Coded Wavelet OFDM Based Communication System for Rayleigh Fading Channel

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ABSTRACT: Orthogonal frequency division multiplexing (OFDM) technique mitigates the effects of fading channels. However, due to its low spectral efficiency and signal interference it becomes disadvantageous. Therefore, Wavelet based OFDM (WOFDM) system is proposed in literature that offers better spectral efficiency with added robustness against fading and interference suppression. This paper evaluates the performance of the special features of convolutionally coded WOFDM system i-e vanishing moments and wavelet filters with convolutionally coded DFT-OFDM system with varying doppler shifts in time varying frequency selective rayleigh fading channel. The simulation results show that WOFDM outperforms DFT-OFDM system due to its special property of cancelling any possible distortion and aliasing effects through its wavelet filter banks.

Keywords: MCM, WOFDM, FFT-OFDM, ISI, ICI

1 INTRODUCTION

Multi-carrier modulation (MCM) is a spectral efficient transmission technique. It is widely used for high speed data communication because it makes the communication system more robust against channel and multipath fading effects [1, 2]. Orthogonal Frequency Division Multiplexing (OFDM) is one of the mature and cost-effective MCM techniques that utilize Fast Fourier Transform (FFT) for data modulation [3, 4]. FFT-OFDM transmits the data reasonably well with improved spectral efficiency in multipath fading environment due to its overlapped and orthogonal sub-carriers [2-6]. Therefore, FFT-OFDM system is effectively applied in both wireless and wired technologies. Wireless OFDM system is implemented in various standards such as HiperLAN/2 and IEEE 802.11. In addition to this, wired OFDM system is successfully employed in ADSL, ADSL2+, VDSL, etc [7]. Another potential communication system, LTE is also likely to use OFDM as its basic modulation scheme in future [5].

Even though conventional OFDM system has proved its worth for many decades, it is accompanied by some disadvantages. The performance of an OFDM system can be degraded due to time synchronization errors, increased amount of inter modulation distortion called high Peak-to-Average Power Ratio (PAPR), phase noise and most importantly high energy side-lobes that are only 15 dB lower than the main lobe [1,2,7-11]. In case of frequency offset, the power leakage due to these side-lobes can generate harmful inter-carrier interference (ICI) [4,9]. Researchers have worked extensively to solve these issues and have proposed utilizing Wavelet Transform (WT) in conjunction with OFDM to provide solutions to these problems that are published in [1,2,4,9,10]. Wavelet based OFDM (WOFDM) system has emerged as an efficient MCM technique in the field of digital modulation [1,11]. It covers nearly all the pros of conventional OFDM system such as robustness against multipath fading, bandwidth efficiency and gives improved performance regarding typical OFDM drawbacks such PAPR, carrier frequency offset and synchronization errors [6,8,11].

The basic theory of conventional OFDM and WOFDM system share many similarities in terms of their functions [1, 3,10]. Both of these techniques have orthogonal sub-carriers, the overlapping of which makes them spectrally efficient [3, 10]. But at the same time, they have also some distinctive features that make them different from each other. In conventional OFDM system, sub-carriers overlap in frequency domain, whereas in WOFDM system, sub-carriers overlap in both time and frequency domain [1,3,11]. Overlapping in time domain is due to the fact that the waveforms used in WOFDM system are longer than the transform duration of one symbol. WOFDM symbols fulfil the property of double shift orthogonality and therefore, their overlapping does not cause ISI and does not require Cyclic Prefixing (CP) [3,10,11]. Hence, making WOFDM system more bandwidth efficient compared to conventional OFDM system [3,6,11].

Wavelet used in WOFDM system offers flexibility and alterations in its design parameters and therefore, makes itself more robust against synchronization errors, PAPR, ISI and ICI without compromising bandwidth efficiency [1-3, 11]. WOFDM is globally standardized as IEEE P1901 for high speed power line communication (PLC) devices with data rate greater than 100 Mbps at the physical layer. However, for wireless communication, WOFDM is not yet standardized [12].

This research work investigates the Bit Error Rate (BER) performance of FFT-OFDM and WOFDM system using IEEE 802.11a standard in time varying frequency selective Rayleigh fading channel with applied coding and Least Square (LS) channel estimation technique. For WOFDM system, Daubechies (db) wavelet with varying vanishing moments is used in simulations for analysis. This paper in Section 2, converses about the system model of FFT-OFDM system. Section 3 and 4 gives the concept of Discrete Wavelet Transform (DWT) and its implementation using filter banks respectively. Section 5 describes the system model of WOFDM. Section 6 discusses the applications of WOFDM in wireless communication. The results with different simulation scenarios for FFT-OFDM and WOFDM system models are discussed in Section 7, followed by conclusion in Section 8.

