tabasum and Khan [6] to the stratified sampling. The mean square errors for the ratio estimator $\bar{Y}^*_{Rd}$ and product estimator $\bar{Y}^*_{Pd}$ are given respectively as,

$$MSE(\bar{Y}^*_{Rd}) \approx \sum_{h=1}^{L} P_h^2 \begin{pmatrix} \lambda'_h S^2_{yh} + \lambda_h \left(S^2_{yh} + R_h^2 S^2_{xh} - 2R_h S_{xyh} \right) \\ +\lambda^*_h \left(S^2_{yh(2)} + R_h^2 S^2_{xh(2)} - 2R_h S_{xyh(2)} \right) \end{pmatrix} \qquad (1.5)$$

and

$$MSE(\bar{Y}^*_{Pd}) \approx \sum_{h=1}^{L} P_h^2 \begin{pmatrix} \lambda'_h S^2_{yh} + \lambda_h \left(S^2_{yh} + R_h^2 S^2_{xh} + 2RS_{xyh} \right) \\ +\lambda^*_h \left(S^2_{yh(2)} + R_h^2 S^2_{xh(2)} + 2RS_{xyh(2)} \right) \end{pmatrix} \qquad (1.6)$$

where S_{xyh} and $S_{xyh(2)}$ are the co-variances from respondents and non-respondents respectively with $R_h = \bar{Y}_h / \bar{X}_h$, $\lambda_h = \left(\dfrac{1}{n_h} - \dfrac{1}{N_h} \right)$, $\lambda'_h = \left(\dfrac{1}{n'_h} - \dfrac{1}{N_h} \right)$, $\lambda^*_h = \left(\dfrac{k_h - 1}{n_h} \right) W_{h(2)}$.

When there occurs non-response only on study variable, the usual two-phase ratio and product estimators are defined in stratified sampling respectively as,

$$\bar{Y}^*_{Rd\circ} = \bar{y}^*_{st}\bar{x}'_{st} \big/ \bar{x}_{st} \text{, (Ratio estimator)} \qquad (1.7)$$

$$\bar{Y}^*_{Pd\circ} = \bar{y}^*_{st}\bar{x}_{st} \big/ \bar{x}'_{st} \text{, (Product estimator)} \qquad (1.8)$$

$\bar{Y}^*_{Rd\circ}$ is a modified form of Tabasum and Khan [7] to the stratified sampling. The MSE's of the estimators are given respectively as,

$$MSE(\bar{Y}^*_{Rd\circ}) \approx \sum_{h=1}^{L} P_h^2 \left(\lambda'_h S^2_{yh} + \lambda_h \left(S^2_{yh} + R_h^2 S^2_{xh} - 2R_h S_{xyh} \right) + \lambda^*_h S^2_{yh(2)} \right) \qquad (1.9)$$

and

$$MSE(\bar{Y}^*_{Pd\circ}) \approx \sum_{h=1}^{L} P_h^2 \left(\lambda'_h S^2_{yh} + \lambda_h \left(S^2_{yh} + R_h^2 S^2_{xh} + 2R_h S_{xyh} \right) + \lambda^*_h S^2_{yh(2)} \right) \qquad (1.10)$$

Sanaullah et al. [13] proposed exponential-type estimators in the presence of non-response for the situation when means as prior information of the two auxiliary variables are known. In situation of practical importance these prior information are not available it is therefore in the present study, the objective is to observe the situation if prior information of the two auxiliary variables are not known and then propose some generalized exponential chain-type ratio and product estimators in stratified two-phase sampling for dealing with two different cases of non-response.

2. PROPOSED GENERALIZED ESTIMATORS

In this section, we propose some generalized exponential-type chain ratio and chain product estimators for stratified two-phase sampling considering two different situations of non-response described.

Situation-I: when there occurs non-response on study and auxiliary variables in 2nd-Phase.

Following Sanaullah et al [13,22], we have adapted some estimators of population mean for situation-I under stratified two-phase sampling as,

$$\hat{\bar{Y}}^{1}_{er} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} - \bar{x}^*_h \right)}{\sum_{h=1}^{L} P_h \left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} + \bar{x}^*_h \right)} \right) \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{z}'_h - \bar{z}^*_h \right)}{\sum_{h=1}^{L} P_h \left(\bar{z}'_h + \bar{z}^*_h \right)} \right) \tag{2.1}$$

and

$$\hat{\bar{Y}}^{1}_{ep} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{x}^*_h - \bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} \right)}{\sum_{h=1}^{L} P_h \left(\bar{x}^*_h + \bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} \right)} \right) \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{z}^*_h - \bar{z}'_h \right)}{\sum_{h=1}^{L} P_h \left(\bar{z}^*_h + \bar{z}'_h \right)} \right) \tag{2.2}$$

A generalized exponential chain-ratio estimator of population mean for Situation-I under stratified two-phase sampling for non-response is given by

$$\hat{\bar{Y}}^{G}_{e} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\alpha \frac{\sum_{h=1}^{L} P_h \left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} - \bar{x}^*_h \right)}{\sum_{h=1}^{L} P_h \left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} + (a-1)\bar{x}^*_h \right)} \right) \exp\left[\beta \left(\frac{\sum_{h=1}^{L} P_h \left(\bar{z}'_h - \bar{z}^*_h \right)}{\sum_{h=1}^{L} P_h \left(\bar{z}'_h + (b-1)\bar{z}^*_h \right)} \right) \right] \tag{2.3}$$

Where (α, β) being the constant which take values (0,1,-1) in order to design different estimators, and (a, b) are some suitable constants whose values are to be estimated so that *MSE* of $\hat{\bar{Y}}^{G}_{e}$ is minimum.

It is observed that for (α, β, a, b) = ($1, 1, a_r, b_r$), in (2.3), we get a generalized exponential chain ratio-type $\hat{\bar{Y}}^{G}_{er}$ as the member of the class of $\hat{\bar{Y}}^{G}_{e}$. From this class some exponential chain ratio-type estimators are presented in Table 1.

$$\hat{\bar{Y}}^{G}_{er} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} - \bar{x}^*_h \right)}{\sum_{h=1}^{L} P_h \left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} + (a_r-1)\bar{x}^*_h \right)} \right) \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{z}'_h - \bar{z}^*_h \right)}{\sum_{h=1}^{L} P_h \left(\bar{z}'_h + (b_r-1)\bar{z}^*_h \right)} \right) \tag{2.4}$$

and for (α, β, a, b) = ($-1, -1, a_p, b_p$) we get a generalized exponential chain product-type estimators $\hat{\bar{Y}}^{G}_{ep}$ as the member of the class of $\hat{\bar{Y}}^{G}_{e}$. Some exponential chain product-type estimators are presented in Table 2.

$$\hat{\bar{Y}}^{G}_{ep} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{x}^*_h - \bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} \right)}{\sum_{h=1}^{L} P_h \left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} + (a_p-1)\bar{x}^*_h \right)} \right) \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{z}^*_h - \bar{z}'_h \right)}{\sum_{h=1}^{L} P_h \left(\bar{z}'_h + (b_p-1)\bar{z}^*_h \right)} \right) \tag{2.5}$$

Table 1:

Some members of the class of the estimator $\hat{\bar{Y}}^{G}_{e}$

Exponential Chain Ratio-type Estimators $\hat{\bar{Y}}^{G}_{er}$	a	b	α	β
$\hat{\bar{Y}}^{1}_{er} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} - \bar{x}^*_h \right)}{\sum_{h=1}^{L} P_h \left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}^*_h} + \bar{x}^*_h \right)} \right) \exp\left(\frac{\sum_{h=1}^{L} P_h \left(\bar{z}'_h - \bar{z}^*_h \right)}{\sum_{h=1}^{L} P_h \left(\bar{z}'_h + \bar{z}^*_h \right)} \right)$	2	2	1	1

$\hat{\hat{Y}}_{er}^{2}=\sum_{h=1}^{L}P_h\bar{y}_h^{*}\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{x}_h'\frac{\sum_{h=1}^{L}P_h\bar{z}_h'}{\sum_{h=1}^{L}P_h\bar{z}_h^{*}}-\bar{x}_h^{*}\right)}{\sum_{h=1}^{L}P_h\left(\bar{x}_h'\frac{\sum_{h=1}^{L}P_h\bar{z}_h'}{\sum_{h=1}^{L}P_h\bar{z}_h^{*}}+\bar{x}_h^{*}\right)}\right)\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{z}_h'-\bar{z}_h^{*}\right)}{\sum_{h=1}^{L}P_h\bar{z}_h'}\right)$	2	1	1	1
$\hat{\hat{Y}}_{er}^{3}=\sum_{h=1}^{L}P_h\bar{y}_h^{*}\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{x}_h'\frac{\sum_{h=1}^{L}P_h\bar{z}_h'}{\sum_{h=1}^{L}P_h\bar{z}_h^{*}}-\bar{x}_h^{*}\right)}{\sum_{h=1}^{L}P_h\left(\bar{x}_h'\frac{\sum_{h=1}^{L}P_h\bar{z}_h'}{\sum_{h=1}^{L}P_h\bar{z}_h^{*}}\right)}\right)\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{z}_h'-\bar{z}_h^{*}\right)}{\sum_{h=1}^{L}P_h\left(\bar{z}_h'+\bar{z}_h^{*}\right)}\right)$	1	2	1	1

