MHD FLOW AND HEAT TRANSFER THROUGH POROUS MEDIUM OVER AN EXPONENTIALLY SHRINKING PERMEABLE SURFACE

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ABSTRACT: *The magnetohydrodynamic fluid flow through porous medium over an exponentially shrinking porous sheet has been considered with convective heat transfer. Parametric study of the problem provides an insight into the flow and heat transfer phenomena of the problem. The effects of magnetic field strength parameter, parameter of permeability, suction parameter and Prandtl number have been observed on flow velocity and temperature function. The exponential form similarity functions reduce the governing partial differential equations to their ordinary differential form. The quantative output has been evaluated numerically and presented graphically.*

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Key Words: Porous medium, MHD flow, Shrinking sheet, Prandtl number, Suction Parameter.

1. INTRODUCTION

There has been growing interest in MHD flow and heat transfer through porous medium due to the fact that liquid metals that occur in nature and industry are electrically conducting and possess termal properties also. This type of fluid flow has received attention of many researchers due to its applications in technological and engineering problems such as MHD generator; plasma studies, nuclear reactors, geothermal energy extraction. By application of magnetic field hydro magnetic techniques are used for the purification of molten metal from non-metallic inclusion. The MHD heat transfer through a porous medium over an exponentially stretching surface has many applications in industries and technical processes. Cortell [1-2] studied the flow and heat transfer of a fluid through a porous medium over a stretching surface. El-Hakiem et al. [3] examined the effects of magnetic field and double dispersion on mixed convection heat and mass transfer in non-darcy porous medium. Rehman et al [4] studied radioactive heat transfer flow of micropolar fluid with variable heat flux in porous medium. Farooq and Sajjad [5] considered MHD flow and heat transfer through a porous medium over a stretching/shrinking surface with suction. Further Mukhopadhyass et al [6] analyzed the effect of variable fluid viscosity on flow past a heated stretching sheet embedded in porous medium in presence of heat source.

Sakiadis [7-8] was the first to purpose and analyse the surface stretching problem. Cran [9] extended this work for the two dimensional flow over a stretching sheet problem. Wang [10] observed the flow around the shrinking sheet while studying the behaviour of fluid film over an unsteady stretching sheet. Sajjad et al.[11] investigated MHD boundary layer flow and heat transfer for micropolar fluids over a shrinking sheet. Bhattacharyya [12-13] studied the boundary layer flow and heat transfer over an exponentially shrinking sheet and also discussed stagnation point flow and heat transfer over an exponentially shrinking sheet. Sajjad and Farooq[14] considered unsteady MHD flow and heat transfer for Newtonian fluids over an exponentially stretching sheet.

Magyari and Keller [15] focused on heat and mass transfer on boundary layer flow to an exponentially continuous stretching sheet. Elbasheshy [16] studied heat transfer over an exponentially stretching continuous surface with suction, and also added dimension to the problem on exponentially continuous stretching surface that was studied by Ali [17]. Khan [18] discussed the boundary layer viscoelastic fluid flow over an exponentially stretching sheet. Ishak et al [19-20] studied MHD stagnation point flow towards a stretching sheet and also discussed MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. Nadeem et al [21-22] discussed thin film flow of a second grad fluid over a shrinking sheet with variable temperature dependent viscosity and also studied boundary layer flow of nano fluid over an exponentially stretching surface. Ahmad et al. [23] found analytical solution of MHD flow over porous stretching sheet. Jat and Gopi [24] analysed steady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid over an exponentially stretching sheet in the presence of auniform transverse magnetic field with viscous dissipation and radiative heat fluxis studied.

2. MATHEMATICAL ANALYSIS

We consider the viscous electrically conducting fluid flow over a continuous exponentially stretching surface. The flow is steady, two dimensional and incompressible. The Cartesian coordinates are used such that the the stretching surface is along x-axis, the direction of motion and y axis is normal to it. A uniform magnetic field of strength $\boldsymbol{B_0}$ is assumed to be applied perpendicular to the surface. The magnetic Reynolds number is small and the induced magnetic field isneglected.

