EXACT ANALYTICAL SOLUTIONS FOR MAXWELL FLUID OVER AN OSCILLATING PLANE

Kashif Ali Abro*, Asif Ali Shaikh
Department of Basic Sciences and Related Studies, Mehran UET, Jamshoro
*Correspondence should be addressed to Kashif Ali,
kashif.abro@faculty.muet.edu.pk

ABSTRACT: *This analysis deals with the exact analytical solutions for the velocity field $u(y,t)$ and the adequate shear stress $\tau(y,t)$ for Maxwell fluid over the sine and cosine oscillations. The solutions for the velocity field $u(y,t)$ and the adequate shear stress $\tau(y,t)$ for Maxwell fluid are investigated. These solutions are obtained by employing the approach of integral transforms (Fourier Sine and Laplace transforms). Both the results of velocity field $u(y,t)$ and shear stress $\tau(y,t)$ are written in terms of the convolutions theorem, elementary functions and simple integral forms; satisfy all the initial and boundary conditions. Newtonian fluid is the limited case, which is reduced for the expression of velocity field $u(y,t)$ and adequate shear stress $\tau(y,t)$. Finally the distinct variations of parameters are depicted for graphical illustrations in order to meet the different physical situations.*

Key Words: Oscillating plane, Maxwell and Newtonian fluids, Integral Transforms, Graphical illustrations.