2 FOURIER TRANSFORM BASED OFDM SYSTEM MODEL

MCM systems like OFDM are computationally complex because it uses separate modulator and demodulator for each of its sub-carriers [13]. Therefore, for its simple and cost
In this implementation, Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT) algorithms were proposed [13,14].

The fundamental block diagram of OFDM transmitter and receiver using IFFT and FFT is shown in Fig. (1). At the transmitting side, after coding and interleaving, the input serial data stream $S[K]$ is modulated to $R$ bps. The output consists of $N$ parallel streams from $X[0]$ to $X[N-1]$, each of rate $\frac{R}{N}$ bps, where $N$ is the total number of sub-carriers. The streams are converted to time domain using IFFT algorithm that is $x[n] = x[0], x[1], x[2], \ldots, x[N-2], x[N-1]$ defined in eq. (1) as [14].

$$\text{IFFT}\{X[K]\} = x[n] = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} X[K] e^{j2\pi Kn/N} \quad (1)$$

Where, $0 \leq n \leq N-1$. The data streams out of IFFT block are again converted into serial form through parallel to serial converter. In order to avoid ISI, that is the interference between the two neighboring OFDM symbols, Cyclic Prefix (CP) of length $\mu$ is introduced at the start of the neighboring symbols giving new OFDM symbol sequences of length $N + \mu$, $X_{OFDM}[n] = x[N - \mu], x[N - \mu + 1], \ldots, x[N - 1], x[0], \ldots, x[N - 1]$, where $\mu$ is the length of the appended CP [14].

Data symbols after the addition of CP are equivalent to channel length, sent to the receiver through the wireless channel, where noise is added. The received data symbols, $y_{OFDM}[n] = y[N - \mu], y[N - \mu + 1], \ldots, y[N - 1], y[0], \ldots, y[N - 1]$, from which CP of length $\mu$ is removed and ISI free data is received. These data symbols are converted from serial to parallel format and then FFT algorithm is applied, described in eq. (2) as [14],

$$\text{FFT}\{y[n]\} = S_e[K] = 1/\sqrt{N} \sum_{n=0}^{N-1} y[n] e^{-j2\pi Kn/N} \quad (2)$$

Where, $0 \leq K \leq N-1$. The resulting frequency components out of FFT block are parallel to serial converted and then equalized, demodulated, de-interleaved and decoded to recover the transmitted data stream, $M[K]$, $S_e[K]$ is the estimate of the original transmitted data sequence $S[K]$. The applied FFT based OFDM system is beneficial due to its low computational complexity and cost but due to addition of CP, it is disadvantageous in terms of low bandwidth efficiency [14]. WOFDM and its system model are discussed in the next section.

### 3 DISCRETE WAVELET TRANSFORM

Discrete Wavelet Transform (DWT) is an important tool for time-frequency analysis that provides different time-frequency resolutions at different scales for both high and low pass filters having different cut-off frequencies [15].

Wavelet Transform can be implemented using Quadrature Mirror Filter (QMF) and gives unique signal decomposition, known as sub-band coding [16,17]. Wavelet transform is applicable in wireless communication including channel characterization, mitigation of interference, multiplexing and modulation, multiple access communication [18].

### 4 DWT IMPLEMENTATION USING FILTER BANKS

Discrete Wavelet Transform (DWT) is implemented using filter banks, shown in Fig. (2), where the input signal $x[n]$ contains both high and low frequency components that range between $0 - \pi$. The frequency components are decomposed into high and low frequency signal components by passing them through low pass filter (LPF), $h[n]$ and high pass filter (HPF), $g[n]$ respectively. After filtering, low band decomposed signal contains frequency ranging from $0 - \pi/2$ and high band decomposed signal contains the frequency ranging from $\pi/2 - \pi$. The low frequency components contain approximate signal information, called scaling functions.

