STATE FEEDBACK H_∞ CONTROL FOR DISCRETE 2-D SWITCHED SYSTEMS WITH ACTUATOR SATURATION

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ABSTRACT: *This paper deals with the state feedback H_∞ stabilization problem of discrete-time 2-D (two-dimensional) switched systems subject to actuator saturation. Firstly, a sufficient condition for asymptotical stability and H_∞ disturbance attenuation performance of the system under consideration is developed by using the multiple Lyapunov functional method while the saturation behavior is described by the convex hull. Secondly, a state feedback controller is designed which guarantees that the resulting closed-loop system is asymptotically stable and possess a prescribed H_∞ disturbance attenuation level γ. Finally, an example is provided to show the validity and effectiveness of the proposed methodology.*

Keywords: asymptotical stability; 2-D systems; switched systems; state feedback; H_∞ performance; actuator saturation.

1. INTRODUCTION

During the past few decades' 2-D (two-dimensional) systems have greatly attracted the attention of researchers due to their increasing application in many areas such as, digital signal processing (DSP), electrical transmission lines, digital image processing and process control [1-3]. Roesser model, FM-model (Fornasini-Marchesini model) and Attasi model are well known state space models for 2-D systems. The stability problem of 2-D discrete systems was investigated by the authors in (see [4-8] and references cited therein).

On the other hand, a switched system is a hybrid system which consists of several subsystems and a switching rule that orchestrates the switching among them. Switched systems have applications in many areas like, automotive control, power electronics, switching power supplies and air traffic controls [9, 10]. Lyapunov stability theory is very effective tool for stability and stabilization of control systems [11-13]. The common Lyapunov functional method, the multiple Lyapunov functional method and the average dwell time approach are the three well known techniques for dealing with switched systems [14-16]. The switch phenomenon was also proposed to occur among 2-D discrete systems in [17], where the switching law was considered as Markovian jumping one. Researchers in [18] have solved stability and control problem of 2-D switched systems by the common Lyapunov functional method and the multiple Lyapunov functional method.

Moreover, actuators are subject to saturation in practical control systems due to physical limitations and safety constraints. Actuator saturation may cause poor performance of the system or it can even lead to the system instability. Stability and control problem of 2-D discrete systems with actuator saturation was solved in [19-21].

Furthermore, H_∞ control theory has been extensively used by researchers as system robustness analysis tool. Robust H_∞ filter design for 2-D discrete systems was presented in [22]. Robust l_2-gain control problem of 2-D nonlinear stochastic systems with time-varying delays and actuator saturation was solved in [23]. Researchers in [24] have presented solution to the problem of stability analysis of 2-D discrete linear system described by the FM second model with actuator saturation. Recently the problem of H_∞ control for 2-D switched systems with time-varying delays and actuator saturation was studied in [25, 26]. Duan and Xiang in [27] have solved the stability and H_∞ control of 2-D switched delay free systems.

Motivated by the results presented in [27], authors in this paper aim to solve H_∞ stabilization problem for 2-D discrete switched delay free systems with actuator saturation, which to the best of our knowledge has not been fully investigated till date and deserves further attention.

The main contributions of this paper can be summarized as: 1) A new stability condition for 2-D discrete switched systems with actuator saturation has been presented along with H_∞ performance. 2) A state feedback H_∞ controller has also been presented, which guarantee that the resulting closed-loop system is asymptotically stable and has a prescribed H_∞ disturbance level γ. All results are formulated in terms of linear matrix inequalities (LMIs) which are therefore easy to solve.

The remainder of this paper is organized as follows. In Section 2, problem formulation and some necessary lemmas are given. In Section 3, main results are described. All these sufficient conditions are derived in terms of linear matrix inequalities (LMI). In section 4, an example is illustrated to show the effectiveness of derived results. In section 5 concluding remarks are given.

Nomenclature

M^T	Transpose of M.
$\|.\|$	Euclidean norm.
I	The identity matrix.
Z_+	Set of all nonnegative integers.
$R, R^k, R^{m\times n}$	The real numbers, real k vectors, real $m\times n$ matrices.
$sat(\cdot)$	Saturation function.
$\sup(.)$	Supremum function.
$diag\{a_i\}$	Diagonal matrix with the diagonal elements a_i, $i=1,2,\dots,n$.
X^{-1}	Inverse of X.
$co\{\cdot\}$	The convex hull
$(*)$	Term in matrix that is induced by symmetry.