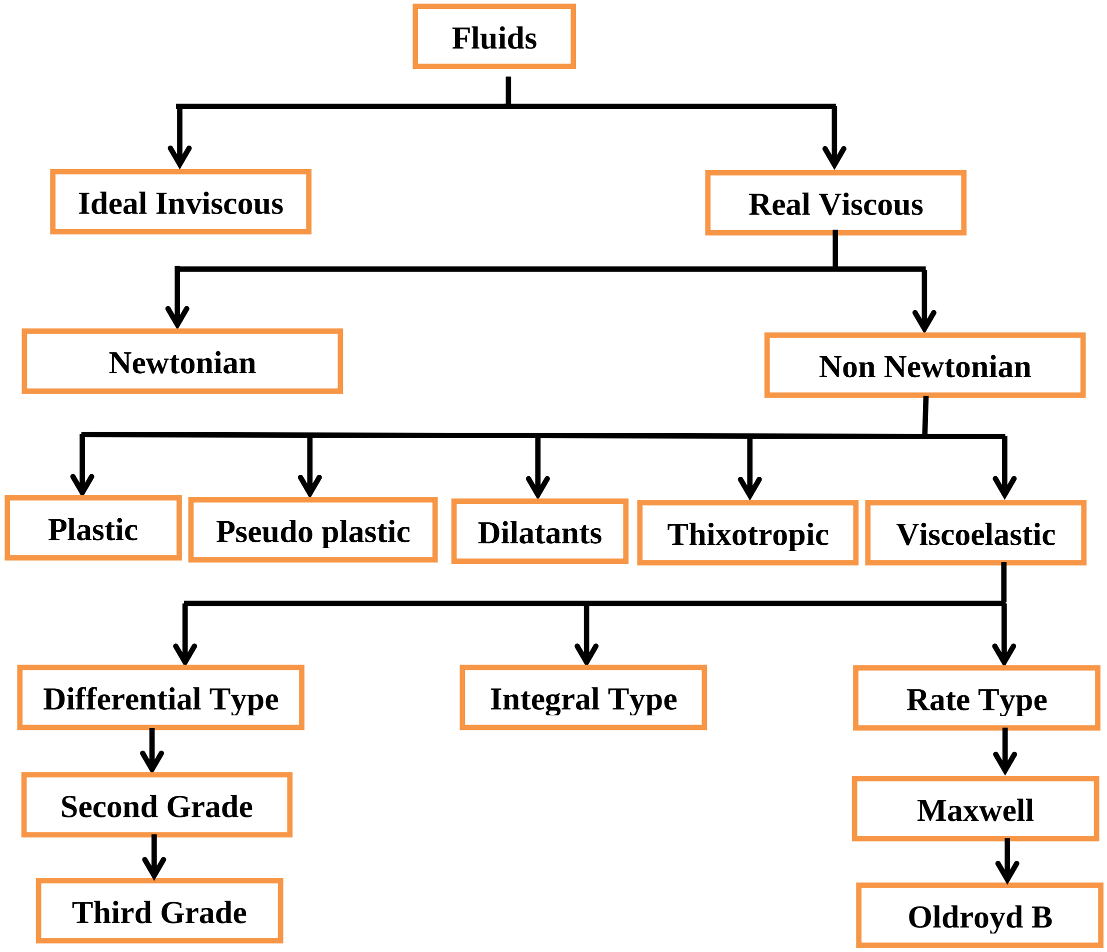
List of symbols

$u_S(y,t)$	Velocity field for sine oscillations	ξ	Fourier sine transform parameter
$\tau_S(y,t)$	Shear stress for cosine oscillations	q	Laplace transform parameter
$u_C(y,t)$	Velocity field for cosine oscillations	U	Non zero constant
$\tau_C(y,t)$	Shear stress for cosine oscillations	ω	Frequency
$u_N(y,t)$	Velocity field for Newtonian fluid	t	Time parameter
$\tau_N(y,t)$	Shear stress for Newtonian fluid	ρ	Density of the fluid
$H(t)$	Heaviside function	λ	Relaxation time parameter
μ	Viscosity of the fluid	ν	Kinematic viscosity

1. INTRODUCTION

The flow of fluid of Newtonian type is governed by Navier-Stokes equations and also these equations have become the best mechanism to recognize the flows of Newtonian fluid. The limited numbers of exact and general solutions are present in the literature for the Navier-Stokes equations. For the sack of exactness and approximations, exact solutions are important because they characterize the basic behavior of flows. On the other hand, the study of non-Newtonian fluid is definitely of great challenging, interesting and inspiring due to the different phenomena of flows of real life applications, technological advancement and in many areas just like: oil exploitation, food industry, pumping of blood, polymer fluids, suspension and colloidal solutions and many more others. As the Maxwell fluid lies in the category of viscoelastic fluid has gotten the Superior consideration and major attention. The linear relationship between shear stress and shear rate can be predicted by Maxwell model which have small and large dimensionless relaxation time. For the sack of Maxwell fluid lying in the category of viscoelastic fluid, we have shown a figure below:

The vast study is present in literature for Maxwell fluid; we shall mention here some closed and recent papers [6, 11, 14, 15, 16, 20, 22,] in Maxwell fluid for oscillating motions. Generally the velocity field is the dependent case of shear stress, and almost the governing partial differential equations are derived for the slip and no slip effects or conditions depending upon the shear stress. The viscoelastic fluid for the unsteady motions between two side perpendicular walls for oscillating shear stress has been discussed in [8]. Furthermore some relevant results regarding the Maxwell and Newtonian fluids for the motion of oscillating plate are in the references [1, 9, 10]. Oscillating flows of Maxwell and Newtonian fluid are of great interest can be found in the references [26-29] with slip and no slip effects and conditions. Moreover; For the Maxwell fluid, the exact solution for first problem of stoke's is discussed in [12, 13, 17, 25]. The purpose of this analysis is to investigate the exact analytical solutions for Maxwell fluid over the sine and cosine oscillations in the consideration and assumption of $y > 0$ for the Cartesian coordinate system $Oxyz$. The solutions are obtained by solving the governing differential equations by using



the approach of integral transforms (Fourier sine and Laplace transform) for the Maxwell fluid over an oscillating plane. These solutions satisfy the initial $u(y,0)=\frac{\partial u(y,0)}{\partial t}=0,\ \tau(y,0)=0$ and boundary conditions $u(0,t)=UH(t)sin\omega t\ \ or\ \ UH(t)cos\omega t,\ t\geq 0.$ The similar solutions are reduced and particularized for the motion of Newtonian fluid. In order to meet the physical aspects, the above descriptions are concluded under the numerical discussion, graphical illustrations and results.