Table 2:

Some members of the class of the estimator $\hat{\hat{Y}}_{e}^{G}$

Exponential Chain product-type Estimators $\hat{\hat{Y}}_{ep}^{G}$	a	b	α	β
$\hat{\hat{Y}}_{ep}^{1}=\sum_{h=1}^{L}P_h\bar{y}_h^{*}\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{x}_h^{*}-\bar{x}_h'\frac{\sum_{h=1}^{L}P_h\bar{z}_h'}{\sum_{h=1}^{L}P_h\bar{z}_h^{*}}\right)}{\sum_{h=1}^{L}P_h\left(\bar{x}_h^{*}+\bar{x}_h'\frac{\sum_{h=1}^{L}P_h\bar{z}_h'}{\sum_{h=1}^{L}P_h\bar{z}_h^{*}}\right)}\right)\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{z}_h^{*}-\bar{z}_h'\right)}{\sum_{h=1}^{L}P_h\left(\bar{z}_h^{*}+\bar{z}_h'\right)}\right)$	2	2	-1	-1
$\hat{\hat{Y}}_{ep}^{2}=\sum_{h=1}^{L}P_h\bar{y}_h^{*}\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{x}_h^{*}-\bar{x}_h'\frac{\sum_{h=1}^{L}P_h\bar{z}_h'}{\sum_{h=1}^{L}P_h\bar{z}_h^{*}}\right)}{\sum_{h=1}^{L}P_h\left(\bar{x}_h'\frac{\sum_{h=1}^{L}P_h\bar{z}_h'}{\sum_{h=1}^{L}P_h\bar{z}_h^{*}}\right)}\right)\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{z}_h^{*}-\bar{z}_h'\right)}{\sum_{h=1}^{L}P_h\left(\bar{z}_h^{*}+\bar{z}_h'\right)}\right)$	2	1	-1	-1
$\hat{\hat{Y}}_{ep}^{3}=\sum_{h=1}^{L}P_h\bar{y}_h^{*}\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{x}_h^{*}-\bar{x}_h'\frac{\sum_{h=1}^{L}P_h\bar{z}_h'}{\sum_{h=1}^{L}P_h\bar{z}_h^{*}}\right)}{\sum_{h=1}^{L}P_h\left(\bar{x}_h^{*}+\bar{x}_h'\frac{\sum_{h=1}^{L}P_h\bar{z}_h'}{\sum_{h=1}^{L}P_h\bar{z}_h^{*}}\right)}\right)\exp\left(\frac{\sum_{h=1}^{L}P_h\left(\bar{z}_h^{*}-\bar{z}_h'\right)}{\sum_{h=1}^{L}P_h\bar{z}_h'}\right)$	1	2	-1	-1

In order to obtain the bias and MSE expressions, we consider

$$\bar{y}_h^{*}=\bar{Y}_h\left(1+e_{oh}^{*}\right),\ \bar{x}_h^{*}=\bar{X}_h\left(1+e_{1h}^{*}\right),\ \bar{z}_h^{*}=\bar{Z}_h\left(1+e_{2h}^{*}\right)$$

$$\bar{x}_h'=\bar{X}_h\left(1+e_{1h}'\right),\ \bar{z}_h'=\bar{Z}_h\left(1+e_{2h}'\right)$$

such that $E(e_i^{*})=E(e_i')=0$, for $(i=0,1,2)$,

$$E(e_o^{*})^2=\frac{1}{\bar{Y}^2}\sum_{h=1}^{l}P_h^2\left(\lambda_h S_{yh}^2+\lambda_h^{*}S_{yh2}^2\right)=C_{yy}^{*}$$

$$E(e_1')^2=\frac{1}{\bar{X}^2}\sum_{h=1}^{l}P_h^2\left(\lambda_h' S_{xh}^2\right)=C_{xx}'$$

$$E(e_2^{*})^2=\frac{1}{\bar{Z}^2}\sum_{h=1}^{l}P_h^2\left(\lambda_h S_{zh}^2+\lambda_h^{*}S_{zh2}^2\right)=C_{zz}^{*}$$

$$E(e_1^{*})^2=\frac{1}{\bar{X}^2}\sum_{h=1}^{l}P_h^2\left(\lambda_h S_{xh}^2+\lambda_h^{*}S_{xh2}^2\right)=C_{xx}^{*}$$

$$E(e_o^{*}.e_2^{*})=\frac{1}{\bar{Y}\bar{Z}}\sum_{h=1}^{l}P_h^2\left(\lambda_h S_{yzh}+\lambda_h^{*}S_{yzh2}\right)=C_{yz}^{*}$$

$$E(e_0^{*}.e_1^{*})=\frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{l}P_h^2\left(\lambda_h S_{xyh}+\lambda_h^{*}S_{xyh2}\right)=C_{xy}^{*}$$

$$E(e_o^{*}.e_1')=\frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{l}P_h^2\left(\lambda_h' S_{yxh}\right)=C_{xy}'$$

$$E(e_1'.e_2^{*})=\frac{1}{\bar{Z}\bar{X}}\sum_{h=1}^{l}P_h^2\left(\lambda_h' S_{xzh}\right)=C_{xz}'$$

where

$$\Delta_{C_{xx}^{*}}=C_{xx}^{*}-C_{xx}'\quad\text{and}\quad\Delta_{C_{xy}^{*}}=C_{xy}^{*}-C_{xy}'$$

The estimator (2.3) can be expressed in the form of e's as

$$\hat{\bar{\bar{Y}}}_e^{G}=\sum_{h=1}^{L}P_h\bar{Y}_h(1+e_{0h})\exp\left(\alpha\frac{\sum_{h=1}^{L}P_h\left(\bar{X}_h(1+e'_{1h})\frac{\sum_{h=1}^{L}P_h\bar{Z}_h(1+e'_{2h})}{\sum_{h=1}^{L}P_h\bar{Z}_h(1+e_{2h})}-\bar{X}_h(1+e_{1h})\right)}{\sum_{h=1}^{L}P_h\left(\bar{X}_h(1+e'_{1h})\frac{\sum_{h=1}^{L}P_h\bar{Z}_h(1+e'_{2h})}{\sum_{h=1}^{L}P_h\bar{Z}_h(1+e_{2h})}+(a-1)\bar{X}_h(1+e_{1h})\right)}\right)$$

$$\exp\left(\beta\frac{\sum_{h=1}^{L}P\left(\bar{Z}_h(1+e'_{2h})-\bar{Z}_h(1+e_{2h})\right)}{\sum_{h=1}^{L}P\left(\bar{Z}_h(1+e'_{2h})+(b-1)\bar{Z}_h(1+e_{2h})\right)}\right) \qquad (2.6)$$

Expanding the right hand side of (2.6) up to the order n^{-1} of approximation, we have

$$\hat{\bar{\bar{Y}}}_e^{G}-\bar{Y}\approx\bar{Y}\left[e_0+\left(\begin{array}{l}\frac{\alpha\left(e'_1-e_1+e'_2-e_2+e_2^2+e'_1e'_2-e'_1e_2-e'_2e_2+e_0e'_1+e_0e'_2-e_0e_1-e_0e_2\right)}{a}+\frac{\beta\left(e'_2-e_2+e_0e'_2-e_0e_2\right)}{b}\\+\frac{\alpha\left(e_1'^2+e_2'^2+e_1^2+e_2^2+2e'_1e'_2-2e'_1e_1-2e'_1e_2-2e'_2e_1-2e'_2e_2+2e_1e_2+(a-1)(e'_1e_1+e'_2e_1-e_2e_1-e_1^2)\right)}{a^2}\\+\frac{\alpha\left(e_1'^2+e_2'^2+e_1^2+e_2^2+2e'_1e'_2-2e'_1e_1-2e'_1e_2-2e'_2e_1-2e'_2e_2+2e_1e_2\right)}{a^2}\\-\frac{\beta\left(e_2'^2-e'_2e_2+(b-1)(e'_2e_2-e_2^2)\right)}{b^2}+\frac{\alpha\beta\left(e'_1e'_2+e_2'^2-e'_2e_2-e_1e'_2-e'_1e_2-e'_2e_2+e_2^2\right)}{ab}\end{array}\right)\right] \qquad (2.7)$$

