IMPROVED ALGORITHM FOR HARMONIC LOAD FLOW SOLUTION IN RADIAL DISTRIBUTION NETWORKS

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Abstract: *The aim of this paper is to present an improved algorithm to solve harmonic load flow problem in balanced radial distribution systems with laterals, which have nonlinear load. The method is efficient and easy to implement. This method is iterative and allows the evaluation of both, voltage (rms) values and phase angles of the fundamental voltage and harmonic voltage for each bus. The proposed method in this paper can be solved harmonic load flow problem iteratively from two sets of recursive equations. The first set concern the determination of the branches current by going up the line (backward sweep). The second one allows us to determine the nodes voltage by going down the line (forward sweep). The total harmonic distortion can also be calculated easily.*

Keywords: *Harmonic load flow, Radial distribution network, Nonlinear load, THD.*

1. INTRODUCTION

Many researchers are used different search algorithms for solving optimization problems in power system [1-6].

Load flow problem has a important role in power system design and analysis. Also, the harmonic load flow is used for small-signal analysis, that is necessary for controller design and stability analysis. Moreover, in the startup a transient program, Initializing variables are important. Those accurate steady-state initializations are received from accurate harmonic load flow [7].

Researchers are proposed a number of harmonic load flow methods [8]. In general, these methods can be divided into three categories:

Time-domain methods [9-15].

Frequency-domain methods [16-24].

Hybrid time–frequency domain methods [25, 26].

The absence of limitation in load flow problem is one of the main disadvantages of the time-domain method (such as active and reactive power demands can be assumed specified constant values) at the fundamental frequency. A time-domain program needs to a harmonic power flow algorithm for the steady-state initialization. In [27], has been presented a time-domain based transient-state analysis that has been utilized the EMPT program for harmonic analysis. Perkins *et al.* [28] have implemented an initialization algorithm based on a time-domain steady-state analysis; that, this method is introduced in ref. [8, 9]. In ref. [9], the nonlinear components are modeled by means of piecewise linearization.

However, most of the harmonic power flow methods are used from the frequency domain [29]. The accuracy of the solution of this method depends on the number of harmonics included in the calculation process. In addition, the number of harmonics also determines the dimensions of the system, which affects the overall simulation time.

Xia and Heydt [16,17] have developed the conventional power flow analysis for harmonic load flow and nonlinear component in the formulation Are considered. However, this method only allows specific harmonics to be taken into account. Researchers have developed this harmonic load flow to three phase networks [23, 30].

Arrillaga *et al.* [14] have represented how modeling of nonlinear component in the three-phase frame of reference in the harmonic load flow. Valcárcel and Mayordomo [31] have solved this problem based on two separate Newton-Raphson equations. That, one of them for the three-phase conventional power flow and the other for calculation the harmonics injected by the nonlinear loads.

This paper presents a new frequency-domain harmonic power-flow algorithm. This algorithm can be used for load flow problem in radial distribution feeders with laterals. In this paper, a simple method for numbering the branches and nodes are used [32]. Then, based on a new algorithm, the matrix of nodes connected to different branches of the network is calculated so that the nodes beyond each branch are determined. Using this matrix and power nodes, the current of branches are calculated. Then the nodes voltage drop at each harmonic can be determined. This process is done for all the harmonics respectively. In this paper, three-phase diode bridge rectifier is modeled, and considered in the analysis of the load flow.

2. COMPONENTS MODEL

In an industrial distribution system, the main equipment that needs to be considered in the harmonic analysis is distribution cable, transformer, nonlinear load, capacitor and inductor etc. Instead of using the very accurate model, some practical and approximated models for industrial use are employed in this paper [16, 17 and 22].

2.1. Distribution Cable

Some corrections need to be applied to the resistance and the internal inductance of a distribution cable at high frequency and several modifications have been developed to cope with this purpose. However, a simple model is enough for a practical harmonic analysis. If the line impedance of fundamental frequency is given by:

$$Z_L^{(h)} = R_L + jhX_L \qquad (1)$$

2.2. Three-Phase Rectifier

Many commercial and industrial appliances require direct current for their operation. The three-phase diode bridge rectifier (Fig. 1) has become a popular power source for these appliances because of its reduced cost and relatively low sensitivity to supply voltage variations under normal

operating conditions. But, these converters have been one of the most important elements in the harmonic of grid.