2. The Governing Partial Differential Equations

Under consideration of incompressible Maxwell fluid is given by the constitutive equations [62-71]

$$T=-pI+S\ \ S+\lambda\frac{\delta S}{\delta t}=\mu A_1, \qquad (2.1)$$

where Cauchy stress tensor **T**, −p**I** denotes the indeterminate spherical stress, **S** is the extra stress tensor, λ is the relaxation time, μ the dynamic viscosity and the first Rivlin Ericksen tensor A_1 is expressed as

$$A_1=L+L^T,$$

$$\frac{\delta S}{\delta t}=\frac{dS}{dt}-LS-SL^T=\dot{S}-LS-SL^T, \qquad (2.3)$$

is the upper convected derivative . For the problem under consideration I assume a velocity field **V** and an extra-stress tensor **S** of the form

$$V=V(y,t)=u(y,t)\mathbf{i},\qquad S=S(y,t), \qquad (2.4)$$

Where, the unit vector is denoted by **i** along the x-coordinate direction. For these flows the constraint of incompressibility is automatically satisfied. Here the model characterized by the constitutive equations contains as a special case the Newtonian fluid model for $\lambda\rightarrow 0$. If the fluid is at rest up to the moment *t* = 0, then

$$V=(y,0)=0,\qquad S=(y,0)=0, \qquad (2.5)$$

and Equations (2.1) and (2.4) imply $S_{yy}=S_{yz}=S_{zz}=S_{xz}=0$, and gives

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\tau(y,t)=\mu\frac{\partial u(y,t)}{\partial y}, \qquad (2.6)$$

In the absence of body forces, the balance of linear momentum reduces to

$$\frac{\partial\tau(y,t)}{\partial t}-\frac{\partial p}{\partial x}=\rho\frac{\partial u(y,t)}{\partial t}, \qquad (2.2) \qquad (2.7)$$

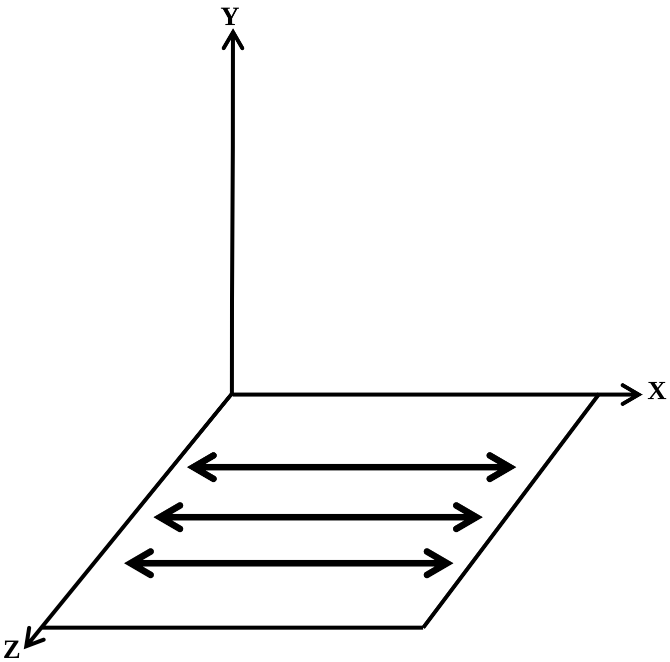


Figure 3.1: Geometry of the Problem.

where $\tau(y,t) = S_{xy}(y,t)$ are the non-zero shear stresses. Eliminating τ between Equations (2.6) and (2.7), we have the governing partial differential equations under the expression

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u(y,t)}{\partial y} = -\frac{1}{\rho}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u(y,t)}{\partial^2 t}; \quad y,t>0, \qquad (2.8)$$

Where, the kinematic viscosity of the fluid is $\nu = {}^{\mu}/_{\rho}$. In the absence of pressure gradient the governing differential equations for an incompressible Maxwell fluid performing the same motion are

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial^2 t}, \qquad (2.9)$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\tau = \mu\frac{\partial u}{\partial y}. \qquad (2.10)$$

$u(0,t) = UH(t)\,sin\omega t \;\; or \;\; UH(t)\,cos\omega t.$

Consider an incompressible Maxwell fluid occupying the space above a flat plate perpendicular to the *y*-axis. Initially, the fluid is at rest and at the moment $t = 0^+$ the plate is impulsively brought to the constant velocity *U* in its plane. Due to the tangential shear stress, the fluid above the plate is slowly and gradually moved. The relevant problem under initial and boundary conditions is

$$u(y,0) = \frac{\partial u(y,0)}{\partial t} = 0, \quad \tau(y,0) = 0, \quad y>0, \qquad (2.11)$$

$$u(0,t) = UH(t)sin\omega t \quad or \quad UH(t)cos\omega t \qquad t \geq 0. \qquad (2.12)$$