The bias and the *MSE* equations of $\hat{\bar{\bar{Y}}}_e^{G}$ upto the order n^{-1} are obtained respectively as

$$\text{Bias}\,(\hat{\bar{\bar{Y}}}_e^{G})\approx\bar{Y}\left(\begin{array}{l}\frac{\alpha\left(C^*_{xx}-C'_{zz}-\Delta_{C^*_{xy}}-\Delta_{C^*_{yz}}\right)}{a}-\frac{\beta\Delta_{C^*_{yz}}}{b}+\frac{\beta(b-1)\Delta_{C^*_{zz}}}{b^2}+\frac{\alpha\beta\left(\Delta_{C^*_{xz}}+\Delta_{C^*_{zz}}\right)}{ab}\\-\frac{\alpha\left(\Delta_{C^*_{xx}}+\Delta_{C^*_{zz}}+2\Delta_{C^*_{xz}}-(a-1)(\Delta_{C^*_{xz}}+\Delta_{C^*_{xx}})+\Delta_{C^*_{zz}}+\Delta_{C^*_{xx}}+2\Delta_{C^*_{xz}}\right)}{a^2}\end{array}\right) \qquad (2.8)$$

and

$$MSE(\hat{\bar{\bar{Y}}}_e^{G})\approx\bar{Y}^2\left(C^*_{yy}+\frac{\alpha^2\left(\Delta_{C^*_{xx}}+\Delta_{C^*_{zz}}+2\Delta_{C^*_{xz}}\right)}{a^2}+\frac{\beta^2\Delta_{C^*_{zz}}}{b^2}-\frac{2\alpha\left(\Delta_{C^*_{xy}}+\Delta_{C^*_{yz}}\right)}{a}-\frac{2\beta\Delta_{C^*_{yz}}}{b}+\frac{2\alpha\beta\left(\Delta_{C^*_{xz}}+\Delta_{C^*_{zz}}\right)}{ab}\right) \qquad (2.9)$$

The optimum values of a and b are obtained respectively as

$$a=\frac{\alpha\left(\left(\Delta_1^*\right)^2-\Delta_3^*\Delta_{C_{zz}^*}\right)}{\Delta_1^*\Delta_{C_{yz}^*}-\Delta_4^*\Delta_{C_{zz}^*}} \text{ and } \quad b=\frac{\beta\left(\left(\Delta_1^*\right)^2-\Delta_3^*\Delta_{C_{zz}^*}\right)}{\Delta_1^*\Delta_4^*-\Delta_3^*\Delta_{C_{yz}^*}} \tag{2.10}$$

Where

$\Delta_1^*=\Delta_{C_{xz}^*}+\Delta_{C_{zz}^*}$, $\quad \Delta_2^*=\Delta_{C_{xx}^*}+\Delta_{C_{zz}^*}$,

$\Delta_3^*=\Delta_2^*+2\Delta_{C_{xz}^*}$ and $\Delta_4^*=\Delta_{C_{xy}^*}+\Delta_{C_{yz}^*}$

Substitution for the optimal values of a and b in (2.9) yields the minimum value of *MSE* of $\hat{\bar{Y}}_e^G$ as

$$\min .MSE(\hat{\bar{Y}}_e^G)\approx\bar{Y}^2\left(C_{yy}^*-\frac{\Delta_3^*\left(\Delta_{C_{yz}^*}\right)^2+\left(\Delta_4^*\right)^2\Delta_{C_{zz}^*}-2\Delta_1^*\Delta_4^*\Delta_{C_{yz}^*}}{\Delta_3^*\Delta_{C_{zz}^*}-\left(\Delta_1^*\right)^2}\right) \tag{2.11}$$

The *MSE* expressions for the estimators $\hat{\bar{Y}}_{er}^G$, and $\hat{\bar{Y}}_{ep}^G$ can be obtained by putting (α, β, a, b) = $(1,1,a_r,b_r)$ and $(-1,-1,a_p,b_p)$ in (2.9) respectively as

$$MSE(\hat{\bar{Y}}_{er}^G)\approx\bar{Y}^2\left(\begin{array}{l}C_{yy}^*+\frac{\left(\Delta_{C_{xx}^*}+\Delta_{C_{zz}^*}+2\Delta_{C_{xz}^*}\right)}{a_r^2}+\frac{\Delta_{C_{zz}^*}}{b_r^2}\\-\frac{2\left(\Delta_{C_{xy}^*}+\Delta_{C_{yz}^*}\right)}{a_r}-\frac{2\Delta_{C_{yz}^*}}{b_r}+\frac{2\left(\Delta_{C_{xz}^*}+\Delta_{C_{zz}^*}\right)}{a_rb_r}\end{array}\right) \tag{2.12}$$

and

$$MSE(\hat{\bar{Y}}_{ep}^G)\approx\bar{Y}^2\left(\begin{array}{l}C_{yy}^*+\frac{\left(\Delta_{C_{xx}^*}+\Delta_{C_{zz}^*}+2\Delta_{C_{xz}^*}\right)}{a_p^2}+\frac{\Delta_{C_{zz}^*}}{b_p^2}\\+\frac{2\left(\Delta_{C_{xy}^*}+\Delta_{C_{yz}^*}\right)}{a_p}+\frac{2\Delta_{C_{yz}^*}}{b_p}+\frac{2\left(\Delta_{C_{xz}^*}+\Delta_{C_{zz}^*}\right)}{a_pb_p}\end{array}\right) \tag{2.13}$$

The *MSE's* of $\hat{\bar{Y}}_{er}^G$, and $\hat{\bar{Y}}_{ep}^G$ are minimum respectively for

$$a_r=\frac{\left(\left(\Delta_1^*\right)^2-\Delta_3^*\Delta_{C_{zz}^*}\right)}{\Delta_1^*\Delta_{C_{yz}^*}-\Delta_4^*\Delta_{C_{zz}^*}} \text{ and } \quad b_r=\frac{\left(\left(\Delta_1^*\right)^2-\Delta_3^*\Delta_{C_{zz}^*}\right)}{\Delta_1^*\Delta_4^*-\Delta_3^*\Delta_{C_{yz}^*}} \tag{2.14}$$

and

$$a_p=\frac{-\left(\left(\Delta_1^*\right)^2-\Delta_3^*\Delta_{C_{zz}^*}\right)}{\Delta_1^*\Delta_{C_{yz}^*}-\Delta_4^*\Delta_{C_{zz}^*}} \text{ and } \quad b_p=\frac{-\left(\left(\Delta_1^*\right)^2-\Delta_3^*\Delta_{C_{zz}^*}\right)}{\Delta_1^*\Delta_4^*-\Delta_3^*\Delta_{C_{yz}^*}} \tag{2.15}$$

The minimum values of the *MSE's* of $\hat{\bar{Y}}_{er}^G$, and $\hat{\bar{Y}}_{ep}^G$ may be obtained as

$$\min .MSE(t_{er}^G)=\min .MSE(t_{ep}^G)=\min .MSE(t_e^G)\approx\bar{Y}^2\left(C_{yy}^*-\frac{\Delta_3^*\left(\Delta_{C_{yz}^*}\right)^2+\left(\Delta_4^*\right)^2\Delta_{C_{zz}^*}-2\Delta_1^*\Delta_4^*\Delta_{C_{yz}^*}}{\Delta_3^*\Delta_{C_{zz}^*}-\left(\Delta_1^*\right)^2}\right) \tag{2.16}$$

The *Bias* and *MSE* expressions for the estimators reported in Table 1-2 may be obtained directly by putting (α, β, a, b) = (1,1,2,2), (1,1,2,1), (1,1,1,2), (-1,-1,2,2), (-1,-1,2,1), (-1,-1,1,2) in (2.8) and (2.9) respectively as

$$\text{Bias }(\hat{\bar{Y}}_{er}^1)\approx\bar{Y}\left(\frac{\left(C_{xx}^*-C_{zz}'-\Delta_{C_{xy}^*}-2\Delta_{C_{yz}^*}\right)}{2}-\frac{\left(\Delta_{C_{xx}^*}+4\Delta_{C_{zz}^*}+4\Delta_{C_{xz}^*}\right)}{4}\right) \tag{2.17}$$

$$\text{Bias }(\hat{\bar{Y}}_{er}^2)\approx\bar{Y}\left(\frac{\left(C_{xx}^*-C_{zz}'-\Delta_{C_{xy}^*}-3\Delta_{C_{yz}^*}\right)}{2}-\frac{\left(\Delta_{C_{xx}^*}+4\Delta_{C_{zz}^*}+5\Delta_{C_{xz}^*}\right)}{4}\right) \tag{2.18}$$

$$\text{Bias }(\hat{\bar{Y}}_{er}^3)\approx\bar{Y}\left(\frac{2\left(C_{xx}^*-C_{zz}'-\Delta_{C_{xy}^*}\right)-3\Delta_{C_{yz}^*}}{2}-\frac{\left(8\Delta_{C_{xx}^*}+11\Delta_{C_{zz}^*}+18\Delta_{C_{xz}^*}\right)}{4}\right) \tag{2.19}$$

$$\text{Bias }(\hat{\bar{Y}}_{ep}^1)\approx-\bar{Y}\left(\begin{array}{c}\frac{\left(C_{xx}^*-C_{zz}'-\Delta_{C_{xy}^*}-2\Delta_{C_{yz}^*}\right)}{2}\\-\frac{\left(\Delta_{C_{xx}^*}+4\Delta_{C_{zz}^*}+4\Delta_{C_{xz}^*}\right)}{4}\end{array}\right) \tag{2.20}$$