The l_2-norm of any 2-D signal $w(i,j)$ is given by:

$$\|w\|_2=\sqrt{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[w^T(i,j)w(i,j)\right]}.$$

where $w(i,j)$ belongs to $l_2\{[0,\infty),[0,\infty)\}$, if $\|w\|_2<\infty$.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following discrete 2-D switched system in

Roesser model with actuator saturation:

$$x^{+}(i,j) = A_{\alpha(i,j)}x(i,j) + B_{\alpha(i,j)}w(i,j) + E_{\alpha(i,j)}sat(u(i,j)).$$

$$z(i,j)=C_{\alpha(i,j)}x(i,j)+D_{\alpha(i,j)}w(i,j)+F_{\alpha(i,j)}sat(u(i,j)),$$

$$i,j=0,1,2\cdots \quad (1)$$

With,

$$x^{+}(i,j) = \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix}, \quad x(i,j)=\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix},$$

$$A_{\alpha(i,j)}=\begin{bmatrix} A_{11}^{\alpha(i,j)} & A_{12}^{\alpha(i,j)} \\ A_{21}^{\alpha(i,j)} & A_{22}^{\alpha(i,j)} \end{bmatrix}, \quad B_{\alpha(i,j)}=\begin{bmatrix} B_1^{\alpha(i,j)} \\ B_2^{\alpha(i,j)} \end{bmatrix},$$

$$E_{\alpha(i,j)}=\begin{bmatrix} E_1^{\alpha(i,j)} \\ E_2^{\alpha(i,j)} \end{bmatrix}, \quad C_{\alpha(i,j)}=\begin{bmatrix} C_1^{\alpha(i,j)} & C_2^{\alpha(i,j)} \end{bmatrix},$$

$$F_{\alpha(i,j)}=\begin{bmatrix} F_1^{\alpha(i,j)} \\ F_2^{\alpha(i,j)} \end{bmatrix}.$$

Where $x^h(i,j)\in R^{n_1}$ and $x^v(i,j)\in R^{n_2}$ are the horizontal state and the vertical state respectively, $x(i,j)$ is the whole state in R^n with $n=n_1+n_2$, $w(i,j)\in R^q$ is the disturbance input which belongs to $l_2\{[0,\infty),[0,\infty)\}$, $u(i,j)\in R^m$ is the controlled input, $z(i,j)\in R^p$ is the controlled output and $\alpha(i,j): Z_+\times Z_+ \to \underline{N}=\{1,2,...,N\}$ is the switching signal. N is the number of subsystems. $\alpha(i,j)=k$, $k\in\underline{N}$, denotes that the k-th subsystem is activated. A_k, B_k, C_k, D_k, E_k and F_k are real matrices with appropriate dimensions. i and j are integers in Z_+. The boundary condition is satisfied if $\|X(0)\|_2<\infty$ with $X(0)$ defined as follows:

$$X(0)=\left[x^h(0,0),\, x^h(0,1),\, x^h(0,2),\, \cdots,\, x^v(0,0),\, x^v(1,0),\, x^v(2,0),\, \cdots\right]^T. \quad (2)$$

The saturation function $sat(\cdot): R^m \to R^m$ is defined as:

$$sat(u)=\left[sat(u_1),\ sat(u_2),\ \cdots,sat(u_m)\right]^T. \quad (3)$$

With input defined as: $u=[u_1,\ u_2,\ \cdots,u_m]^T\in R^m$. For $p=1,2,...,m$, a standard saturation function can be described as: $sat(u_p)=sign(u_p)\min\{1,|u_p|\}$.

