APPLICATION OF VOLTERRA SERIES AND PARTICLE SWARM OPTIMIZATION (PSO) ALGORITHM IN ANALYSIS AND ESTIMATION OF MULTI-INPUT MULTI-OUTPUT (MIMO) NON-LINEAR SYSTEMS

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ABSTRACT: *In this paper, the application of Volterra series is presented in analysis and estimation of non-linear multi-input multi-output systems as a black-box based on actual data. Theories and methods that are presented for understanding the dynamic behavior of non-linear systems, cover a limited area of non-linear systems. Volterra series is one of the quite comprehensive approaches for analyzing and estimating non-linear systems. This series is a powerful mathematical tool for analyzig nonlinear systems which expands convolution integral of linear systems to non-linear mode. Some of tuning parameters are calculated by PSO algorithm in order to improve the identified system. Besides, in this paper the algorithm is implemented on a mechanical system of CD player device arm to fully show the validity of the proposed method.*

Keywords- Volterra series, nonlinear system identification, SVD algorithm, PSO algorithm, mechanical system of CD player device arm.

Nomenclature

Constant

y : Output

u : input

N : Set of natural numbers

Variables

M : memory of the system

L : degree of system

h : Kernel of Volterra

n : Sample

t : time (sec)

e : error

U : Data Matrix

θ : Parameter vector

Indices

k, i, j : Degree

1. INTRODUCTION

Linear theories are the basis of improvement for most of the control systems in different fields of study. However, there are some dynamic behaviors that cannot be explained by linear system theories. Such behaviors occur in non-linear systems. There are different methods to analyze such systems like chaos method [1,2], non-linear time series analysis method [3-7], multiple comparison method [8], Homotopy analysis method and Harmonics equilibrium method [9-12]. Besides, Voterra series is one of the quite comprehensive approaches for estimating non-linear systems. Volterra series is an expansion for convolution in linear mode. It was first presented by Vito volterra in 1887 [13, 14]. After that it was used widely in identification of non-linear systems which one of the examples is presented in [15]. There are too many published documents about this issue that try to use Volterra series for system identification or a method for calculating the kernels of this series. For instance, [16] presents the elimination of Gaussian noise using Volterra series and [17] presents the application of Volterra series in identification and modeling of synchronous generator. Identification of Volterra series kernels with different methods are presented in [18] and higher-order kernels of Volterra series are estimated using reproduced kernels Hilbert space [19, 20]. Problems of modeling in SISO systems and MIMO are discussed, respectively in [21] and [22].

In this paper, SISO Volterra series are presented in section 2 and section 2-1 illustrates the calculation method for kernels or factors of SISO Volterra series. MIMO Volterra series are presented in section 3 and section 3-1 discusses the estimation of kernels for MIMO Volterra series. Section 4 investigates mechanical system of CD player device arm and simulation results are presented in section 5.

2. Single-input single-output (SISO) Voterra series

Volterra series is in fact an expansion of convolution in a linear system. This series is well suited for modeling non-linear systems. One of the features of Volterra series is that it is linear toward its own factors (kernels) [23, 30] which is expressed in equation (1) for L degree Volterra series.

$$y(n) = y_0 + \sum_{m_1=0}^{\infty} h_1(m_1)u(n-m_1) + \sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty} h_2(m_1,m_2).u(n-m_1).u(n-m_2) + ... \tag{1}$$

h_n is rank n kernel of Volterra. In fact $h_1(t)$ shows impulse function of linear system and $h_n(t)$ is impulse function of a n-dimensional non-linear system. In equation (1), there are too many numbers of unknowns, which causes complexity and waste of time. To solve this problem, it has to be considered that maybe, output is not extremely dependent to more previous inputs. Besides, series can be approximated with only kernels of degree 1 to L. considering abovementioned explanations, second-order Volterra series (L=2) is changed as follows:

$$y(n) = y_0 + \sum_{m_1=0}^{M-1} h_1(m_1)u(n-m_1) + \sum_{m_1=0}^{M-1}\sum_{m_2=0}^{M-1} h_2(m_1,m_2).u(n-m_1).u(n-m_2) \tag{2}$$

where $0 \le (M-1) \le n$ is always true assuming causal and zero input and output in times before zero. M is the memory of Volterra series which is related to the dynamics of the system and L is the degree of Volterra series which most of the time its values are chosen based on trial and error method.