Additionally, the natural environment of conditions with $H(t)$ is the Heaviside function.

$$u(y,t), \quad \frac{\partial u(y,t)}{\partial t} \to 0 \; as \; y \to \infty \quad and \quad t>0, \qquad (2.13)$$

have to be also satisfied. They are consequences of the fact that the fluid is at rest at infinity and there is no shear in the free stream.

3. CALCULATION OF THE VELOCITY FIELD

3.1 The case $u(0,t) = UH(t)sin\omega t$

In order to solve the governing partial differential equation (2.9), possessing in mind the initial and boundary conditions (2.11) and (2.12), we shall apply the Fourier sine transforms w.r.t spatial variable. Consequently multiplying both sides of (2.9) $\sqrt{2/\pi}\,\sin(y\xi)$, integrating the result with respect to *y* from 0 to infinity, we attain

$$\left\{\frac{\partial u_s(\xi,t)}{\partial t} + \lambda\frac{\partial^2 u_s(\xi,t)}{\partial^2 t}\right\} = \left\{-\nu\xi^2 u_s(\xi,t) + \sqrt{\frac{2}{\pi}}\nu\xi\, UH(t)sin\omega t\right\}, \qquad (3.1)$$

Where $H(t)$ is the Heaviside function, $u_s(\xi,t)$ is the Fourier sine transform and has to satisfy the initial conditions

$$u_s(\xi,0) = \frac{\partial u_s(\xi,0)}{\partial t} = 0, \quad \xi>0, \qquad (3.2)$$

By applying the Laplace transform to (3.1) and having in mind the initial conditions (2.11), we find that

$$\overline{u_s}(\xi,q) = \sqrt{\frac{2}{\pi}}\frac{U\nu\xi\omega}{(q^2+\omega^2)[\lambda q^2+q+\nu\xi^2]}, \qquad (3.3)$$

Now, for a more suitable representation of the final results, we rewrite (3.3) in the following equivalent form

$$\overline{u_s}(\xi,q)=\frac{U\omega}{\xi}\sqrt{\frac{2}{\pi}}\left[\frac{1}{(q^2+\omega^2)}-\frac{q(1+\lambda q)}{(q^2+\omega^2)[\lambda q^2+q+\nu\xi^2]}\right], \tag{3.4}$$

Inverting (3.4) by means of the Fourier sine formula, we can write $\overline{u_s}(\xi,q)$ as

$$\bar{u}(y,q)=\frac{2U\omega}{\pi}\int_0^\infty\frac{\sin(y\xi)}{\xi}\left[\frac{1}{(q^2+\omega^2)}-\frac{q(1+q\lambda)}{(q^2+\omega^2)[\lambda q^2+q+\nu\xi^2]}\right]d\xi, \tag{3.5}$$

Finally in order to have the expression for velocity field $u(y,t)=\mathcal{L}^{-1}\{\bar{u}(y,q)\}$, we apply the inverse Laplace transform to (3.5). We find for the velocity field, the following simple expression is

$$u_S(y,t)=U\,H(t)\,sin\omega t-\frac{2\,U\,H(t)\,\omega}{\pi\,\lambda\,(q_1-q_2)}\int_0^\infty\int_0^t\frac{\sin(y\xi)}{\xi}\,cos\omega(t-u)$$
$$\times\{(1+\lambda q_1)e^{q_1u}-(1+\lambda q_2)e^{q_2u}\}\,d\xi\,du. \tag{3.6}$$

Where

$$q_1,q_2=-\frac{(1)\pm\sqrt{(1)^2-4\lambda(\nu\xi^2)}}{2\lambda} \tag{3.7}$$

are the roots of the algebraic equation $\lambda q^2+q+\nu\xi^2=0$.