By implementing the closed loop state feedback control law: $u(i,j)=K_{\alpha(i,j)}x(i,j)$ with:

$K_{\alpha(i,j)}=[K_1^{\alpha(i,j)}\ K_2^{\alpha(i,j)}]$, where $K_1^{\alpha(i,j)}\in R^{m\times n_1}$, $K_2^{\alpha(i,j)}\in R^{m\times n_2}$ are the controller gain matrices to be designed. The corresponding closed-loop system is:

$$x^{+}(i,j)=A_{\alpha(i,j)}x(i,j)+B_{\alpha(i,j)}w(i,j)+E_{\alpha(i,j)}sat(K_{\alpha(i,j)}x(i,j)),$$

$$z(i,j)=C_{\alpha(i,j)}x(i,j)+D_{\alpha(i,j)}w(i,j)+F_{\alpha(i,j)}sat(K_{\alpha(i,j)}x(i,j)),$$

$$i,j=0,1,2\cdots. \quad (4)$$

Let Θ be the set of all diagonal matrices in $R^{n\times n}$ with diagonal elements that are either 1 or 0. For example, if $n=2$, then

$$\Theta=\{D_1,D_2,D_3,D_4\}=\left\{\begin{pmatrix}0&0\\0&0\end{pmatrix},\begin{pmatrix}0&0\\0&1\end{pmatrix},\begin{pmatrix}1&0\\0&0\end{pmatrix},\begin{pmatrix}1&0\\0&1\end{pmatrix}\right\}.$$

There are 2^n elements D_p in Θ, and for every $p=1,...,2^n$, $D_p^-=I_n-D_p$ is also an element in Θ.

For taking actuator saturation nonlinearity we will embed $sat(K_{\alpha(i,j)}x(i,j))$ within a convex hull of a group of linear feedbacks. For a positive definite matrix $P\in R^{n\times n}$ and a scalar $\delta>0$, an ellipsoid $\Omega(P,\delta)$ is defined as:

$$\Omega(P,\delta)=\left\{x(i,j)\in R^n : x(i,j)^T Px(i,j)\le\delta, D\in Z^+\right\}.$$

Define the following polyhedral set $L(H)$:

$$L(H):=\left\{x(i,j)\in R^n : |H_l x(i,j)|\le 1, l=1,2,...,m\right\},$$

Where H_l is the l-th row of matrix H.

Remark 1 In this paper, it is assumed that switching occurs only at each sampling point of i or j. The switching sequence can be described as,

$$((i_0,j_0),\alpha(i_0,j_0)),((i_1,j_1),\alpha(i_1,j_1)),\cdots,((i_\kappa,j_\kappa),\alpha(i_\kappa,j_\kappa)),\cdots$$

with (i_κ,j_κ) denotes the κ-th switching instant. It should be noted that the value of $\alpha(i,j)$ only depends on $i+j$ (see [17]).

Definition 1 [1] Discrete 2-D switched system (1) is asymptotically stable under switching signal $\alpha(i,j)$ if $\sup_{i,j}\|x(i,j)\|<\infty$ and $\lim_{i,j\to\infty} x(i,j)=0$ under $w(i,j)=0$ and any boundary condition $\sup_j\|x^h(0,j)\|<\infty$ and $\sup_i\|x^v(i,0)\|<\infty$.

Definition 2 [1] Given a positive scalar γ, and two symmetric and positive definite matrices R_1 and R_2, the discrete 2-D switched system (1) is said to have a prescribed H_∞ disturbance attenuation level γ if it is asymptotically stable and satisfies:

$$J=\sup_{0\ne w(i,j)\in l_2}\frac{\|z\|_2^2}{\|w\|_2^2+\sum_{i=0}^{\infty}x^{vT}(i,0)R_2x^v(i,0)+\sum_{j=0}^{\infty}x^{hT}(0,j)R_1x^h(0,j)}<\gamma^2 \quad (5)$$

Remark 2 For the case when the boundary condition is known to be zero, the inequality (5) reduces to

$$J_0=\sup_{0\ne w(i,j)\in l_2}\frac{\|z\|_2^2}{\|w\|_2^2}<\gamma^2, \quad (6)$$

Lemma 1 [24] Given K and $H\in R^{m\times n}$, then

$$sat(Kx(i,j))\in co\left\{(D_pK+D_p^-H)x(i,j), p=1,2,...,2^m\right\},\ \forall x(i,j)\in R^n, \quad (7)$$

that satisfy $\left|H_l x(i,j)\right| \le 1$, for H_l, $l = 1,2,...,m$. $co\{\square\}$, is convex hull.