Bias $(\hat{\bar{Y}}_{er}^{2})$

$$\approx -\bar{Y}\left(\frac{\left(C_{xx}^{*}-C_{zz}'-\Delta_{C_{xy}^{*}}-3\Delta_{C_{yz}^{*}}\right)}{2}-\frac{\left(\Delta_{C_{xx}^{*}}+4\Delta_{C_{zz}^{*}}+5\Delta_{C_{xz}^{*}}\right)}{4}\right) \quad (2.21)$$

$$\text{Bias }(\hat{\bar{Y}}_{er}^{3}) \approx -\bar{Y}\left(\frac{2\left(C_{xx}^{*}-C_{zz}'-\Delta_{C_{xy}^{*}}\right)-3\Delta_{C_{yz}^{*}}}{2}-\frac{\left(8\Delta_{C_{xx}^{*}}+11\Delta_{C_{zz}^{*}}+18\Delta_{C_{xz}^{*}}\right)}{4}\right) \quad (2.22)$$

and

$$MSE(\hat{\bar{Y}}_{er}^{1}) \approx \bar{Y}^{2}\left(C_{yy}^{*}+\frac{\left(\Delta_{C_{xx}^{*}}+2\Delta_{C_{zz}^{*}}+2\Delta_{C_{xz}^{*}}\right)}{4}-\left(\Delta_{C_{xy}^{*}}+2\Delta_{C_{yz}^{*}}\right)+\frac{\left(\Delta_{C_{xz}^{*}}+\Delta_{C_{zz}^{*}}\right)}{2}\right) \quad (2.23)$$

$$MSE(\hat{\bar{Y}}_{er}^{2}) \approx \bar{Y}^{2}\left(C_{yy}^{*}+\frac{\left(\Delta_{C_{xx}^{*}}+5\Delta_{C_{zz}^{*}}+2\Delta_{C_{xz}^{*}}\right)}{4}-\left(\Delta_{C_{xy}^{*}}+3\Delta_{C_{yz}^{*}}\right)+\left(\Delta_{C_{xz}^{*}}+\Delta_{C_{zz}^{*}}\right)\right) \quad (2.24)$$

$$MSE(\hat{\bar{Y}}_{er}^{3}) \approx \bar{Y}^{2}\left(C_{yy}^{*}+\frac{\left(4\Delta_{C_{xx}^{*}}+5\Delta_{C_{zz}^{*}}+8\Delta_{C_{xz}^{*}}\right)}{4}-\left(2\Delta_{C_{xy}^{*}}+\Delta_{C_{yz}^{*}}\right)+\left(\Delta_{C_{xz}^{*}}+\Delta_{C_{zz}^{*}}\right)\right) \quad (2.25)$$

$$MSE(\hat{\bar{Y}}_{ep}^{1}) \approx \bar{Y}^{2}\left(C_{yy}^{*}+\frac{\left(\Delta_{C_{xx}^{*}}+2\Delta_{C_{zz}^{*}}+2\Delta_{C_{xz}^{*}}\right)}{4}+\left(\Delta_{C_{xy}^{*}}+2\Delta_{C_{yz}^{*}}\right)+\frac{\left(\Delta_{C_{xz}^{*}}+\Delta_{C_{zz}^{*}}\right)}{2}\right) \quad (2.26)$$

$MSE(\hat{\bar{Y}}_{ep}^{2})$

$$\cong \bar{Y}^{2}\left(C_{yy}^{*}+\frac{\left(\Delta_{C_{xx}^{*}}+5\Delta_{C_{zz}^{*}}+2\Delta_{C_{xz}^{*}}\right)}{4}+\left(\Delta_{C_{xy}^{*}}+3\Delta_{C_{yz}^{*}}\right)+\left(\Delta_{C_{xz}^{*}}+\Delta_{C_{zz}^{*}}\right)\right) \quad (2.27)$$

$MSE(\hat{\bar{Y}}_{ep}^{3})$

$$\approx \bar{Y}^{2}\left(C_{yy}^{*}+\frac{\left(4\Delta_{C_{xx}^{*}}+5\Delta_{C_{zz}^{*}}+8\Delta_{C_{xz}^{*}}\right)}{4}+\left(2\Delta_{C_{xy}^{*}}+\Delta_{C_{yz}^{*}}\right)+\left(\Delta_{C_{xz}^{*}}+\Delta_{C_{zz}^{*}}\right)\right) \quad (2.28)$$

Situation-II: when there occurs non-response only on study variable.

A generalized exponential chain-ratio estimator of population mean under stratified two-phase sampling for situation-II of non-response is given by

$$\hat{\bar{Y}}_{e}^{G\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^{*} \exp\left(\alpha \frac{\sum_{h=1}^{L} P_h\left(\bar{x}_h' \frac{\sum_{h=1}^{L} P_h \bar{z}_h'}{\sum_{h=1}^{L} P_h \bar{z}_h} - \bar{x}_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{x}_h' \frac{\sum_{h=1}^{L} P_h \bar{z}_h'}{\sum_{h=1}^{L} P_h \bar{z}_h} + (a^{\circ}-1)\bar{x}_h\right)}\right) \exp\left(\beta \frac{\sum_{h=1}^{L} P_h\left(\bar{z}_h' - \bar{z}_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{z}_h' + (b^{\circ}-1)\bar{z}_h\right)}\right) \quad (2.29)$$

where (α, β) being the constant which take values (0,1,-1) in order to design different estimators, and $(a_{\circ}, b_{\circ})$ are some suitable constants whose values are to be estimated so that *MSE* of $\hat{\bar{Y}}_{e}^{G\circ}$ is minimum.

For $(\alpha, \beta, a^{\circ}, b^{\circ})$ = $(1,1,a_r^{\circ},b_r^{\circ})$ and $(-1,-1,a_p^{\circ},b_p^{\circ})$ in (2.29) we get generalized exponential chain ratio-type estimator $\hat{\bar{Y}}_{er}^{G\circ}$ and chain product-type estimator $\hat{\bar{Y}}_{ep}^{G\circ}$, as the member of the class of $\hat{\bar{Y}}_{e}^{G\circ}$.

$$\hat{\bar{Y}}_{er}^{G\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^{*} \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}_h' \frac{\sum_{h=1}^{L} P_h \bar{z}_h'}{\sum_{h=1}^{L} P_h \bar{z}_h} - \bar{x}_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{x}_h' \frac{\sum_{h=1}^{L} P_h \bar{z}_h'}{\sum_{h=1}^{L} P_h \bar{z}_h} + (a_r^{\circ}-1)\bar{x}_h\right)}\right) \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}_h' - \bar{z}_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{z}_h' + (b_r^{\circ}-1)\bar{z}_h\right)}\right) \quad (2.30)$$

and

$$\hat{\bar{Y}}_{ep}^{G\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}_h - \bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h}\right)}{\sum_{h=1}^{L} P_h\left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h} + (a_p^\circ - 1)\bar{x}_h\right)}\right) \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}_h - \bar{z}'_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{z}'_h + (b_p^\circ - 1)\bar{z}_h\right)}\right) \qquad (2.31)$$

The proposed estimators for situation-II in (2.29)-(2.31) follow naturally in exactly the same fashion as that for situation-I. In addition, the relation between a° & b° is the same as that for case-I. It is therefore the class of estimators for situation-II in Table 3 and Table 4 may be presented in the same way as the estimators reported in Table 1 and Table 2 for situation-I.

Table 3:

Some members of the class of the estimator $\hat{\bar{Y}}_1^{G\circ}$

Exponential Chain Ratio-type Estimators	a°	b°	α	β
$\hat{\bar{Y}}_{er}^{1\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h} - \bar{x}_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h} + \bar{x}_h\right)}\right) \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}'_h - \bar{z}_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{z}'_h + \bar{z}_h\right)}\right)$	2	2	1	1
$\hat{\bar{Y}}_{er}^{2\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h^*} - \bar{x}_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h^*} + \bar{x}_h\right)}\right) \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}'_h - \bar{z}_h\right)}{\sum_{h=1}^{L} P_h \bar{z}'_h}\right)$	2	1	1	1
$\hat{\bar{Y}}_{er}^{3\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h} - \bar{x}_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h}\right)}\right) \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}'_h - \bar{z}_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{z}'_h + \bar{z}_h\right)}\right)$	1	2	1	1

Table 4:

Some members of the class of the estimator $\hat{\bar{Y}}_2^{G\circ}$

Exponential Chain Product-type Estimators	a°	b°	α	β
$\hat{\bar{Y}}_{ep}^{1\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}_h - \bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h}\right)}{\sum_{h=1}^{L} P_h\left(\bar{x}_h + \bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h}\right)}\right) \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}_h - \bar{z}'_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{z}_h + \bar{z}'_h\right)}\right)$	2	2	-1	-1
$\hat{\bar{Y}}_{ep}^{2\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}_h - \bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h^*}\right)}{\sum_{h=1}^{L} P_h\left(\bar{x}_h + \bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h^*}\right)}\right) \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}_h - \bar{z}'_h\right)}{\sum_{h=1}^{L} P_h \bar{z}'_h}\right)$	2	1	-1	-1
$\hat{\bar{Y}}_{ep}^{3\circ} = \sum_{h=1}^{L} P_h \bar{y}_h^* \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{x}_h - \bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h}\right)}{\sum_{h=1}^{L} P_h\left(\bar{x}'_h \frac{\sum_{h=1}^{L} P_h \bar{z}'_h}{\sum_{h=1}^{L} P_h \bar{z}_h}\right)}\right) \exp\left(\frac{\sum_{h=1}^{L} P_h\left(\bar{z}_h - \bar{z}'_h\right)}{\sum_{h=1}^{L} P_h\left(\bar{z}_h + \bar{z}'_h\right)}\right)$	1	2	-1	-1