3.2 The case $u(0,t)=UH(t)cos\omega t$

Employing the same procedure, we have investigated the solution

$$u_C(y,t)=U\,H(t)\,cos\omega t-\frac{2\,U\,H(t)\,\omega}{\pi\,\lambda\,(q_1-q_2)}\int_0^\infty\int_0^t\frac{\sin(y\xi)}{\xi}\,sin\omega(t-u)$$
$$\times\{(1+\lambda q_1)e^{q_1u}-(1+\lambda q_2)e^{q_2u}\}\,d\xi\,du. \tag{3.8}$$

4. Calculation of the Shear Stress

4.1 The case $u(0,t)=UH(t)sin\omega t$

The corresponding shear stress can be found by apply Laplace transform to equation (2.10), we find that

$$\bar{\tau}(y,q)=\frac{\mu}{(1+\lambda q)}\,\frac{\partial\bar{u}(y,q)}{\partial y}, \tag{4.1}$$

where $\bar{\tau}(y,q)$ is the Laplace transform of $\tau(y,t)$, Now differentiating partially equation (3.5) with respect to y, we get

$$\frac{\partial\bar{u}(y,q)}{\partial y}=\frac{2U\omega}{\pi}\int_0^\infty\cos(y\xi)\left[\frac{1}{(q^2+\omega^2)}-\frac{q(1+q\lambda)}{(q^2+\omega^2)[\lambda q^2+q+\nu\xi^2]}\right]d\xi, \tag{4.2}$$

Employing equation (4.2) in (4.1),

$$\bar{\tau}(y,q)=\frac{\mu}{(1+\lambda q)}\left[\frac{2U\omega}{\pi}\int_0^\infty\cos(y\xi)\left\{\frac{1}{(q^2+\omega^2)}-\frac{q(1+q\lambda)}{(q^2+\omega^2)[\lambda q^2+q+\nu\xi^2]}\right\}d\xi\right], \tag{4.3}$$

On simplification equation (4.3)

$$\bar{\tau}(y,q)=\frac{2U\mu\omega}{\pi}\int_0^\infty\cos(y\xi)\left\{\frac{1}{(q^2+\omega^2)}-\frac{q}{(q^2+\omega^2)[\lambda q^2+q+\nu\xi^2]}\right\}d\xi, \tag{4.4}$$

Applying inverse Laplace transform in (4.4) we get

$$\tau_S(y,t)=-\frac{2\,U\,H(t)\omega\,\mu}{\pi\,\lambda\,(q_1-q_2)}\int_0^\infty\int_0^t\cos(y\xi)\,cos\omega(t-u)(e^{q_1u}-e^{q_1u})d\xi\,du \tag{4.5}$$

are obtained.

4.2 The case $u(0,t)=UH(t)sin\omega t$

We have investigated the corresponding shear stress by applying similar manners, we get

$$\tau_C(y,t)=-\frac{2\,U\,H(t)\omega\,\mu}{\pi\,\lambda\,(q_1-q_2)}\int_0^\infty\int_0^t\cos(y\xi)\,sin\omega(t-u)(e^{q_1u}-e^{q_1u})d\xi\,du \tag{4.6}$$

5. LIMITING CASE

5.1. Newtonian Fluid

Taking the limit $\lambda\to 0$ into equations (3.6), (4.5), (3.8) and (4.6) and using following facts or equations (5.1), (5.2) and (5.3)

$$\lim_{\lambda\to 0}q_1=-\nu\xi^2, \tag{5.1}$$

$$\lim_{\lambda\to 0}q_2=-\infty, \tag{5.2}$$

$$\lim_{\lambda\to 0}\lambda(q_1-q_2)=1 \tag{5.3}$$

the solutions for Newtonian fluid for the velocity field and the shear stress

$$u_{SN}(y,t)=UH(t)sin\omega t-\frac{2\,U\,H(t)\,\omega}{\pi}\int_0^\infty\int_0^t\frac{\sin(y\xi)}{\xi}cos\omega(t-u)$$
$$\times\,e^{-\nu\xi^2u}\,d\xi\,du, \tag{5.4}$$

$$\tau_{SN}(y,t)=-\frac{2\,U\,H(t)\,\omega\,\mu}{\pi}\int_0^\infty\int_0^t\cos(y\xi)\,cos\omega(t-u)\,e^{-\nu\xi^2u}\,d\xi\,du. \tag{5.5}$$