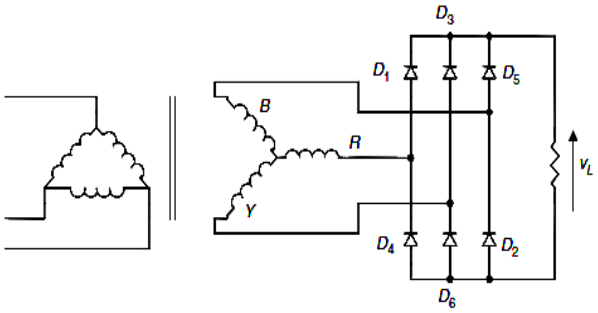

Figure. 1. Three-phase diode bridge rectifier.

A three-phase rectifier is characterized by a very inductive DC side configuration relative to the AC system side. Under these conditions, the DC current is reasonably constant and the converter acts as a source of harmonic voltage on the DC side and of harmonic current on the AC side.

Six-pulse rectification (and inversion) is obtained from three-phase two-way configurations. That, in this rectifier the phase currents consists of periodic positive rectangular pulses of width $w = 2\pi/3$, repeating at the supply frequency. The waveform of the phase current is shown in Fig. 2. If the actual DC current be equal to I_d, then the current in phase a is [16]:

$$i_a = \frac{2\sqrt{3}}{\pi} I_d(\cos(\omega t) + \frac{1}{5}\cos(5\omega t) - \frac{1}{7}\cos(7\omega t) + \frac{1}{11}\cos(11\omega t) - \frac{1}{13}\cos(13\omega t) + \frac{1}{17}\cos(17\omega t) - \frac{1}{19}\cos(19\omega t)...) \quad (2)$$

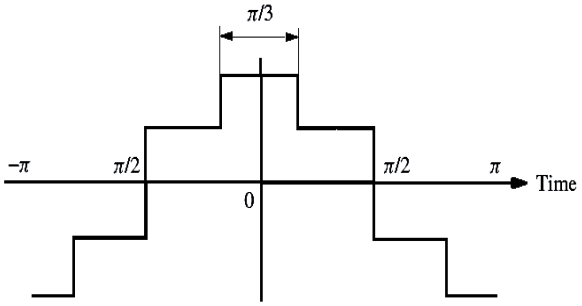

Figure. 2. Time-domain representation of a six-pulse waveform with delta–star transformer.

3. NODES LOCATED BEYOND EACH BRANCH

In this paper, a method based on backward and forward sweeps is used for solving the load flow problem in balanced radial distribution systems. A simple method of [32] is used here for numbering the branches and nodes. Using a new algorithm, the nodes are identified beyond each branch of the network as a matrix called *NBEB* matrix.

To identify the nodes beyond each branch (*NBEB*), the following algorithm is used for numbering the nodes and branches [11]. The first bus (source node) is numbered as bus number 0. Buses should be numbered on the main feeder to the end, respectively. Thereafter, the lateral branches following from the nearest lateral branch relative to the source node (if two branches have the same position relative to the reference bus, starting from each one is arbitrary) and then buses following the last bus in the main feeder are respectively numbered. Then, similarly, the buses are numbered at other lateral branches following the last bus at the last lateral branch. Numbering of branches is also similar to the numbering of buses.

The network shown in Fig. 3, presents numbering scheme of buses and branches according to the algorithm. Table 1 lists start and end nodes of each branch.

Fig. 4 shows the proposed algorithm for determining nodes beyond each branch of the network. This algorithm receives the total number of branches, number of the last node in the main feeder, number of lateral branches and number of start and end nodes of each lateral branch. The output of this algorithm is a matrix named *NBEB*. The rows of this matrix correspond to the branch network. For the i^{th} branch, non-zero values of *NBEB (i, j)* are the number of buses beyond the i^{th} branches (*M (i)* is equal to the total number of buses beyond the i^{th} branch). For example, for the network shown in Fig.1, the following equations can be written:

$$NBEB(3,1) = 3;\ NBEB(3,2) = 4;\ NBEB(3,3) = 7$$

$$NBEB(3,4) = 8;\ M(3) = 4;\ NBEB(3,j) = 0,\ j = 4,5,6,7$$

And, the *NBEB* matrix, derived using this method, is as follows:

$$NBEB = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 3 & 4 & 0 & 6 & 0 & 8 & 0 & 10 & 0 \\ 3 & 4 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 5 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

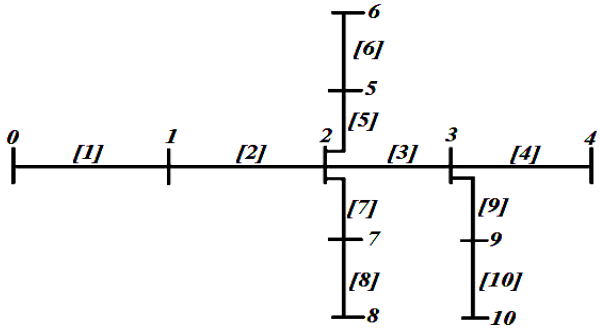

Figure. 3. Nodes and branches numbering scheme.