When $x \in L(H)$, it follows that $sat\left(K_{\alpha(i,j)} x(i,j)\right)$ satisfies the condition (7) of Lemma 1. Then substituting (7) to system (4), and noticing the relationship between convex combination and its vertex, we can obtain the following representation for $p = 1, 2, \cdots, 2^m$.

$$x^+(i,j) = A^p_{\alpha(i,j)} x(i,j) + B_{\alpha(i,j)} w(i,j),$$

$$z(i,j) = C^p_{\alpha(i,j)} x(i,j) + D_{\alpha(i,j)} w(i,j), \tag{8}$$

where,

$$A^p_{\alpha(i,j)} = A_{\alpha(i,j)} + E_{\alpha(i,j)} D_P K_{\alpha(i,j)} + E_{\alpha(i,j)} D_P^- H_{\alpha(i,j)}.$$

$$C^p_{\alpha(i,j)} = C_{\alpha(i,j)} + F_{\alpha(i,j)} D_P K_{\alpha(i,j)} + F_{\alpha(i,j)} D_P^- H_{\alpha(i,j)}.$$

3. MAIN RESULTS

In this section, we investigate the problem of controller design for system (1) to ensure that closed-loop system (4) is asymptotically stable and has H_∞ disturbance attenuation level γ.

3.1. H_∞ PERFORMANCE ANALYSIS

In this subsection, the problem of H_∞ performance analysis of 2-D switched systems is discussed.

Theorem 1 Given a positive scalar γ, and two symmetric positive definite matrices R_1 and R_2, if there exist a set of block-diagonal, symmetric and positive definite matrices $P_i = diag\left\{P_1^k, P_2^k\right\} > 0$, and matrix H_k, $k \in \underline{N}$, such that

$$\begin{bmatrix} -P_k & P_k(A_l + E_l D_P K_l + E_l D_P^- H_l) & P_k B_l & 0 \\ * & -P_l & 0 & (C_l + F_l D_P K_l + F_l D_P^- H_l)^T \\ * & * & -\gamma^2 I & D_l^T \\ * & * & * & -I \end{bmatrix} < 0,$$

$$\forall (k,l) \in \underline{N} \times \underline{N}. \tag{9a}$$

$$P_k < \gamma^2 diag(R_1, R_2). \tag{9b}$$

$$\Omega(P_k, 1) \in L(H_k). \tag{9c}$$

then 2-D switched system (4) is asymptotically stable and has a prescribed H_∞ disturbance attenuation level γ for any arbitrary sequence of switching.

Proof When $x(i,j) \in \Omega(P_i, 1)$, then we choose the Lyapunov function candidate for system (4) as follows:

$$V(i,j) = x^T(i,j) P_{\alpha(i,j)} x(i,j), \tag{10}$$

where,

$$P_{\alpha(i,j)} = diag\{P_1^{\alpha(i,j)}, P_2^{\alpha(i,j)}\}. \tag{11}$$

$$\Delta V(i,j) = V^+(i,j) - V(i,j). \tag{12}$$

Set, $w(i,j) = 0$, along the trajectory of system (4), we have

$$\Delta V(i,j) = \left[x^{hT}(i+1,j) \quad x^{vT}(i,j+1)\right] \begin{bmatrix} P_1^{\alpha(i+1,j)} & 0 \\ 0 & P_2^{\alpha(i,j+1)} \end{bmatrix} \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} - \left[x^{hT}(i,j), x^{vT}(i,j)\right] \begin{bmatrix} P_1^{\alpha(i,j)} & 0 \\ 0 & P_2^{\alpha(i,j)} \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}. \tag{13}$$

Note that $\alpha(i+1,j) = \alpha(i,j+1)$, then (13) can be written as

$$\Delta V(i,j) = x^{+\mathrm{T}}(i,j) P_{\alpha(i+1,j)} x^+(i,j) - x^{\mathrm{T}}(i,j) P_{\alpha(i,j)} x(i,j). \tag{14}$$

There is no loss of generality if we assume that k-th subsystem is activated at $i+j$, $k \in \underline{N}$ that is $\alpha(i,j) = k$. When no switching occurs at $i+j+1$, in other words k-th subsystem is still active at $i+j+1$, one can obtain $\alpha(i,j) = \alpha(i+1,j) = k$, then one has

$$\Delta V(i,j) = x^{+T}(i,j) P_k x^+(i,j) - x^T(i,j) P_k x(i,j). \tag{15}$$

From, (9a) it follows that k-th subsystem is asymptotically stable.