$$u_{CN}(y,t)=UH(t)cosh\omega t-\frac{2\,U\,H(t)\,\omega}{\pi}\int_0^\infty\int_0^t\frac{\sin(y\xi)}{\xi}sin\omega(t-u)$$
$$\times\,e^{-\nu\xi^2u}\,d\xi\,du, \tag{5.6}$$

$$\tau_{CN}(y,t)=-\frac{2\,U\,H(t)\,\omega\,\mu}{\pi}\int_0^\infty\int_0^t\cos(y\xi)\,sin\omega(t-u)\,e^{-\nu\xi^2u}\,d\xi\,du. \tag{5.7}$$

are achieved.

6. Numerical Results and Discussion

The main objectives of this analysis is to establish the the exact analytical solutions for Maxwell fluid over the sine and cosine oscillations under the assumption of $y>0$ for the Cartesian coordinate system $Oxyz$. The solutions are obtained by solving the governing partial differential equations by using the technique of integral transforms (Fourier sine and Laplace transform) for the Maxwell fluid over an oscillating plane. These solutions satisfy the initial $u(y,0)=\frac{\partial u(y,0)}{\partial t}=0,\ \tau(y,0)=0$ and boundary conditions $u(0,t)=UH(t)sin\omega t\ \ or\ UH(t)cos\omega t,\ t\geq 0.$ The general solutions are reduced and particularized for the motion of Newtonian fluid when $\lambda\to 0$ performing the similar motion. In brevity, some results are obtained for physical aspects and features from the diagrams of the velocity field and the shear stress against the distinct values of material constants $t,\ \lambda,\ \nu,\ \omega,\ y$. The following major findings indicate the results categorically:

- The solutions are investigated for both sine as well as cosine oscillations satisfying the initial and boundary conditions.
- The solutions (3.6), (4.5), (3.8) and (4.6) are specialized for Newtonian fluid when $\lambda \to 0$.
- As the velocity field and the shear stress of fluid is decreasing function with respect time t and amplitude ω.
- The relaxation time λ has the point of intersecting on the motion of fluid for viscoelasticity.
- It should be noted that the velocity field of fluid is increasing function along with the shear stress with respect kinematic viscosity ν.
- It is noted that for different values of y the motion of fluid effect is oscillating.

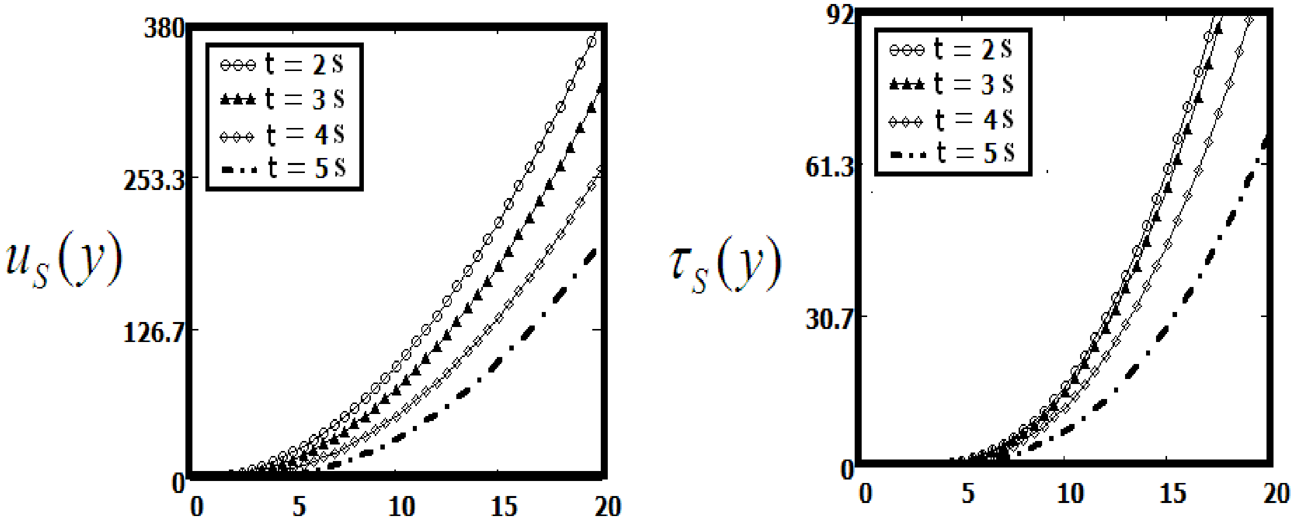


Figure 1: Profiles of the velocity field $u_s(y,t)$ and the shear stress $\tau_s(y,t)$ for Maxwell fluid given by equations (3.6) and (3.8), for $U = 1,\ \nu = 0.63,\ \mu = 1.52,\ \lambda = 2, \omega = 2$ and different values of *t*.