TABLE 1.
FEEDER CONNECTIVITY

Branch	Start node of branch	End node of branch
1	0	1
2	1	2
3	2	3
4	3	4
5	2	5
6	5	6
7	2	7
8	7	8
9	3	9
10	9	10

4. LOAD FLOW EQUATIONS

It is assumed that the three-phase radial distribution network is balanced. The load flow of the radial distribution network for any specific harmonic can be solved iteratively by two sets of recursive equations. The first set should determine of the branches current by going up the line (backward sweep). The second one determines the nodes voltage by going down the line (forward sweep).

Consider the i^{th} branch network. Starting bus of the i^{th} branch and the end bus of the branch are, respectively, called SE (i) and RE(i) (Power is entered to the branch through starting bus.).

Assume that: $P_{Load}^{(h)}(k)$ and $Q_{Load}^{(h)}(k)$ are active and reactive load powers at node k and for h^{th} harmonic; $P_{Loss}^{(h)}(k)$ and $Q_{Loss}^{(h)}(k)$ are active and reactive power losses at the k^{th} branch and for h^{th} harmonic. Therefore, $P^{(h)}(RE(i))$ and $Q^{(h)}(RE(i))$ are active and reactive powers at the end of the branch i, as follows:

$$P^{(h)}(RE(i)) = \sum_{k=BNC(i,1)}^{BNC(i,M(i))} \left[P_{Load}^{(h)}(k)\right] + \sum_{k=BNC(i,2)}^{BNC(i,M(i))} P_{Loss}^{(h)}(k)$$
$$Q^{(h)}(RE(i)) = \sum_{k=BNC(i,1)}^{BNC(i,M(i))} \left[Q_{Load}^{(h)}(k)\right] + \sum_{k=BNC(i,2)}^{BNC(i,M(i))} Q_{Loss}^{(h)}(k) \quad (3)$$

By this equation, the i^{th} branch active and reactive power losses at h^{th} harmonic are calculated. Therefore, active and reactive powers at the beginning of the i^{th} branch and for h^{th} harmonic can be calculated by the following equation:

$$P_{Loss}^{(h)}(i) = \frac{P^{(h)}(RE(i))^2 + Q^{(h)}(RE(i))^2}{\left|V_i^{(h)}\right|^2} r_i^{(h)}$$
$$Q_{Loss}^{(h)}(i) = \frac{P^{(h)}(RE(i))^2 + Q^{(h)}(RE(i))^2}{\left|V_i^{(h)}\right|^2} x_i^{(h)} \quad (4)$$

$$P^{(h)}(SE(i)) = P^{(h)}(RE(i)) + P_{Loss}^{(h)}(i)$$
$$Q^{(h)}(SE(i)) = Q^{(h)}(RE(i)) + Q_{Loss}^{(h)}(i) \quad (5)$$

where, $r_i^{(h)}(x_i^{(h)})$ is the resistance (reactance) of the i^{th} branch and for h^{th} harmonic.

The current flowing through the i^{th} branch and for h^{th} harmonic is given by the following equation:

$$I^{(h)}(i) = \frac{P^{(h)}(SE(i)) - jQ^{(h)}(SE(i))}{V^{(h)}(SE(i))^*} \quad (6)$$

Therefore, the voltage of the bus at the end of the branch (i) is written, as follows:

$$V^{(h)}(RE(i)) = V^{(h)}(SE(i)) - (r_i^{(h)} + jx_i^{(h)})I^{(h)}(i) \quad (7)$$