When l-th subsystem is activated at $i+j+1$ that is, $\alpha(i+1,j) = l$, we get

$$\Delta V(i,j) = x^{+T}(i,j) P_k x^+(i,j) - x^T(i,j) P_l x(i,j). \tag{16}$$

It follows (9a), the Lyapunov function is decreasing at switching instants. Therefore, closed-loop system (4) with $w(i,j) = 0$ is asymptotically stable.

For any nonzero $w(i,j) \in l_2\{[0,\infty),[0,\infty)\}$ closed-loop system (4) has a prescribed H_∞ disturbance attenuation level γ. For this purpose, we introduce

$$\delta(i,j) = \Delta V(i,j) + z^T(i,j) z(i,j) - \gamma^2 w^T(i,j) w(i,j). \tag{17}$$

That is,

$$\delta(i,j) = \chi^T(i,j) \psi \chi(i,j). \tag{18}$$

Where, $\chi(i,j) = \left[x^T(i,j) \quad w^T(i,j)\right]^T$ and

$$\psi = \begin{bmatrix} A^{pT}_{\alpha(i,j)} P_{\alpha(i+1,j)} A^p_{\alpha(i,j)} - P_{\alpha(i,j)} + C^{pT}_{\alpha(i,j)} C^p_{\alpha(i,j)} & A^{pT}_{\alpha(i,j)} P_{\alpha(i+1,j)} B_{\alpha(i,j)} + C^{pT}_{\alpha(i,j)} D_{\alpha(i,j)} \\ B^T_{\alpha(i,j)} P_{\alpha(i+1,j)} A^p_{\alpha(i,j)} + D^T_{\alpha(i,j)} C^p_{\alpha(i,j)} & B^T_{\alpha(i,j)} P_{\alpha(i+1,j)} B_{\alpha(i,j)} + D^T_{\alpha(i,j)} D_{\alpha(i,j)} - \gamma^2 I \end{bmatrix}.$$

Applying Schur's Lemma it follows from (9c) that,

$$\psi < 0. \tag{19}$$

That implies,

$$\Delta V(i,j) + z^T(i,j) z(i,j) - \gamma^2 w^T(i,j) w(i,j) < 0. \tag{20}$$

It follows that,

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(z^T(i,j)z(i,j)-\gamma^2 w^T(i,j)w(i,j)\right)<\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(-\Delta V(i,j)\right). \tag{21}$$

From (15) one can obtain

$$\begin{aligned}\Delta V(i,j) &= x^{hT}(i+1,j)P_1^{\alpha(i+1,j)}x^h(i+1,j)-x^{hT}(i,j)P_1^{\alpha(i,j)}x^h(i,j)\\ &\quad + x^{vT}(i,j+1)P_2^{\alpha(i,j+1)}x^v(i,j+1)-x^{vT}(i,j)P_2^{\alpha(i,j)}x^v(i,j).\end{aligned} \tag{22}$$

Denoting,

$$M = x^{hT}(i+1,j)P_1^{\alpha(i+1,j)}x^h(i+1,j)-x^{hT}(i,j)P_1^{\alpha(i,j)}x^h(i,j).$$

$$N = x^{vT}(i,j+1)P_2^{\alpha(i,j+1)}x^v(i,j+1)-x^{vT}(i,j)P_2^{\alpha(i,j)}x^v(i,j). \tag{23}$$

From (23) it should be noted that $\alpha(i+1,j)=\alpha(i,j+1)$, since the value of $\alpha(i,j)$ only depends on $(i+j)$. For any positive scalars $n_1, n_2 \in Z_+$. We can write

$$\begin{aligned}&\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}\Delta V(i,j)=\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}M+\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}N\\ &<\sum_{j=0}^{n_2}\left(x^{hT}(n_1+1,j)P_1^{\alpha(n_1+1,j)}x^h(n_1+1,j)-x^{hT}(0,j)P_1^{\alpha(0,j)}x^h(0,j)\right)\\ &\quad+\sum_{i=0}^{n_1}\left(x^{vT}(i,n_2+1)P_2^{\alpha(i,n_2+1)}x^v(i,n_2+1)-x^{vT}(i,0)P_2^{\alpha(i,0)}x^v(i,0)\right).\end{aligned} \tag{24}$$