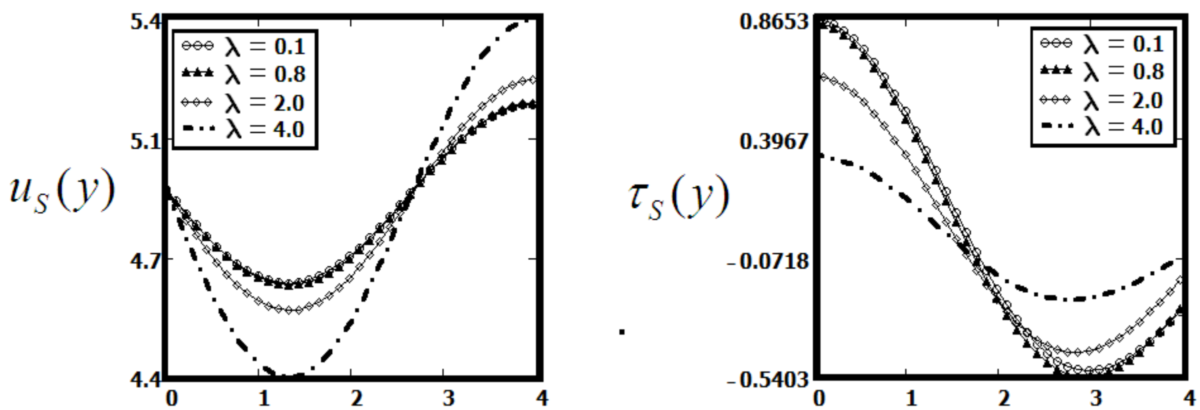


Figure 2: Profiles of the velocity field $u_s(y,t)$ and the shear stress $\tau_s(y,t)$ for Maxwell fluid given by equations (3.6) and (3.8), for $U = 1,\ \nu = 0.63,\ \mu = 1.52,\ t = 2\ s,\ \omega = 2$ and different values of λ.

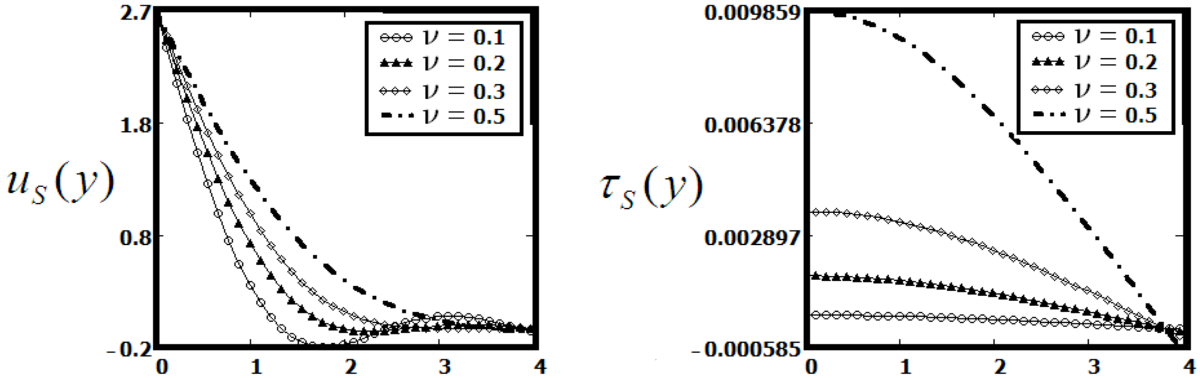


Figure 3: Profiles of the velocity field $u_s(y,t)$ and the shear stress $\tau_s(y,t)$ for Maxwell fluid given by equations (3.6) and (3.8), for $U = 1,\ \rho = 2.413,\ \mu = 1.52,\ t = 2\ s,\ \lambda = 2$ and different values of ν.