In order to obtain approximations to the bias and *MSE* for stratified two-phase sampling for situation-II of non-response, let us define

$$\bar{x}_h = \bar{X}_h\left(1+e_{1h}\right), \qquad \bar{y}_h^* = \bar{Y}_h\left(1+e_{oh}^*\right),$$

$$\bar{z}_h = \bar{Z}_h\left(1+e_{2h}\right),$$

$$\bar{x}_h = \frac{\sum_{i=1}^{n_h} x_{hi}}{n_h}, \text{ and } \qquad \bar{z}_h = \frac{\sum_{i=1}^{n_h} z_{hi}}{n_h}$$

$$E(e_o^*)^2 = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L} P_h^2\left(\lambda_h S_{yh}^2 + \lambda_h^* S_{yh2}^2\right) = C_{yy}^*,$$

$$E(e_1^2) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} P_h^2 \lambda_h S_{xh}^2 = C_{xx}$$

$$E(e_2)^2 = \frac{1}{\bar{Z}^2}\sum_{h=1}^{L} P_h^2 \lambda_h S_{zh}^2 = C_{zz}$$

$$E(e_o^*.e_2) = \frac{1}{\overline{YZ}}\sum_{h=1}^{L} P_h^2\left(\lambda_h S_{yzh} + \lambda_h^* S_{yzh2}\right) = C_{xz}^*$$

$$E(e_0^*.e_1) = \frac{1}{\overline{YX}}\sum_{h=1}^{L} P_h^2\left(\lambda_h S_{xyh} + \lambda_h^* S_{xyh2}\right) = C_{xy}^*$$

$$E(e_1'.e_2) = \frac{1}{\overline{ZX}}\sum_{h=1}^{L} P_h^2 \lambda_h' S_{xzh} = C_{xz}' \text{ where}$$

$$\Delta_{C_{xx}} = C_{xx} - C_{xx}' \text{ and } \quad \Delta_{C_{xy}} = C_{xy} - C_{xy}'$$

The bias and the *MSE* equations of $\hat{\hat{Y}}_{e\circ}^{G}$ are obtained respectively as

$$\text{Bias}\,(\hat{\hat{Y}}_e^{G\circ}) \approx \bar{Y}\left(\begin{array}{c}\dfrac{\alpha\left(C_{xx} - C_{zz}' - \Delta_{C_{xy}} - \Delta_{C_{yz}}\right)}{a_\circ} - \dfrac{\beta\Delta_{C_{yz}}}{b_\circ} + \dfrac{\beta(b-1)\Delta_{C_{zz}}}{b_\circ^2} + \dfrac{\alpha\beta\left(\Delta_{C_{xz}} + \Delta_{C_{zz}}\right)}{a_\circ b_\circ} \\ -\dfrac{\alpha\left(\Delta_{C_{xx}} + \Delta_{C_{zz}} + 2\Delta_{C_{xz}} - (a-1)(\Delta_{C_{xz}} + \Delta_{C_{xx}}) + \Delta_{C_{zz}} + \Delta_{C_{xx}} + 2\Delta_{C_{xz}}\right)}{a_\circ^2}\end{array}\right) \tag{2.32}$$

and

$$MSE(\hat{\hat{Y}}_e^{G\circ}) \approx \bar{Y}^2\left(C_{yy}^* + \frac{\alpha^2\left(\Delta_{C_{xx}} + \Delta_{C_{zz}} + 2\Delta_{C_{xz}}\right)}{a_\circ^2} + \frac{\beta^2\Delta_{C_{zz}}}{b_\circ^2} - \frac{2\alpha\left(\Delta_{C_{xy}} + \Delta_{C_{yz}}\right)}{a_\circ} - \frac{2\beta\Delta_{C_{yz}}}{b_\circ} + \frac{2\alpha\beta\left(\Delta_{C_{xz}} + \Delta_{C_{zz}}\right)}{a_\circ b_\circ}\right) \tag{2.33}$$

The optimum values of $a_\circ$ and $b_\circ$ are obtained as

$$a_\circ = \frac{\alpha\left(\left(\Delta_1\right)^2 - \Delta_3\Delta_{C_{zz}}\right)}{\Delta_1\Delta_{C_{yz}} - \Delta_4\Delta_{C_{zz}}} \text{ and } b_\circ = \frac{\beta\left(\left(\Delta_1\right)^2 - \Delta_3\Delta_{C_{zz}}\right)}{\Delta_1\Delta_4 - \Delta_3\Delta_{C_{yz}}} \tag{2.34}$$

Where

$$\Delta_1 = \Delta_{C_{xz}} + \Delta_{C_{zz}}, \quad \Delta_2 = \Delta_{C_{xx}} + \Delta_{C_{zz}}, \quad \Delta_3 = \Delta_2 + 2\Delta_{C_{xz}}$$

$$\text{and} \quad \Delta_4 = \Delta_{C_{xy}} + \Delta_{C_{yz}}$$

Substitution for the optimal values of $a_\circ$ and $b_\circ$ in (2.33) yields the minimum value of *MSE* of $\hat{\hat{Y}}_{e\circ}^{G}$ as

$$\min.MSE(\hat{\hat{Y}}_e^{G\circ}) \approx \bar{Y}^2\left(C_{yy}^* - \frac{\left(\begin{array}{c}\Delta_3\left(\Delta_{C_{yz}}\right)^2 + \left(\Delta_4\right)^2\Delta_{C_{zz}} \\ -2\Delta_1\Delta_4\Delta_{C_{yz}}\end{array}\right)}{\Delta_3\Delta_{C_{zz}} - \left(\Delta_1\right)^2}\right) \tag{2.35}$$

Similarly the *MSE's* of $\hat{\hat{Y}}_{er}^{G\circ}$, and $\hat{\hat{Y}}_{ep}^{G\circ}$ are minimum respectively for

$$a_r^\circ = \frac{\left(\left(\Delta_1\right)^2 - \Delta_3\Delta_{C_{zz}}\right)}{\Delta_1\Delta_{C_{yz}} - \Delta_4\Delta_{C_{zz}}} \text{ and } \quad b_r^\circ = \frac{\left(\left(\Delta_1\right)^2 - \Delta_3\Delta_{C_{zz}}\right)}{\Delta_1\Delta_4 - \Delta_3\Delta_{C_{yz}}} \tag{2.36}$$

and

$$a_p^\circ = \frac{-\left(\left(\Delta_1\right)^2 - \Delta_3\Delta_{C_{zz}}\right)}{\Delta_1\Delta_{C_{yz}} - \Delta_4\Delta_{C_{zz}}} \text{ and } b_p^\circ = \frac{-\left(\left(\Delta_1\right)^2 - \Delta_3\Delta_{C_{zz}}\right)}{\Delta_1\Delta_4 - \Delta_3\Delta_{C_{yz}}} \tag{2.37}$$

The minimum values of the *MSE's* of $\hat{\hat{Y}}_{er}^{G\circ}$, and $\hat{\hat{Y}}_{ep}^{G\circ}$ may be obtained as

$$\min.MSE(t_{er}^{G\circ}) = \min.MSE(t_{ep}^{G\circ}) = \min.MSE(t_{e}^{G\circ})$$

$$\approx \bar{Y}^2\left(C_{yy}^* - \frac{\Delta_3\left(\Delta_{C_{yz}}\right)^2 + \left(\Delta_4\right)^2\Delta_{C_{zz}} - 2\Delta_1\Delta_4\Delta_{C_{yz}}}{\Delta_3\Delta_{C_{zz}} - \left(\Delta_1\right)^2}\right) \quad (2.38)$$

The Bias and MSE expressions for the estimators reported in Table 3-4 may be obtained directly by putting ($\alpha, \beta, a_\circ, b_\circ$) = (1,1,2,2), (1,1,2,1), (1,1,1,2), (-1,-1,2,2), (-1,-1,2,1), (-1,-1,1,2) in (2.32) and (2.33) respectively.