When, $n_1, n_2=\infty$, then we obtain

$$\begin{aligned}&\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(z^T(i,j)z(i,j)-\gamma^2 w^T(i,j)w(i,j)\right)\\ &<\sum_{j=0}^{\infty}\left(x^{hT}(0,j)P_1^{\alpha(0,j)}x^h(0,j)\right)+\sum_{i=0}^{\infty}\left(x^{vT}(i,0)P_2^{\alpha(i,0)}x^v(i,0)\right).\end{aligned} \tag{25}$$

From (9b) and (9c) we know that $P_1^k<\gamma^2 R_1$ and $P_2^k<\gamma^2 R_2$, $\forall k\in \underline{N}$. Then (25) becomes

$$\begin{aligned}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(z^T(i,j)z(i,j)\right)&<\gamma^2\Bigg(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(w^T(i,j)w(i,j)\right)\\ &\quad+\sum_{j=0}^{\infty}\left(x^{hT}(0,j)R_1x^h(0,j)\right)+\sum_{i=0}^{\infty}\left(x^{vT}(i,0)R_2x^v(i,0)\right)\Bigg).\end{aligned} \tag{26}$$

From (26), it can be obtained that,

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(z^T(i,j)z(i,j)\right)=\left\|z(i,j)\right\|_2^2,$$

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(w^T(i,j)w(i,j)\right)=\left\|w(i,j)\right\|_2^2. \tag{27}$$

That is

$$\left\|z(i,j)\right\|_2^2<\gamma^2\left(\left\|w(i,j)\right\|_2^2+\sum_{j=0}^{\infty}\left(x^{hT}(0,j)R_1x^h(0,j)\right)+\sum_{i=0}^{\infty}\left(x^{vT}(i,0)R_2x^v(i,0)\right)\right). \tag{28}$$

This completes the proof.

3.2. H_∞ CONTROLLER DESIGN

In this subsection, the H_∞ controller design problem of 2-D switched systems is addressed.

Theorem 2 Consider discrete 2-D switched system (1) with actuator saturation, for given positive scalar γ and symmetric positive definite matrices, $R=diag\{R_1,R_2\}$, if there exist a set of positive definite matrices $X^k=diag\{X_1^k,X_2^k\}$ and matrices Y^k, W^k with $k\in\underline{N}$, satisfying

$$\begin{bmatrix}-X^k & \Phi & B_l & 0\\ * & -X^l & 0 & \Pi\\ * & * & -\gamma^2 I & D_l^T\\ * & * & * & -I\end{bmatrix}<0,\quad \forall(k,l)\in\underline{N}\times\underline{N}. \tag{29a}$$

$$X^l>\gamma^{-2}R^{-1},\ \forall l\in\underline{N}. \tag{29b}$$

$$\begin{bmatrix}1 & W^l\\ * & X^l\end{bmatrix}\geq 0. \tag{29c}$$

with,

$$\Phi=A_lX^l+E_lD_pY^l+E_lD_p^-W^l,\ \Pi=(C_lX^l+F_lD_pY^l+F_lD_p^-W^l)^T,$$

$$K_l=[Y_1^l(X_1^l)^{-1}\quad Y_2^l(X_2^l)^{-1}]=[K_1^l\quad K_2^l],$$
$$H_l=[W_1^l(X_1^l)^{-1}\quad W_2^l(X_2^l)^{-1}]=[H_1^l\quad H_2^l].$$

Then the corresponding closed-loop system (4) is said to be asymptotically stable and possesses H_∞ disturbance attenuation level γ.