2.1. Calculating the kernels (h_i) of SISO Volterra series

Calculation of ker nels is the most important issue in analysis and identification of systems using Volterra series. Identification of non-linear systems by Volterra series is in fact identifying its kernels. Some methods to do this are recursive algorithm [24], utilizing Gaussian input [24], Gradient-based search [25], Cross correlation method [26], Hilbert space reproduced kernel method [19, 20], expansion with orthogonal functions [27-30] etc. Besides, in this paper kernels are considered symmetric to reduce the number of parameters:

$$h_n^{sym}(k_1,k_2,...,k_n)=\tfrac{1}{n!}\sum h_n(k_1,k_2,...,k_n) \qquad (3)$$

3. Multi-input multi-output (MIMO) Volterra series

The relation for Volterra series for a MIMO system is as follows [23]:

$$y(k)=h_0+\sum_{i=1}^{L}\sum_{j_1=1}^{n}\cdots\sum_{j_i=1}^{n}\sum_{m_1=0}^{M-1}\cdots\sum_{m_i=m_{i-1}}^{M-1}h_{j_1,j_2,...,j_i}(m_1,...,m_i)\times\prod_{e=1}^{i}u_{j_e}(k-m_e) \qquad (4)$$

where $u=[u_1(k),...,u_1(k),...,u_n(k)]$ is input vector and $y(k)$ output and $h_{j_1,j_2,...,j_i}(m_1,...,m_i)$ kernels of Volterra series. Besides, L is the degree of non-linear Volterra series and M is the memory of series which is related to the dynamics of the system. Number of parameters in equation (4) is:

$$n_{MISO}=1+\sum_{i=1}^{L}n^i\,\frac{(M-1+i)!}{(M-1)!i!} \qquad (5)$$

3.1. MIMO Volterra series

MIMO system can be considered of combination of some MISO subsystems. Therefore, modeling of MIMO is modeling of these subsystems. Now the Volterra series equation for MIMO system is changed into equation (6) considering n inputs and s outputs [23]:

$$y_s(k)=h^s{}_0+\sum_{i=1}^{L}\sum_{j_1=1}^{n}\cdots\sum_{j_i=1}^{n}\sum_{m_1=0}^{M-1}\cdots\sum_{m_i=m_{i-1}}^{M-1}h^s{}_{j_1,j_2,...,j_i}(m_1,...,m_i)\times\prod_{e=1}^{i}u_{j_e}(k-m_e) \qquad (6)$$

where $h^s{}_{j_1,j_2,...,j_i}(m_1,...,m_i)$ is i_{th} degree kernel related to $y_s(k)$ output of Volterra series. Besides, L is the degree of non-linear Volterra series and $h^s{}_0$ static gain related to output $y_s(k)$. Number of parameters in equation (6) is:

$$n_{MIMO}=\left(1+\sum_{i=1}^{L}n^i\,\frac{(M-1+i)!}{(M-1)!i!}\right)*S \qquad (7)$$

3.2. Estimation of kernels for MIMO Volterra series

As explained earlier, one of the features of Volterra series is linearity with respect to its factors (kernels) that causes this series to change into a linear regression as equation (8) [30].

$$y=U\theta+e \qquad (8)$$

where y is output vector and θ factors vector and e noise or model error and U a known matrix of linear and non-linear inputs. Now the value of parameter vector can be obtained by different identification methods in a way that modeling error $\|y-\hat{y}\|^2$ is minimized.

One of such methods is linear least square method where best parameters that minimize the modeling error are calculated by following equation:

$$\hat{\theta}_{LS}=(U^TU)^{-1}U^Ty \qquad (9)$$

One of the problems of least square method is impropriety of matrix U^TU that reduces the accuracy and increases the error of estimation. There are some reasons for this problem [21]:

1. Input is not exciting enough and not able to excite all the modes of the system
2. The considered rank of model is more than actual rank of system
3. System is identified in closed-loop operation.

For compensating this problem in least square method, inversing U^TU is not allowed and alternatively other techniques are employed. In this paper, Singular Value Decomposition (SVD) method is chosen for this purpose. Besides, equation (10) is a criterion for feasibility assessment of model under the name of residual of normalized output error [29, 31, 32]:

$$J^S{}_{dB}=10\log\left(\sum_{t=1}^{M}|\varepsilon(t)|^2 \Big/ \sum_{t=1}^{M}|y^S(t)|^2\right) \qquad (10)$$

where $\varepsilon(t)$ is the error between actual and estimated output at instant t, where if identification is performed successfully, this residual has to be contained of noise energy added to the data.

Now a wide range of non-linear MIMO systems can be identified. The only obstacle is to select the value of M and L. Although the real values of these parameters are too big, smaller values are used to avoid complexity and to reduce the identification time if the smaller values present agreeable error. Most of the times, these parameters are obtained by trial and error method in literatures. In this paper, these parameters are calculated by PSO algorithm for minimizing criterion function of error squares sum in equation (11).

$$J=\sum_{i=0}^{N-1}(y(i)-\hat{y}(i))^2 \qquad (11)$$

Where $y(i)$ is actual output and $\hat{y}(i)$ estimated output at instant i.