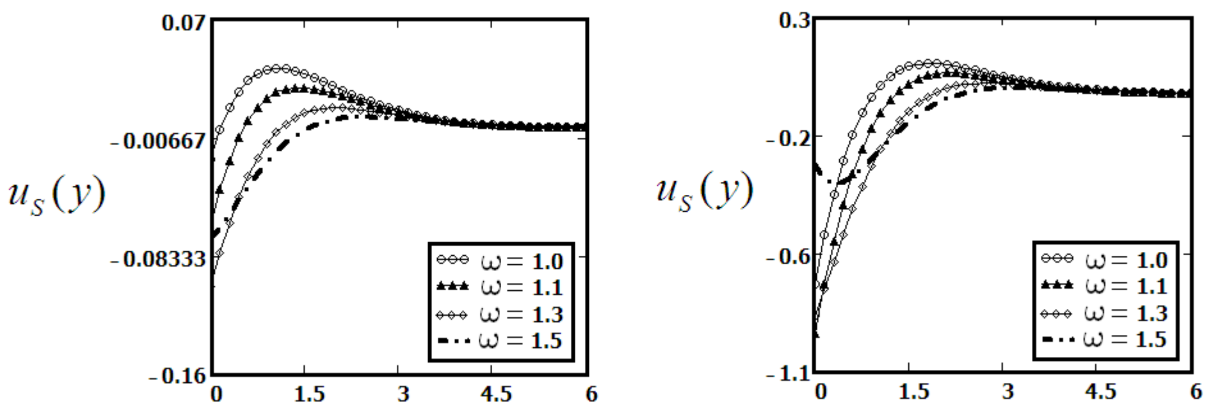


Figure 4: Profiles of the velocity field $u_s(y,t)$ and the shear stress $\tau_s(y,t)$ for Maxwell fluid given by equations (3.6) and (3.8), for $U = 1,\ \nu = 0.63,\ \mu = 1.52,\ t = 2\ s,\ \lambda = 2$ and different values of ω.

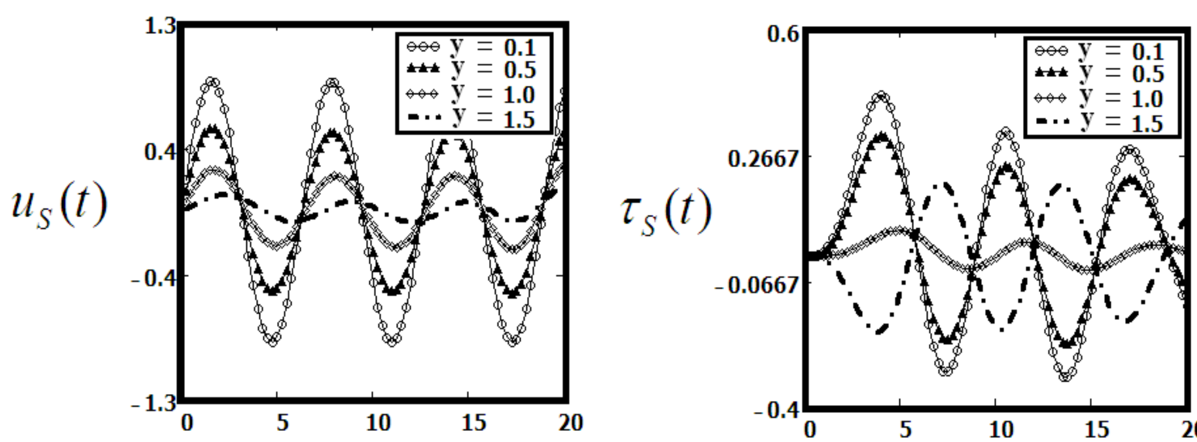


Figure 5: Profiles of the velocity field $u(y,t)$ and shear stress $\tau(y,t)$ for MHD Maxwell fluid given by Eqs. (2.18) and (2.28), for $U = 1,\ \nu = 0.63,\ \mu = 1.52,\ t = 2\ s,\ n = 2$ and different values of y.

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