3. Efficiency Comparisons

Now we compare the proposed generalized estimators with usual Hansen and Hurwitz's [1] unbiased estimator $\bar{y}_{st}^*$ and some other considered estimators under two different cases as

a)- Situation-I

$$\left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{er}^1) < MSE(\bar{y}_{st}^*) \\ \text{if} \quad \dfrac{\Delta_3 + \Delta_{C_{zz}^*}}{2\left(2\Delta_4 + 2\Delta_{C_{yz}^*} - \Delta_1\right)} < 1 \end{array} \right\rangle \text{ and } \left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{er}^1) < MSE(\bar{Y}_{Rd}^*) \\ \text{if} \quad \dfrac{\Delta_3 + \Delta_{C_{zz}^*}}{2\left[\left(2\Delta_4 + 2\Delta_{C_{yz}^*} - \Delta_1\right) + 2\left(\Delta_{C_{xx}^*} - 2\Delta_{C_{xy}^*}\right)\right]} < 1 \end{array} \right\rangle \quad (3.1)$$

$$\left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{er}^2) < MSE(\bar{y}_{st}^*) \\ \text{if} \quad \dfrac{\Delta_3 + 4\Delta_{C_{zz}^*}}{4\left(\Delta_4 + 2\Delta_{C_{yz}^*} - \Delta_1\right)} < 1 \end{array} \right\rangle \text{ and } \left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{er}^2) < MSE(\bar{Y}_{Rd}^*) \\ \text{if} \quad \dfrac{\Delta_3 + 4\Delta_{C_{zz}^*}}{4\left[\left(\Delta_4\gamma + 2\Delta_{C_{yz}^*} - \Delta_1\right) + \left(\Delta_{C_{xx}^*} - \Delta_{C_{xy}^*}\right)\right]} < 1 \end{array} \right\rangle \quad (3.2)$$

$$\left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{er}^3) < MSE(\bar{y}_{st}^*) \\ \text{if} \quad \dfrac{4\Delta_3 + \Delta_{C_{zz}^*}}{4\left(2\Delta_4 + \Delta_{C_{yz}^*} - \Delta_1\right)} < 1 \end{array} \right\rangle \text{ and } \left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{er}^3) < MSE(\bar{Y}_{Rd}^*) \\ \text{if} \quad \dfrac{4\Delta_3 + \Delta_{C_{zz}^*}}{4\left[\left(2\Delta_4 + \Delta_{C_{yz}^*} - \Delta_1\right) + \left(\Delta_{C_{xx}^*} - \Delta_{C_{xy}^*}\right)\right]} < 1 \end{array} \right\rangle \quad (3.3)$$

$$\left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{er}^G) < MSE(\bar{y}_{st}^*) \\ \text{if} \quad \dfrac{2\Delta_1\Delta_4\Delta_{C_{yz}^*}}{\Delta_3\left(\Delta_{C_{yz}^*}\right)^2 + \left(\Delta_4\right)^2\Delta_{C_{zz}^*}} < 1 \end{array} \right\rangle \text{ and } \left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{er}^G) < MSE(\bar{Y}_{Rd}^*) \\ \text{if} \quad \dfrac{-\left(\Delta_3\left(\Delta_{C_{yz}^*}\right)^2 + \left(\Delta_4\right)^2\Delta_{C_{zz}^*} - 2\Delta_1\Delta_4\Delta_{C_{yz}^*}\right)}{\left(\Delta_3\Delta_{C_{zz}^*} - \left(\Delta_1\right)^2\right)\left(\Delta_{C_{xx}^*} + 2\Delta_{C_{xy}^*}\right)} < 1 \end{array} \right\rangle \quad (3.4)$$

ii)- Chain-Product Estimators

$$\left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{ep}^1) < MSE(\bar{y}_{st}^*) \\ \text{if} \quad -\dfrac{\Delta_3 + \Delta_{C_{zz}^*}}{2\left(2\Delta_4 + 2\Delta_{C_{yz}^*} + \Delta_1\right)} > 1 \end{array} \right\rangle \text{ and } \left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{ep}^1) < MSE(\bar{Y}_{Pd}^*) \\ \text{if} \quad \dfrac{\Delta_3 + \Delta_{C_{zz}^*}}{2\left[2\left(\Delta_{C_{zz}^*} + 2\Delta_{C_{xy}^*}\right) - \left(2\Delta_4 + 2\Delta_{C_{yz}^*} + \Delta_1\right)\right]} < 1 \end{array} \right\rangle \quad (3.5)$$

$$\left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{ep}^2) < MSE(\bar{y}_{st}^*) \\ \text{if} \quad -\dfrac{\Delta_3 + 4\Delta_{C_{zz}^*}}{4\left(\Delta_4 + 2\Delta_{C_{yz}^*} + \Delta_1\right)} > 1 \end{array} \right\rangle \text{ and } \left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{ep}^2) < MSE(\bar{Y}_{Pd}^*) \\ \text{if} \quad \dfrac{\Delta_3 + 4\Delta_{C_{zz}^*}}{4\left[\left(\Delta_{C_{xx}^*} + \Delta_{C_{xy}^*}\right) - \left(\Delta_4 + 2\Delta_{C_{yz}^*} + \Delta_1\right)\right]} < 1 \end{array} \right\rangle \quad (3.6)$$

$$\left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{ep}^3) < MSE(\bar{y}_{st}^*) \\ \text{if} \quad -\dfrac{4\Delta_3 + \Delta_{C_{zz}^*}}{4\left(2\Delta_1\gamma + \Delta_{C_{yz}^*} + \Delta_1\right)} > 1 \end{array} \right\rangle \text{ and } \left\langle \begin{array}{l} MSE(\hat{\hat{\bar{Y}}}{}_{ep}^3) < MSE(\bar{Y}_{Pd}^*) \\ \text{if} \quad \dfrac{4\Delta_3 + \Delta_{C_{zz}^*}}{4\left[\left(\Delta_{C_{xx}^*} + 2\Delta_{C_{xy}^*}\right) - \left(2\Delta_4 + \Delta_{C_{yz}^*} + \Delta_1\right)\right]} < 1 \end{array} \right\rangle \quad (3.7)$$

$$\left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\hat{\bar{Y}}}{}^{G}_{ep}) < MSE(\bar{y}^{*}_{st}) \\ \dfrac{2\Delta_1\Delta_4\Delta_{C^*_{yz}}}{\Delta_3\left(\Delta_{C^*_{yz}}\right)^2+\left(\Delta_4\right)^2\Delta_{C^*_{zz}}}<1 \end{array} \right\rangle \quad \text{and}$$

$$\left\langle \text{if} \quad \begin{array}{c} MSE(\hat{\hat{\bar{Y}}}{}^{G}_{ep}) < MSE(\bar{Y}^{*}_{Pd}) \\ \dfrac{-\left(\Delta_3\left(\Delta_{C^*_{yz}}\right)^2+\left(\Delta_4\right)^2\Delta_{C^*_{zz}}-2\Delta_1\Delta_4\Delta_{C^*_{yz}}\right)}{\left(\Delta_3\Delta_{C^*_{zz}}-\left(\Delta_1\right)^2\right)\left(\Delta_{C^*_{xx}}+2\Delta_{C^*_{xy}}\right)}<1 \end{array} \right\rangle \tag{3.8}$$

The estimators proposed in situation-I of non-response will be more efficient if the above conditions found to be true. Similar comparisons can be made for the estimators proposed in situation-II of non-response.

4. EMPIRICAL STUDY

In order to examine the performance of proposed estimators under stratified two-phase sampling, we have taken two different stratified populations as,

Population-I: [Source: Koyuncu and Kadilar, 2009]

We consider no of teachers as study variable *(Y)*, no of students as auxiliary variable *(X)*, and no of classes in primary and secondary schools as another auxiliary variable *(Z)* for 923 districts at six 6 regions (1: Marmara, 2: Agean, 3: Mediterranean, 4: Central Anatolia, 5: Black Sea, and 6: East and Southeast Anatolia) in Turkey in 2007. The summery statistics for population-I given in Table A5 shows that study variable *Y* is positively correlated with both auxiliary variables *X* and *Z*, it is therefore this population is suitable for ratio type estimators.

Population-II:[source: detailed livelihood assessment of flood affected districts of Pakistan September 2011, Food Security Cluster, Pakistan]

We consider food expenditure as study variable *(Y)*, household earn as auxiliary variable *(X)*, and total expenditure in May (2011) as another auxiliary variable *(Z)* for (6940) male and (1678) female households in flood affected districts of Pakistan September 2011. The summery statistics for population-II given in Table A5 shows that study variable *Y* is negatively correlated with both auxiliary variables *X* and *Z*, it is therefore this population is suitable for product type estimators.

The comparison of proposed generalized exponential chain-ratio and chain-product estimators with respect to modified Hansen and Hurwitz's [1] have been made with modified Tabasum and Khan's (6,7) ratio and product estimators respectively. We used Neyman allocation for allocating the samples to different strata.

MSE's for proposed estimators for situation-I and situation-II are given in and Table A3 and Table A4 respectively. From Table A1 and Table A2, it is observed that proposed class of generalized exponential chain ratio-type ($\hat{\hat{\bar{Y}}}{}^{G}_{er}$, $\hat{\hat{\bar{Y}}}{}^{G\circ}_{er}$) and chain product-type estimators ($\hat{\hat{\bar{Y}}}{}^{G}_{ep}$, $\hat{\hat{\bar{Y}}}{}^{G\circ}_{ep}$) under both situations of non-response are more efficient than modified Hansen and Hurwitz's [1] estimator $\bar{y}^{*}_{st}$ for the population mean $\bar{Y}$. It is further observed that generalized exponential chain ratio-type estimators $\hat{\hat{\bar{Y}}}{}^{G}_{er}$ is more efficient than modified form of the ratio estimators $\hat{\hat{\bar{Y}}}_{Rd}$ under situation-I and $\hat{\hat{\bar{Y}}}{}^{G\circ}_{er}$ is more efficient than $\hat{\hat{\bar{Y}}}_{Rd_\circ}$ under situation-II. From A3 It is also observed that *PREs* for the proposed estimators increases as inverse sampling rate k_h increases under situation-I.