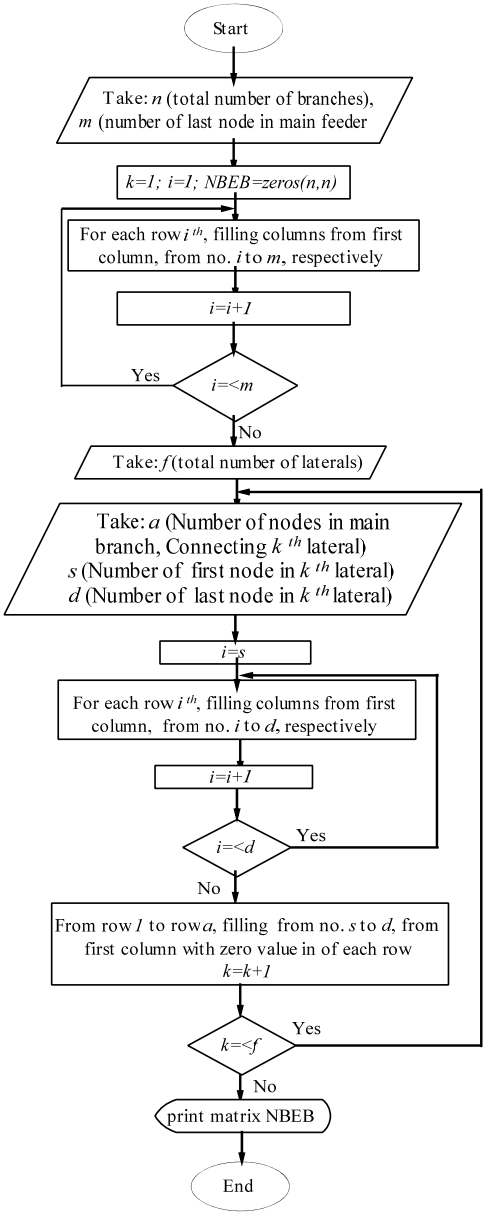

Figure.4. Node determination after branch (NBEB matrix)

In this method, the voltage rms value and phase-angle of the reference bus (bus number zero) are considered respectively 1.0 pu. and zero. Also in the first iteration, the voltage rms

value and phase-angle of the all buses are assumed to be, respectively, 1.0 pu. and zero.

5. DEVELOPED ALGORITHM

In this paper, a program to solve the load flow problem is developed in MATLAB environment. This program, as shown in Fig.5, receives the load, line and nonlinear load; then it makes matrix NBEB. In next step for the fundamental frequency and any harmonic, starting from the last branch, the crossing active and reactive powers from branch *i* and in the same harmonic according to the equation (3) and power losses at branch *i* and in the same harmonic on the basis of the voltage at the sending end node and according to the equation (4) is calculated. Then the crossing active and reactive powers from branch *i* and in the same harmonic according to the equation (5) is calculated. Then, the current in the branch *i* and in the same harmonic according to (6) on the basis of the voltage at the sending-end node is calculated. In next step, the voltage at the receiving-end node of branch *i* and in the same harmonic, according to (7) is calculated. Then, these steps are repeated for branch *i-1* and the mentioned steps are repeated to achieve the source node. Then, the convergence rate is compared. This process is done for all the harmonics respectively. The effect of three-phase rectifier in the load flow equations has been considered as follows:

In the first step, to calculate the fundamental frequency of bus voltage, the power of rectifier is modeled as a load (Fig. 6). Based on the fundamental voltage and following equation, the current of equivalent load is calculated.

$$I_K^{(1)} = \frac{P_K}{V_k^{(1)}} \qquad (8)$$

So that, according to equation (2) the harmonic components in the other harmonics can be achieved. And in the next step power of any non-linear load in each harmonic Proportional to component harmonic can be obtained.

6. SIMULATION RESULTS

The proposed method was tested in the 33-bus system [33] shown in Fig. 7. In this example, the base values are 2300 kVA and 12.66 kV. This case contains a three-phase diode bridge rectifier at the node 17 that itsActive power consumption is 150 KW. This rectifier consists of harmonic components based on that the harmonics can be modeled as a constant current load. This value of the harmonic components based on Equation 2, are a multiple of the fundamental current. Then, based on load flow equations, at the first stage, fundamental voltage of each bus and current equivalent to each load are calculated. In the next stage, for each harmonic based on the fundamental current, amount of load at each harmonic is calculated. Then, based on load flow algorithms (shown in Fig. 5), the harmonic voltage at each bus is determined. This process is done for all the harmonics respectively.