![Fig.1. FFT-OFDM Transmitter and Receive](image-url)

The high frequency components contain detailed information, called wavelet functions [17,19]. After
decomposition, the two signals are downsampled by two to satisfy the Nyquist criteria [19]. The decomposition at this stage is called single level decomposition. High and low frequency component decomposition is represented mathematically in form of eq. (3) and eq. (4) respectively as [18],

\[ y_{\text{high}}[k] = \sum_n x[n]g[2k - n] \]  
\[ y_{\text{low}}[k] = \sum_n x[n]h[2k - n] \]

Where, \( y_{\text{high}}[k] \) is the resultant high frequency component, \( y_{\text{low}}[k] \) is the low frequency component, \( g[n] \) is high pass analysis filter and \( h[n] \) is low pass analysis filter. For second level decomposition, high band decomposed signal is ignored and the low band decomposed signal ranging from \( 0 - \pi/2 \) is further decomposed using the same procedure of filtering and downsampling. This process is iteratively repeated until the desired time-frequency resolution is attained [16], [19].

In DWT, the signal can be reconstructed back through inverse procedure that is through upsampling and then filtering operations, shown in eq. (5) and eq. (6) as [18],

\[ x[n]_{\text{high}} = \sum_n y_{\text{high}}[k]g[2k - n] \]  
\[ x[n]_{\text{low}} = \sum_n y_{\text{low}}[k]h[2k - n] \]

Where, \( h[n] \) is the low pass synthesis filter and \( g[n] \) is the high pass synthesis filter.

According to the property of Quadrature Mirror Filter (QMF), both analysis and synthesis low and high pass filters that are used for signal decomposition and reconstruction are inverse conjugate of each other, as shown in eq. (7) and eq. (8) as [20],

\[ h[n] = g^*[n] \]  
\[ g[n] = h^*[n] \]

\[ \text{Fig.2. Filterbank implementation of DWT} \]

5 SYSTEM MODEL OF WOFDM

When the idea of OFDM system comes together with Discrete Wavelet Transform (DWT) it yields the concept of WOFDM, thereby showing advantageous features in both time and frequency domain. These features specifically include Orthogonality and multi-resolution analysis. Orthogonal basis function composed of wavelets reduces the interference that was found in FFT-OFDM system due to its sines-cosines basis function [21]. The working model of Wavelet based OFDM system is shown in fig. (3).

The transmitter side of WOFDM system offers encoding followed by an interleaving along with modulation and Inverse Wavelet Transform (IWT). The Wavelet Transform does not require CP since its carriers overlap both in time and frequency domain [22]. The output \( s_0[n] \) after inverse Wavelet Transform is represented through eq. (9) and eq. (10) as scaling and wavelet functions, \( v_0[n] \) and \( u_0[n] \) respectively as [19],

\[ v_0[n] = \frac{1}{\sqrt{M}} \sum_{j=0}^{M-1} W_\psi(j_0, k) \varphi_{j_0,k}[n] \]  
\[ u_0[n] = \sum_{j=0}^{M-1} \sum_{k=0}^{\infty} W_\psi(j, k) \psi_{j,k}[n] \]  
\[ s_0[n] = v_0[n] + u_0[n] \]

Where, \( W_\psi(j_0,k) \) is the input signal to scaling function, \( W_\psi(j,k) \) is the input signal to wavelet function, \( M \) is the total number of carriers, \( n \) is the time parameter with range \( 0 \leq n \leq M - 1 \), \( j, j_0 \) are the scale parameters, \( k \) is the shift parameter, \( \psi_{j,k}[n] \) is the transform kernel for wavelet function and \( \varphi_{j_0,k}[n] \) is the transform kernels for scaling function. The number of branches, \( M \) in which the serial data is decomposed in WOFDM depends on \( m \) number of iterations or levels, i.e. \( M = 2^m \) [22].

At the receiving side, after Wavelet Transform (WT), \( s_1[n] \) can be represented in form of scaling and wavelet functions through eq. (12) and eq. (13) respectively as [19],

\[ W_\psi(j, k) = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} s_1[n] \psi_{j,k}[n] \]  
\[ W_\psi(j, k) = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} s_1[n] \psi_{j,k}[n] \]

Where, \( s_1[n] = s_0[n] * h[n] + N_{\text{AWGN}}[n] \). The final data bits are recovered after performing equalization, demodulation, de-interleaving and decoding. WOFDM is advantageous when compared to other transforms because the characteristics of the filter design could be changed according to the requirement, thus offering greater flexibility in its transform [22, 23].

6 APPLICATIONS OF WOFDM IN WIRELESS COMMUNICATION

Wavelet Transform based OFDM System (WOFDM) has many momentous applications in the field of wireless communication. Such as, channel characterization, interference mitigation, de-noising, modulation and multiplexing [24].
The existing working model of wireless communication system with time invariant channels are performing sufficiently well as compared to the system models with time variant channels [25]. Therefore, in order to improve the performance of the wireless system with time variant channels, WOFDM system can be utilized to model these channels in an efficient manner due to its unique property of time-frequency localization [24,25]. WOFDM performs channel modeling through accurate characterization of time variant and frequency selective fading channels [24].