Again from Table A1 and Table A2, it is observed that the proposed generalized exponential chain product-type estimators ($\hat{\hat{\bar{Y}}}{}^{G}_{ep}$, $\hat{\hat{\bar{Y}}}{}^{G\circ}_{ep}$) are more efficient than $\bar{y}^{*}_{st}$ and modified forms of product estimators ($\hat{\hat{\bar{Y}}}_{Pd}$, $\hat{\hat{\bar{Y}}}{}^{\circ}_{Pd}$) under situation-I and situation-II.

5. CONCLUSION

Finally it is concluded that the performance of generalized exponential chain ratio-type and product-type estimators are better on the basis of *PREs*, and therefore the class of generalized exponential chain estimators is proposed for their practical application for both of the situations for non-response.

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TABLES BELOW.

Table A1: Percent relative efficiencies (*PRE*s) of estimators with respect to $\bar{y}_{st}^{*}$ for different values of k_h each at different rate of non-response under Situation-I using three different populations.

W_{h2}	k_h	Population No	$\bar{y}_{st}^{*}$	$\hat{\hat{\bar{Y}}}_{Rd}$	$\hat{\hat{\bar{Y}}}_{er}^{1}$	$\hat{\hat{\bar{Y}}}_{er}^{2}$	$\hat{\hat{\bar{Y}}}_{er}^{3}$	$\hat{\hat{\bar{Y}}}_{er}^{G}$	$\hat{\hat{\bar{Y}}}_{Pd}$	$\hat{\hat{\bar{Y}}}_{ep}^{1}$	$\hat{\hat{\bar{Y}}}_{ep}^{2}$	$\hat{\hat{\bar{Y}}}_{ep}^{3}$	$\hat{\hat{\bar{Y}}}_{ep}^{G}$
10%	2.0	1	100	316.5725	300.00	171.44	80.80	355.02	***	***	***	***	***
		2	100	***	***	***	***	***	64.9538	100.64	73.89	29.64	218.00
	2.5	1	100	343.7379	327.35	180.63	82.71	389.20	***	***	***	***	***
		2	100	***	***	***	***	***	61.8850	97.40	71.07	28.30	198.29
	3.0	1	100	369.9346	353.99	188.99	84.36	422.99	***	***	***	***	***
		2	100	***	***	***	***	***	59.3792	94.80	68.83	27.25	194.05
	3.5	1	100	395.2162	379.88	196.61	85.79	456.29	***	***	***	***	***
		2	100	***	***	***	***	***	57.2944	92.67	67.02	26.41	190.53
20%	2.0	1	100	364.5528	339.02	179.15	79.59	415.91	***	***	***	***	***
		2	100	***	***	***	***	***	65.6052	100.65	73.89	29.64	218.00
	2.5	1	100	412.3668	382.02	189.65	80.71	478.71	***	***	***	***	***
		2	100	***	***	***	***	***	62.9599	97.40	71.07	28.30	198.29
	3.0	1	100	457.3057	422.09	198.34	81.57	539.89	***	***	***	***	***
		2	100	***	***	***	***	***	60.8689	94.80	68.84	27.25	194.05
	3.5	1	100	499.6209	459.59	205.68	82.26	599.60	***	***	***	***	***
		2	100	***	***	***	***	***	59.17469	92.67	67.02	26.41	190.53
30%	2.0	1	100	382.2853	353.89	181.46	79.72	439.14	***	***	***	***	***
		2	100	***	***	***	***	***	76.5228	111.73	83.82	34.93	220.90
	2.5	1	100	436.9706	402.13	192.03	80.80	512.18	***	***	***	***	***
		2	100	***	***	***	***	***	77.3685	112.15	84.21	35.26	202.26
	3.0	1	100	487.7213	446.43	200.54	81.60	582.90	***	***	***	***	***
		2	100	***	***	***	***	***	78.0201	112.46	84.51	35.50	199.73
	3.5	1	100	534.9462	487.22	207.53	82.22	651.39	***	***	***	***	***
		2	100	***	***	***	***	***	78.5378	112.71	84.74	35.70	197.83

Table A2: Percent relative efficiencies (*PRE*s) of estimators with respect to $\bar{y}_{st}^{*}$ for different values of k_h each at different rate of non-response under Situation-II using three different populations.

W_{h2}	k_h	Population No	$\bar{y}_{st}^{*}$	$\hat{\hat{\bar{Y}}}_{Rd}^{\circ}$	$\hat{\hat{\bar{Y}}}_{er}^{1\circ}$	$\hat{\hat{\bar{Y}}}_{er}^{2\circ}$	$\hat{\hat{\bar{Y}}}_{er}^{3\circ}$	$\hat{\hat{\bar{Y}}}_{er}^{G\circ}$	$\hat{\hat{\bar{Y}}}_{Pd}^{\circ}$	$\hat{\hat{\bar{Y}}}_{ep}^{1\circ}$	$\hat{\hat{\bar{Y}}}_{ep}^{2\circ}$	$\hat{\hat{\bar{Y}}}_{ep}^{3\circ}$	$\hat{\hat{\bar{Y}}}_{ep}^{G\circ}$
10%	2.0	1	100	193.8316	186.58	135.68	79.97	205.02	***	***	***	***	***
		2	100	***	***	***	***	***	76.1469	108.968	84.267	36.777	198.44
	2.5	1	100	177.8583	172.30	131.20	81.53	186.29	***	***	***	***	***
		2	100	***	***	***	***	***	77.1852	108.42	85.02	38.14	181.68
	3.0	1	100	166.5321	162.06	127.72	82.87	173.24	***	***	***	***	***
		2	100	***	***	***	***	***	78.1368	107.93	85.71	39.44	174.09
	3.5	1	100	158.0830	154.37	124.94	84.03	163.61	***	***	***	***	***
		2	100	***	***	***	***	***	79.0122	107.50	86.33	40.69	167.79
20%	2.0	1	100	169.0789	164.37	128.53	82.55	176.16	***	***	***	***	***
		2	100	***	***	***	***	***	77.2828	108.37	85.09	38.27	187.09

	2.5	**1**	100	153.8445	150.49	123.48	84.67	158.82	***	***	***	***	***
		2	100	***	***	***	***	***	78.7123	107.65	86.12	40.25	169.88
	3.0	**1**	100	144.1153	141.53	119.95	86.33	147.91	***	***	***	***	***
		2	100	***	***	***	***	***	79.9726	107.04	87.01	42.12	161.52
	3.5	**1**	100	137.3642	135.27	117.34	87.66	140.42	***	***	***	***	***
		2	100	***	***	***	***	***	81.0920	106.53	87.80	43.87	154.96
30%	**2.0**	**1**	100	162.3269	158.24	126.36	83.43	168.43	***	***	***	***	***
		2	100	***	***	***	***	***	80.5834	106.76	87.44	43.06	161.70
	2.5	**1**	100	147.7912	144.92	121.31	85.67	152.02	***	***	***	***	***
		2	100	***	***	***	***	***	82.8178	105.77	88.99	46.76	146.11
	3.0	**1**	100	138.7533	136.56	117.89	87.37	141.95	***	***	***	***	***
		2	100	***	***	***	***	***	84.5909	105.02	90.20	50.01	138.33
	3.5	**1**	100	132.5902	130.82	115.41	88.72	135.15	***	***	***	***	***
		2	100	***	***	***	***	***	86.0324	104.46	91.18	52.88	132.79

Table A3: *MSEs* of $\bar{y}_{st}^{*}$, $\hat{\hat{Y}}_{Rd}$, $\hat{\hat{Y}}_{Pd}$ and proposed estimators for different values of k_h each at different rate of non-response under Situation-I using three different populations.

W_{h2}	k_h	*Population No*	$\bar{y}_{st}^{*}$	$\hat{\hat{Y}}_{Rd}$	$\hat{\hat{Y}}_{er}^{1}$	$\hat{\hat{Y}}_{er}^{2}$	$\hat{\hat{Y}}_{er}^{3}$	$\hat{\hat{Y}}_{er}^{G}$	$\hat{\hat{Y}}_{Pd}$	$\hat{\hat{Y}}_{ep}^{1}$	$\hat{\hat{Y}}_{ep}^{2}$	$\hat{\hat{Y}}_{ep}^{3}$	$\hat{\hat{Y}}_{ep}^{G}$
10%	**2.0**	**1**	2144.00	677.254	714.6653	1250.5901	2653.4464	603.9020	***	***	***	***	***
		2	5.09881	***	***	***	***	***	7.8499	5.1321	6.9233	17.0571	2.4510
	2.5	**1**	2370.93	689.749	724.2836	1312.5835	2866.417	609.1876	***	***	***	***	***
		2	5.40353	***	***	***	***	***	8.7316	5.6632	7.6641	18.9580	2.8708
	3.0	**1**	2597.86	702.248	733.8733	1374.5911	3079.4306	614.1602	***	***	***	***	***
		2	5.70825	***	***	***	***	***	9.6132	6.1948	8.4055	20.8602	3.1385
	3.5	**1**	2824.79	714.745	743.6016	1436.7757	3292.6163	619.0727	***	***	***	***	***
		2	6.01298	***	***	***	***	***	10.4949	6.7256	9.1461	22.7604	3.4051
20%	**2.0**	**1**	2540.35	696.839	749.3190	1417.9597	3191.7864	610.7929	***	***	***	***	***
		2	5.43362	***	***	***	***	***	8.2823	5.3988	7.3536	18.3297	2.4925
	2.5	**1**	2965.45	719.129	749.3190	1417.9597	3191.7864	610.7929	***	***	***	***	***
		2	5.90575	***	***	***	***	***	9.3802	6.0631	8.3095	20.8667	2.9783
	3.0	**1**	3390.55	741.419	776.2618	1563.6545	3674.0082	619.4667	***	***	***	***	***
		2	6.37788	***	***	***	***	***	10.4781	6.7274	9.2653	23.4036	3.2867
	3.5	**1**	3815.66	763.711	803.2763	1709.4545	4156.3209	628.0063	***	***	***	***	***
		2	6.85001	***	***	***	***	***	11.5759	7.3920	10.2216	25.9412	3.5953
30%	**2.0**	**1**	2703.11	707.092	830.2335	1855.1829	4638.5619	636.3692	***	***	***	***	***
		2	6.62876	***	***	***	***	***	8.6625	5.9329	7.9079	18.9754	3.0007
	2.5	**1**	3209.59	734.510	763.8134	1489.6096	3390.5111	615.5480	***	***	***	***	***
		2	7.69847	***	***	***	***	***	9.9504	6.8646	9.1414	21.8357	3.8062
	3.0	**1**	3716.08	761.927	798.1421	1671.3971	3972.3152	626.6575	***	***	***	***	***
		2	8.76816	***	***	***	***	***	11.2383	7.7969	10.3755	24.6981	4.3901
	3.5	**1**	4222.56	789.343	832.4038	1853.0507	4553.9760	637.5133	***	***	***	***	***
		2	9.83786	***	***	***	***	***	12.5263	8.7286	11.6089	27.5587	4.9728