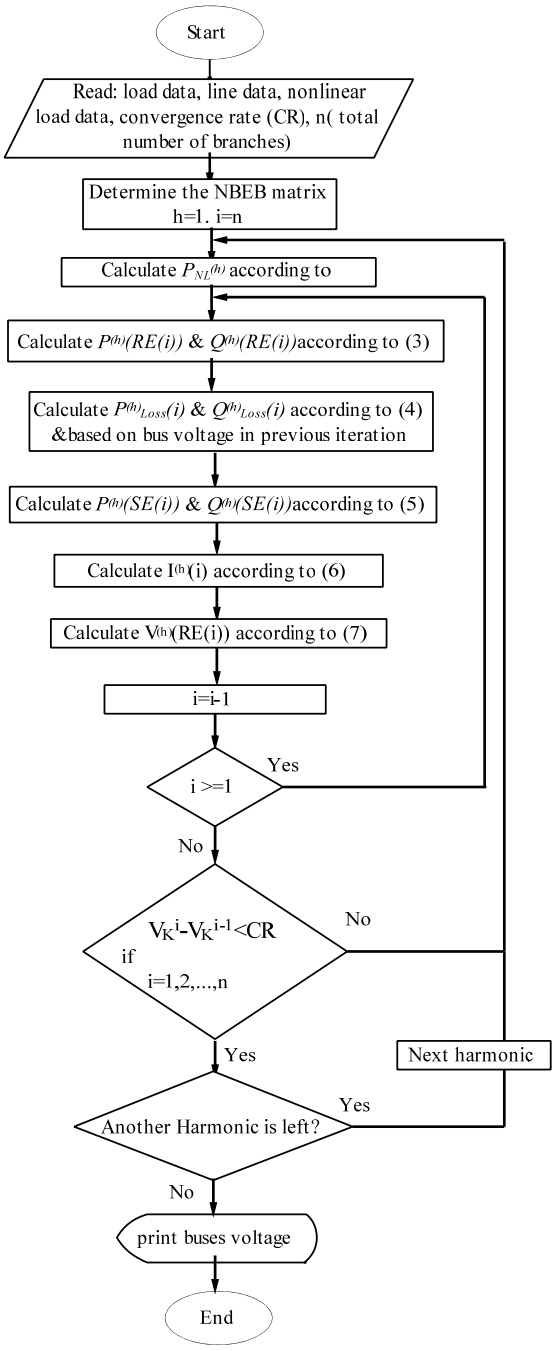

Figure. 5. Algorithms of harmonic load flow solution.

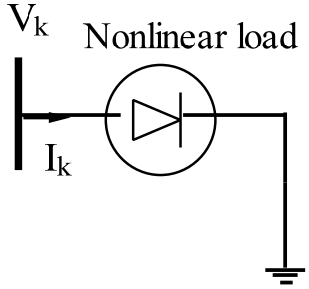

Figure. 6. Modeling of nonlinear load for linear analyze.

TABLE 2.
RESULT OF HARMONIC LOAD FLOW

Node No.	Bus Voltages (p.u)							THD %
	Fund	5th	7th	11th	13th	17th	19th	
0	1	0	0	0	0	0	0	0
1	0.996915	8.867e-05	7.6089e-05	6.4644e-6	6.1567e-5	5.7581e-5	5.6218e-5	2.8581e-6
2	0.982256	0.0005569	0.00047765	4.0559e-5	0.0003862	0.0003611	0.0003525	0.000114
3	0.974357	0.0009045	0.00077575	6.5869e-5	0.0006272	0.0005864	0.0005725	0.000303
4	0.966526	0.0012664	0.00108616	9.2224e-5	0.0008781	0.0008210	0.0008015	0.000600
5	0.947163	0.0024783	0.00214074	0.0001833	0.0017513	0.0016444	0.0016078	0.002411
6	0.943436	0.0034399	0.00299296	0.0002586	0.0024773	0.0023358	0.0022874	0.004796
7	0.937814	0.0056094	0.00487274	0.0004202	0.0040228	0.0037896	0.0037098	0.012750
8	0.930403	0.0069101	0.00599961	0.0005171	0.0049491	0.0046608	0.0045622	0.019465
9	0.923459	0.0082127	0.00712779	0.0006141	0.0058762	0.0055327	0.0054151	0.027661
10	0.922391	0.0083468	0.00724096	0.0006235	0.0059652	0.0056150	0.0054952	0.028555
11	0.920495	0.0086022	0.00745649	0.0006414	0.0061347	0.0057719	0.0056478	0.030297
12	0.912789	0.0106062	0.00919661	0.0007915	0.0075704	0.0071241	0.0069714	0.046506
13	0.909945	0.0117748	0.01022193	0.0008810	0.0084304	0.0079386	0.0077704	0.057760
14	0.907923	0.0126729	0.01100402	0.0009486	0.0090786	0.0085501	0.0083694	0.067113
15	0.905796	0.0136285	0.01183225	0.0010199	0.0097599	0.0091911	0.0089966	0.077760
16	0.902445	0.0164462	0.01430492	0.0012358	0.0118345	0.0111565	0.0109245	0.114472
17	0.901122	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.128900
18	0.996387	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.116576
19	0.992809	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.116996
20	0.992104	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.117079
21	0.991467	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.117155
22	0.978668	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.118687
23	0.971992	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.119502
24	0.968665	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.119912
25	0.945229	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.122885
26	0.942660	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.123220
27	0.931190	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.124738
28	0.922953	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.125851
29	0.919390	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.126339
30	0.915224	0.0174424	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.126914
31	0.914318	0.0174423	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.127039
32	0.914048	0.0174423	0.01516999	0.0013104	0.0125482	0.0118286	0.0115824	0.127076