In wireless communication system, interferences such as Inter Symbol Interference (ISI) and Inter Carrier Interference (ICI) are the major source of signal distortion. WOFDM system through its flexible wide range of waveforms, has the ability to alleviate the signal from the harmful effects of these interference and noise [24,26].

WOFDM system performs modulation and overlapped orthogonal multiplexing through its finite time duration wavelet carriers [23,26]. This orthogonal multiplexing of wavelet subcarriers is significant and leads to the development of a new multiplexing technique, wavelet division multiplexing (WDM) [23,26].

7 SIMULATION RESULTS
The performance of WOFDM system is compared with FFT-OFDM system through MATLAB simulation under identical scenarios. The comparative graphical results for the system models are plotted between BER and Energy per bit to noise density ratio, $\frac{E_b}{N_0}$. The simulation parameters along with the specifications of the standard, IEEE 802.11a are mentioned in table (1).

A. Performance Comparison of FFT-OFDM & WOFDM System Based on Filter Bank Model in Multipath Interference
When parallel streams of low rate data are transmitted through filter bank based conventional and wavelet based OFDM system over multipath wireless channel, the orthogonality of the sub-channel is disturbed and causes ISI and ICI [27]. The response of the multipath channel can be modeled as eq. (14) [27, 28].

$$h_{ch} = c_1 \delta(n) + c_2 \delta(n - \tau) \quad (14)$$

Where, $\tau$ is the excess delay of the channel, $c_1,c_2$ are multipath attenuation constants. The researchers in [27], [28] and [29] discuss the addition of ICI and ISI in the transmitted signal and their respective powers can be calculated as shown in eq. (15) and (16).

$$P_{ISI} = \sum_{k=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \left| h_i(m - \tau)h'_i(m - n_k) \right|^2 \quad (15)$$

$$P_{ICI} = \sum_{j=0}^{N-1} \sum_{k=-\infty}^{+\infty} \left| \sum_{m=-\infty}^{+\infty} h_j(m - \tau)h'_j(m - n_jk) \right|^2 \quad (16)$$

Where, $m, k$ are determined by the length of the filter, $N$ is the total number of sub-channels, $n_i, n_j$ is the sampling rate of the particular $j^{th}$ transmitting and $i^{th}$ receiving branch respectively.

Table 1. Simulation parameters of FFT-OFDM and WOFDM system models

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data sub-carriers</td>
<td>48</td>
</tr>
<tr>
<td>Pilot sub-carriers</td>
<td>4</td>
</tr>
<tr>
<td>FFT length</td>
<td>64</td>
</tr>
<tr>
<td>Length of CP in FFT-OFDM system</td>
<td>16</td>
</tr>
<tr>
<td>Level of WOFDM system</td>
<td>Level-2</td>
</tr>
<tr>
<td>Length of WOFDM data symbol</td>
<td>64 (no CP required)</td>
</tr>
<tr>
<td>Type of Wavelet</td>
<td>Daubechies-2 (db2)</td>
</tr>
<tr>
<td>Modulation technique</td>
<td>16-QAM</td>
</tr>
<tr>
<td>Data rate of DFT-OFDM with 16-QAM</td>
<td>24 Mbps</td>
</tr>
<tr>
<td>Data rate of WOFDM with 16-QAM</td>
<td>30 Mbps</td>
</tr>
<tr>
<td>Error correction code</td>
<td>$\frac{1}{2}$-convolutional code</td>
</tr>
<tr>
<td>Channel Estimation Technique</td>
<td>Comb type Least Square (LS)</td>
</tr>
<tr>
<td>Doppler Shift in Hz</td>
<td>166.56 Hz</td>
</tr>
<tr>
<td>Delay Spread</td>
<td>0.8 $\mu$s</td>
</tr>
<tr>
<td>Transmission carrier frequency</td>
<td>5.26 GHz</td>
</tr>
<tr>
<td>Total available bandwidth</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Bandwidth for each sub-carrier</td>
<td>312.5 KHz</td>
</tr>
</tbody>
</table>

Perfect Synchronization assumed
This section compares the performance of filter bank based OFDM system with wavelet filter bank with reference to interference introduced by multipath fading channel i.e. ICI and ISI. The simulations are performed with \( N = 2 \) and \( 4 \) sub-channels. Daubechies (db-2) and (db-4) are considered for wavelet filter bank model.