Table A4: *MSEs* of $\bar{y}_{st}^{*}$, $\hat{\hat{Y}}_{Rd}^{\circ}$, $\hat{\hat{Y}}_{Pd}^{\circ}$ and proposed estimators for different values of k_h each at different rate of non-response under Situation-II using three different populations.

W_{h2}	k_h	*Population No*	$\bar{y}_{st}^{*}$	$\hat{\hat{Y}}_{Rd}^{\circ}$	$\hat{\hat{Y}}_{er}^{1\circ}$	$\hat{\hat{Y}}_{er}^{2\circ}$	$\hat{\hat{Y}}_{er}^{3\circ}$	$\hat{\hat{Y}}_{er}^{G\circ}$	$\hat{\hat{Y}}_{Pd}^{\circ}$	$\hat{\hat{Y}}_{ep}^{1\circ}$	$\hat{\hat{Y}}_{ep}^{2\circ}$	$\hat{\hat{Y}}_{ep}^{3\circ}$	$\hat{\hat{Y}}_{ep}^{G\circ}$
10%	**2.0**	**1**	2144.00	1106.12	1149.1301	1580.1377	2680.9868	1045.7392	***	***	***	***	***
		2	5.09881	***	***	***	***	***	6.6960	4.6791	6.0507	13.8641	2.5694
	2.5	**1**	2370.93	1333.04	1376.0884	1807.096	2907.9451	1272.6975	***	***	***	***	***
		2	5.40353	***	***	***	***	***	7.0007	4.9839	6.3555	14.1688	2.9741
	3.0	**1**	2597.86	1559.97	1602.9702	2033.9778	3134.8269	1499.5793	***	***	***	***	***
		2	5.70825	***	***	***	***	***	7.3055	5.2886	6.6603	14.4736	3.2789
	3.5	**1**	2824.79	1786.90	1829.9285	2260.9361	3361.7852	1726.5376	***	***	***	***	***
		2	6.01298	***	***	***	***	***	7.6102	5.5933	6.9649	14.7783	3.5835
20%	**2.0**	**1**	2540.35	1502.46	1545.4705	1976.4781	3077.3272	1442.0796	***	***	***	***	***
		2	5.43362	***	***	***	***	***	7.0308	5.0140	6.3856	14.1989	2.9042
	2.5	**1**	2965.45	1927.56	1970.5895	2401.5971	3502.4462	1867.1986	***	***	***	***	***
		2	5.90575	***	***	***	***	***	7.5030	5.4861	6.8577	14.6710	3.4763
	3.0	**1**	3390.55	2352.67	2395.6893	2826.6969	3927.546	2292.2984	***	***	***	***	***
		2	6.37788	***	***	***	***	***	7.9751	5.9582	7.3298	15.1431	3.9484
	3.5	**1**	3815.66	2777.77	2820.7892	3251.7967	4352.6458	2717.3983	***	***	***	***	***
		2	6.85001	***	***	***	***	***	8.4472	6.4304	7.8020	15.6153	4.4206
30%	**2.0**	**1**	2703.11	1665.23	1708.2365	2139.244	3240.0931	1604.8456	***	***	***	***	***
		2	6.62876	***	***	***	***	***	8.2260	6.2090	7.5807	15.3939	4.0992
	2.5	**1**	3209.59	2171.71	2214.7384	2645.746	3746.5951	2111.3475	***	***	***	***	***
		2	7.69847	***	***	***	***	***	9.2957	7.2787	8.6503	16.4636	5.2689
	3.0	**1**	3716.08	2678.19	2721.2212	3152.2288	4253.0779	2617.8303	***	***	***	***	***
		2	8.76816	***	***	***	***	***	10.3654	8.3486	9.7202	17.5335	6.3388
	3.5	**1**	4222.56	3184.67	3227.7232	3658.7307	4759.5798	3124.3323	***	***	***	***	***
		2	9.83786	***	***	***	***	***	11.4351	9.4183	10.7899	18.6032	7.4085

Table A5: Data Statistics for Population-I and Population-II

		Population-I						Population-II	
Stratum (h)		1	2	3	4	5	6	1	2
Stratified Mean, S.Ds and Correlation Coefficients	N_h	127	117	103	*170*	*205*	*201*	6940	*1678*
	n_h	31	21	29	38	22	39	750	181
	n'_h	70	50	75	95	70	90	1874	453
	S_{yh}	883.84	644.92	1033.4	810.58	403.65	711.72	21.4256	22.1319
	S_{xh}	30486.7	15180..77	27549.69	18218.93	8497.77	23094.14	16625.33	12861.40
	S_{zh}	555.58	365.46	612.95	458.03	260.85	397.05	19394.09	16143.74
	$\overline{Y}_h$	703.74	413	573.17	424.66	267.03	393.84	47.9805	48.0556
	$\overline{X}_h$	20804.59	9211.79	14309..30	9478.85	5569.95	12997.59	18746.55	14303.98
	$\overline{Z}_h$	498.28	318.33	431.36	311.32	227.20	313.71	19124.75	14742.47
	ρ_{xyh}	0.9360	0.996	0.994	0.983	0.989	0.965	-0.4777	-0.4406
	ρ_{xzh}	0.9396	0.9696	0.9770	0.9640	0.9670	0.9960	0.9138	0.8035
	ρ_{yzh}	0.9790	0.976	0.984	0.983	0.964	0.983	-0.4422	-0.3547
W_h=10% Non-response	S_{yh2}	510.57	386.77	1872.88	1603.3	264.19	497.84	20.4752	21.7407
	S_{xh2}	9446.93	9198.29	52429.99	34794.9	4972.56	12485.10	18121.44	15492.72
	S_{zh2}	303.92	278.51	960.71	821.29	190.85	287.99	22010.50	20204.85
	ρ_{xy2}	0.9961	0.9975	0.9998	0.9741	0.995	0.9284	-0.4826	-0.5422
	ρ_{xz2}	0.9901	0.9895	0.9964	0.9609	0.9865	0.9752	0.8566	0.7691
	ρ_{yz2}	0.9931	0.9871	0.99716	0.9942	0.985	0.9647	-0.3922	-0.3181
W_h=20% Non-response	S_{yh2}	396.77	406.15	1654.4	1333.35	335.83	903.91	20.7359	22.6272
	S_{xh2}	7439.16	8880.46	45784.78	29219.3	6540.43	28411.44	16155.37	13887.44
	S_{zh2}	244.56	274.42	965.42	680.28	214.49	469.86	19251.39	17323.10
	ρ_{xy2}	0.9954	0.9931	0.996	0.9761	0.9966	0.9869	-0.4870	-0.4880
	ρ_{xz2}	0.9897	0.9884	0.9789	0.9629	0.982	0.9825	0.8845	0.8399

	ρ_{yz2}	0.9898	0.9798	0.9846	0.994	0.9818	0.9874	-0.4293	-0.3304
W_h=30% Non-response	S_{yh2}	500.26	356.95	1383.7	1193.47	289.41	825.24	21.4660	22.4381
	S_{xh2}	14017.994	7812.00	38379.77	26090.6	5611.32	24571.95	16877.33	12852.95
	S_{zh2}	284.4409	247.6279	811.21	631.28	188.30	437.90	19985.52	16007.36
	ρ_{xy2}	0.9639	0.9919	0.9955	0.9801	0.9961	0.9746	-0.4808	-0.4395
	ρ_{xz2}	0.9107	0.9848	0.9771	0.9650	0.9794	0.9642	0.8939	0.8298
	ρ_{yz2}	0.9739	0.9793	0.9839	0.9904	0.9799	0.9829	-0.4347	-0.2823