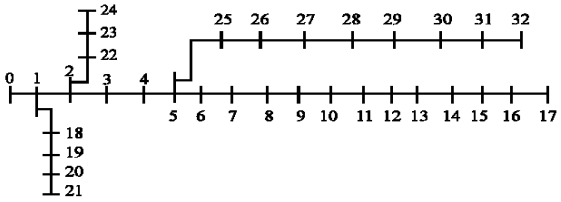

Figure. 7. 33-bus single line diagram.

Table 2 is shown test results of the system including the fundamental bus voltage, harmonic voltage and THD (%) for each bus and harmonic orders and this diagram is plotted in Fig.8.

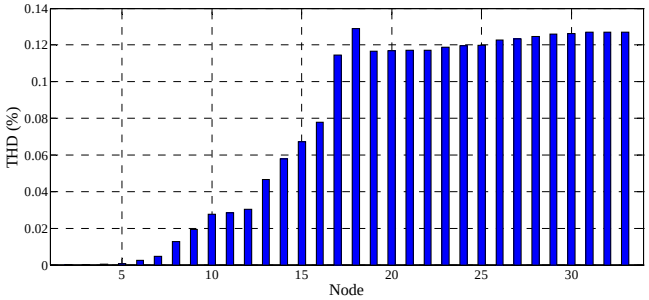

Figure. 8. THD (%) for each bus of 33-bus network.

Active and reactive power losses for each line and harmonic orders in this network is shown in Fig.9 and Fig.10, respectively.

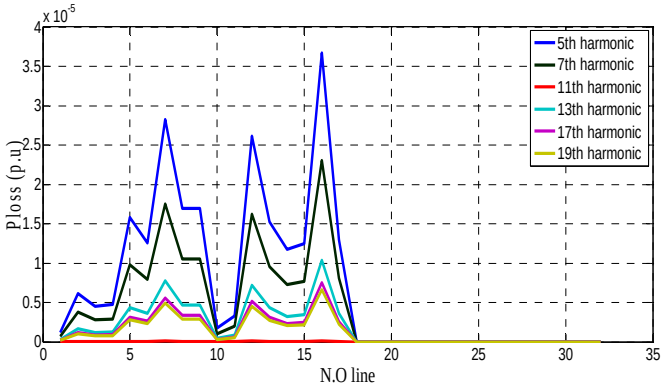

Figure. 9. Active and power losses for each line and harmonic orders.

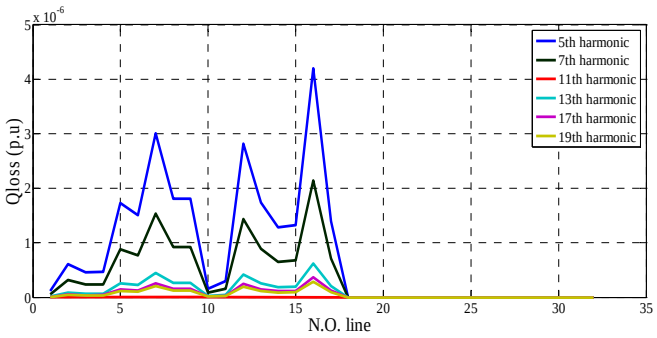

Figure. 10. Reactive and power losses for each line and harmonic orders.

7. CONCLUSION

In this paper, a time-domain harmonic power-flow method is described for steady-state analysis and is developed a new algorithm for calculating the harmonic voltages in a radial distribution network. This method can be easily extended to three-phase networks. This method allows the evaluation of both, voltage (rms) values and phase angles of the fundamental voltage and harmonic voltage for each bus. The proposed load flow algorithm is easy understanding and acceptable computational performance. Also, one of the most important advantages of this method is that it avoids constructing and inverting any network matrix.

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