Comparative performance of WOFDM filter bank and DFT-OFDM filter bank is analyzed for ICI in fig. (4). ICI is introduced by multichannel transmission over multipath communication channel. Fig. (4) shows that in multichannel transmissions the averaged ICI power in case of WOFDM is comparatively less compared to DFT-OFDM system. Moreover, if the length of QMF is increased, the power of ICI is further reduced like Daubechies with length 8 reduces the power of ICI compared to Daubechies with length 4. In general for both DFT-OFDM and WP-OFDM, the power of ICI reduces with an increase in number of sub-channels [20].

Comparison between DFT-OFDM and WOFDM system on the basis of ISI is shown in fig. (5). Averaged ISI power of WOFDM system is comparatively less compared to DFT-OFDM system. The power of averaged ISI can further be reduced by increasing the length of QMF, as the performance of Daubechies with filter length 8 performs better than Daubechies with filter length 4 where, filter length represent wavelet filter coefficients. Overall averaged ISI power increases with an increase in the number of sub-channels [20], [29].

B. Performance Comparison of FFT-OFDM & WOFDM System Based on Coding and Doppler Shifts

Both FFT-OFDM and WOFDM system models are simulated using 1/2 rate convolutional coding followed by interleaving. According to fig. (6), the comparative BER result with reference to coding shows that Wavelet based OFDM system performs much better than FFT based OFDM system. At \( BER \) of \( 10^{-3} \), WOFDM has approximately a marginal \( E_b/N_0 \) difference of about 2.3 dBs compared to FFT-OFDM in frequency selective Rayleigh fading channel with additive white Gaussian noise.

The better performance of WOFDM is attributed to its lower side lobes. Side lobes in the transmitted data symbols of WOFDM is 45 dB lower than its main lobe, while in FFT-OFDM system, the side lobes are just 13 dB lower than its main lobe [7]. It means the energy contained within side lobes in WOFDM is very much lower as compared to FFT-OFDM. Due to these low energy side lobes, interference in frequency selective time varying channel is lower for Wavelet OFDM compared to FFT-OFDM thereby, reducing ICI.

The second comparative scenario deals with the performance of WOFDM and DFT-OFDM systems on the basis of different Doppler shifts. The comparative performance result of the two system models with different velocities are computed in kilometer per hour (km/h). According to the result shown in fig. (7), the BER performance of WOFDM and FFT-OFDM systems is proportional to relative speed found between transmitter.
and receiver. The performance of the two OFDM system models decreases with the increase in velocity. When the velocity is 34 km/h, WOFDM system has approximately a marginal $\frac{E_{b}}{N_{0}}$ difference of about 2.3 dBs, when compared with FFT-OFDM for the same BER of $10^{-3}$. However, for much higher velocity the performance of both FFT-OFDM and WOFDM is poor but still WOFDM performs better than FFT-OFDM system. Conclusively, WOFDM gives improved results compared to FFT-OFDM system due to its lower side-lobe energy.

Another scenario evaluates the performance WOFDM using wavelet’s various vanishing moments i.e. Daubechies (db) wavelet at 2, 8, 16 and 22. The result from fig. 8 shows that at lower $\frac{E_{b}}{N_{0}}$ values i.e. less than 15 dBs, Daubechies wavelet with its varying vanishing moments shows more or less the same behavior. However, with $E_{b}/N_{0}$ values greater than 15 dBs, Daubechies with different vanishing moments give different results. Overall, the performance of db22 is better when compared to db16, db8, db2. The worse performance is observed by db2. Conclusively, Daubechies with higher vanishing moment perform better than Daubechies with lower vanishing moment.

The above result is supported by the fact that the vanishing moment of a wavelet is directly proportional to its smoothness. As the vanishing moment increases, smoothness also increases. When wavelet gets smoother, it expands in time domain. As a result, in frequency domain, side-lobes get contracted and thereby decrease the chance of ICI.

8 CONCLUSION
The perfect reconstruction property of Wavelet filter bank can efficiently be utilized in MCM technique of digital communication, thus formulating WOFDM, which has been proposed in literature to overcome some of the drawbacks of FFT-OFDM technique. This paper presents the motivating factors that promotes the application of WOFDM instead of FFT-OFDM in a frequency selective Rayleigh fading channel. Simulation results have shown that WOFDM performs better when compared with FFT-OFDM system. This shows its special property of cancelling any possible distortion and aliasing effects through its wavelet filter banks. These results enhance the significance attached with WOFDM properties that can be employed in PHY layer LTE communication system.

REFERENCES