The surface is stretched with a velocity

$U = U_0 e^{\frac{x}{L}}$ and T is the temperature of fluid.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\upsilon}{K'} - \frac{\sigma B_0^2}{\rho}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

Where u and v are the velocity components and ρ is the density of the fluid, υ is the kinematic viscosity, C_p is the specific heat with constant pressure, κ is the thermal conductivity of the fluid under consideration, T is the temperature.

The boundary conditions are:

$$u=U_w=U_0 e^{x/L},\ v=-V_0, \qquad T=T_\infty+T_0 e^{2x/L}$$

at $y=0$,

$$u\to 0, \qquad T\to 0 \qquad \text{as } y\to\infty \qquad (4)$$

Where U_0, T_0 and L are the reference velocity, temperature and length .

The equation of continuity (1) is identically satisfied if we choose the stream function ψ (x, y) such that

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x}$$

The momentum and energy equation can be transformed into the corresponding ordinary differential equations by introducing the following similarity transformation:

$$\psi\,(x,y)=\sqrt{2\nu U_0 L}\;e^{x/2L} f(\eta), \qquad \frac{T-T_\infty}{T_0}=e^{2x/L}\theta(\eta),$$

$$\eta=\sqrt{\frac{U_0}{2\nu L}}e^{x/2L}y \qquad (5)$$

Then, the momentum and energy equation (2) and (3) are transformed to:

$$f'''-2(f')^2+ff''-(M+\frac{1}{K})f'=0 \qquad (6)$$

$$\theta''+\Pr\left(f\theta'-f'\theta\right)=0 \qquad (7)$$

The corresponding boundary conditions are:

$$\eta=0, \quad f=S, f'=-1, \theta=1$$
$$\eta\to\infty, f'\to 0, \quad \theta\to 0 \qquad (8)$$

Where prime denote the differentiation with respect to η and dimensionless parameters are: $M=\frac{2\sigma B_0^2 L}{\rho U_0 e^{x/L}}$ Magnetic parameter, $S=\frac{V_0}{\sqrt{2\upsilon U_0 L}}e^{\frac{-1}{2L}}$, suction parameter, $K=\frac{K' U_0^2}{2\upsilon L}e^{\frac{x}{2L}}$ Permeability Parameter, $\Pr=\frac{\mu c_p}{\kappa}$ is Prandtl number.

3. RESULTS AND DISCUSION

The set of ODE's (6) to (8) which is highly non - linear in form, has been solved numerically by using MATHEMATICA. The plots for non-dimensional velocity $f'(\eta)$ and temperature function $\theta(\eta)$ are presented.

The curves for $f'(\eta)$ as in the Fig.1 are plotted to observe the effect of magnetic field. It is noted that M reduces the boundary layer thickness. But Fig.2 shows that the boundary layer thickness is larger for small values of permeability parameter K. The suction parameter S , also decreases the boundary layer thickness as shown in Fig.3.

It is noticed that the magnetic field affects the heat distribution. In Fig.4, the reduction in the magnitude of $\theta(\eta)$ is observed with increase in magnetic strength. The suction from the surface also causes a decrease in the fluid heat distribution as illustrated in Fig.5. The increase in the vaues of Prandtl number Pr significantly decreases the temperature of the fluid as shown in Fig.6.

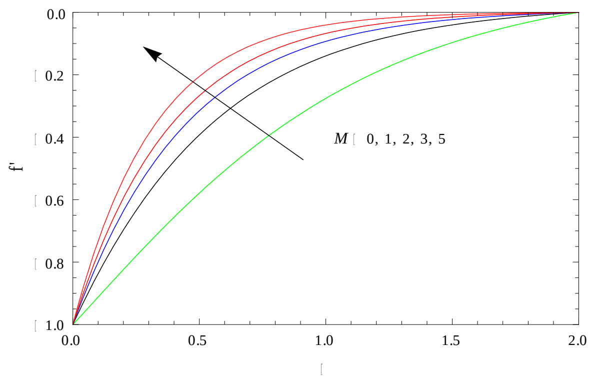

Fig.1:Plot for horizontal velocity f' under the influence of Magnetic field.