4. Mechanical system of CD player device arm

In this paper, the proposed algorithm is validated on a MIMO mechanical system of CD player device arm. Actual data are from [33] where there are 2048 samples of input and output with identification in 10 seconds. This system has two inputs and outputs where inputs are mechanical motive force and outputs are tracking accuracy of arm. Input and output data of system are categorized into two groups, one group is used for training, by which the model is extracted from and the second group is for testing the model.

5. Simulation

As discussed in section 3-2, at first the values of L (degree of system) and M (system memory) are calculated where sum of error squares in equation (11) is minimized. In this paper, PSO algorithm is used to calculate the free parameters where the ranges of free parameters are considered $1 \le L \le 5, 1 \le M \le 40$. After executing the algorithm, optimal values are calculated as $L = 2, M = 10$. Equation of linear regression is made by putting optimal L and M in Volterra series of equation (6) and using learning data (for 1300 data) and as a result an estimation of system parameters are performed and a model is obtained for learning data. Number of parameters of estimated model is calculated by equation (7). Now feasibility of the obtained model is tested by learning data (for 700 data) in which values of outputs and error of identified model and actual outputs are obtained as Figures 1 and 2:

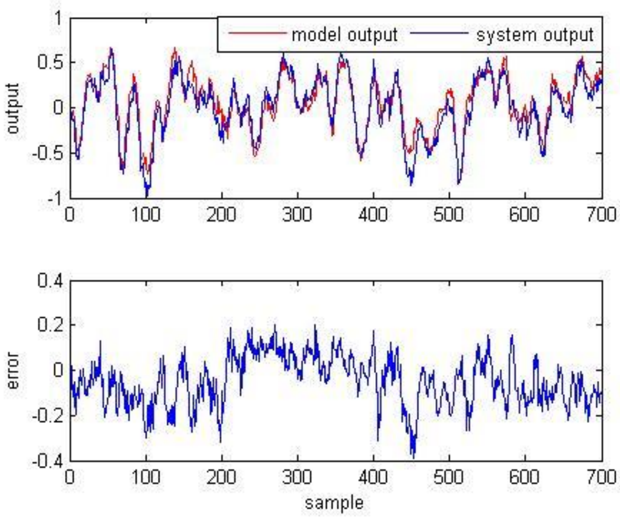

Figure 1. Value of output and first error of identified model and actual system

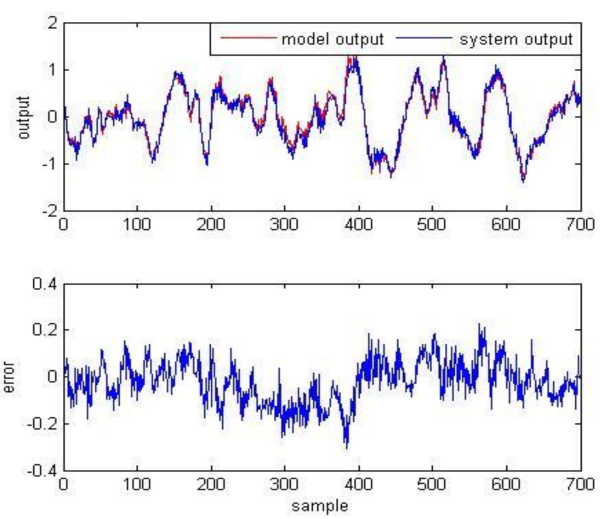

Figure 2. Value of output and second error of identified model and actual system

The results are obtained by criterion of equation (10) and for 1300 learning data and 700 test data as shown I Table 1.

Table 1: Value of error in different outputs considering M=10 and L=2

The maximum of the prediction error	Sum of squared errors	residual normalized output error (dB)	output
0.3880	10.0199	-1,117	one
0.3068	6,2620	-37,9183	two

As deduced from Table 1, the proposed method has appropriate identification error which shows its ability for identifying MIMO system.

6. CONCLUSION

In this paper, Volterra series model was used to analyze and estimate the non-linear multi-input multi-output systems. In the following, SVD method was implemented to avoid the lack of reverse-taking problems or impropriety of regression matrix in specific cases. PSO algorithm was also used to optimally select the memory number and Volterra series degree and finally the presented method was applied to identify the non-linear two-input two-output mechanical system of CD player device arm. Identification results of Volterra series show that the obtained model has appropriate and agreeable accuracy.

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