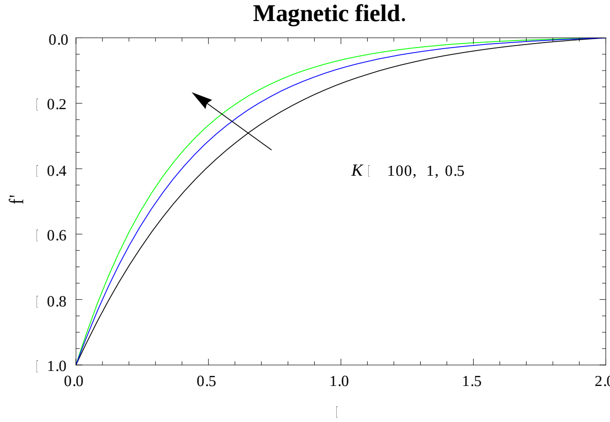

Fig.2: Plot for horizontal velocity f' under the influence of K.

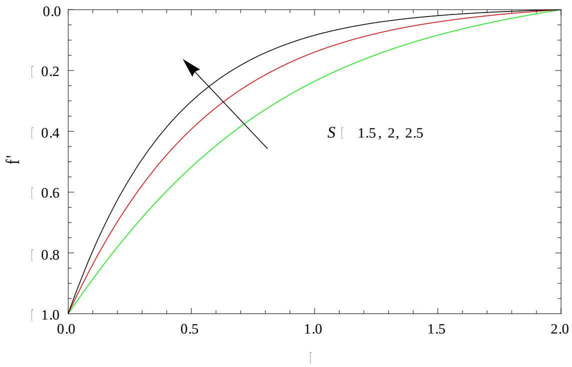

Fig.3: Plot for f' for various values of *S*.

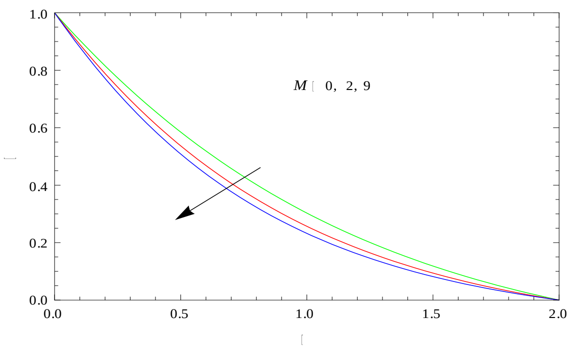

Fig.4: Plot for heat function θ for different values of *M*.

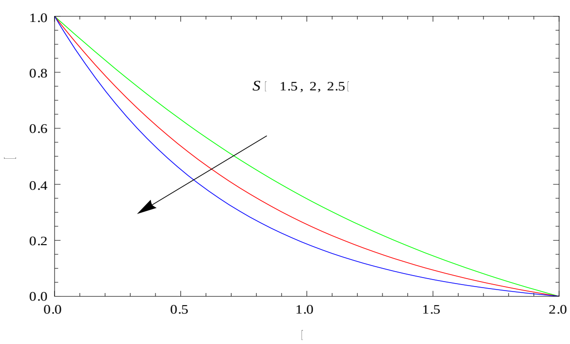

Fig.5: Plot for heat function θ for different values of *S*.

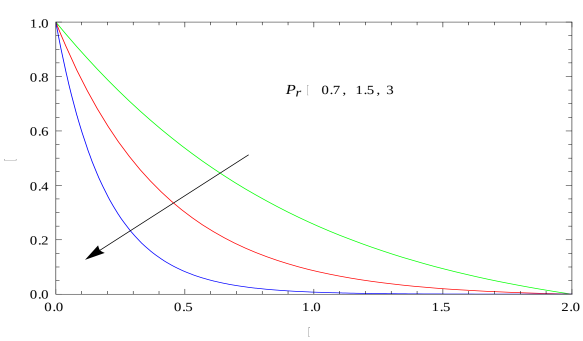

Fig.6: Plot for heat function θ for different values o P_r.

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