Proof Pre- and post- multiplying (9a) by $diag\left((P_k)^{-1},(P_l)^{-1},I,I\right)$ we get

$$\begin{bmatrix}-(P_k^{-1}) & (A_l+E_lD_pK_l+E_lD_P^-H_l)(P_l^{-1}) & B_l & 0\\ * & -(P_l^{-1}) & 0 & (P_l^{-1})(C_l+F_lD_pK_l+F_lD_P^-H_l)^T\\ * & * & -\gamma^2 I & D_l^T\\ * & * & * & -I\end{bmatrix}<0. \tag{30}$$

Let, $X^l=(P_l^{-1})$, $X^k=(P_k^{-1})$, $Y^l=K_l(P_l^{-1})$, $W^k=H_l(P_l^{-1})$. then (30) becomes

$$\begin{bmatrix}-X^k & A_lX^l+E_lD_pY^l+E_lD_p^-W^l & B_l & 0\\ * & -X^l & 0 & (C_lX^l+F_lD_pY^l+F_lD_p^-W^l)^T\\ * & * & -\gamma^2 I & D_l^T\\ * & * & * & -I\end{bmatrix}<0. \tag{31}$$

Let,

$$\Phi = A_l X^l + E_l D_p Y^l + E_l D_P^- W^l,$$

$$\Pi = (C_l X^l + F_l D_p Y^l + F_l D_P^- W^l)^T, \ \forall (k,l) \in \underline{N} \times \underline{N}.$$

It can be obtained that (31) is equivalent to (29). Thus, we can deduce that the given closed-loop system (4) with actuator saturation is asymptotically stable and has H_∞ disturbance attenuation level γ for any arbitrary switching sequence.

This completes the proof.

Remark 3 It should be pointed that the results presented in Theorem 2 can be considered as an extension of results presented in [27] to the problem of 2-D discrete switched systems with actuator saturation. To the best our knowledge, there are no results available on the problem of state feedback H_∞ control of 2-D switched systems with actuator saturation therefore direct comparison is not possible.

4. NUMERICAL EXAMPLE

This section presents the simulation example to ensure the validity of above proposed results.

Consider the following equation which describes some thermal process, for example in chemical reactors, heat exchangers and pipe furnaces [2].

$$\frac{\partial T(x,t)}{\partial x} = -\frac{\partial T(x,t)}{\partial t} - a^{\sigma(x,t)} T(x,t) + b^{\sigma(x,t)} w(x,t) + e^{\sigma(x,t)} sat(u(x,t)). \tag{32}$$

$T(x,t)$ is the temperature of the process at $x(Space) \in [o, x_f]$ and $t(Time) \in [o, \infty)$. $u(x,t)$ is the input function. $a^{\sigma(x,t)}, b^{\sigma(x,t)}$ and $e^{\sigma(x,t)}$ are real coefficients. Initial goal is to represent the above stated 2-D model conversion to 2-D discrete state space Roesser model. Taking,

$$T(i,j) = T(i\Delta x, j\Delta t), \quad w(i,j) = w(i\Delta x, j\Delta t),$$
$$u(i,j) = u(i\Delta x, j\Delta t).$$

$$\frac{\partial T(x,t)}{\partial t} \cong \frac{T(i,j+1) - T(i,j)}{\Delta t}, \quad \frac{\partial T(x,t)}{\partial x} \cong \frac{T(i,j) - T(i-1,j)}{\Delta x}.$$

Then we can obtain following Roesser's model

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{\Delta t}{\Delta x} & 1 - \frac{\Delta t}{\Delta x} - a^{\sigma(x,t)} \Delta t \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + \begin{bmatrix} 0 \\ b^{\sigma(x,t)} \Delta t \end{bmatrix} w(i,j) + \begin{bmatrix} 0 \\ e^{\sigma(x,t)} \Delta t \end{bmatrix} u(i,j).$$

For numerical simulation choose time and the space discretization period as:

Δt = 0.5, Δx = 0.3, Now it is assumed that 2-D discrete switched system has two subsystems with, a^1=0.1, a^2=0.3, b^1=0.2, b^2=0.1, e^1=0.5 and e^2=0.6 then we will have following set of parameters for the approximated process.

Subsystem 1

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1.67 & -0.72 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$

$$C_1 = [0 \ \ -1], \quad D_1 = 1, \quad F_1 = 0.$$

Subsystem 2

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1.67 & -0.82 \end{bmatrix}, E_1 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0.05 \end{bmatrix},$$

$$C_2 = [0 \ \ -1], \quad D_2 = 2, \quad F_2 = 0.$$

Take $\gamma = 10$, $R_1 = 1$, $R_2 = diag(1,1)$, and the disturbance input $w(k) = \frac{1}{i}$, the boundary condition is given as follows:

$$x^h(0,j) = \frac{2}{j}, \quad 0 < j \le 15; \quad x^h(0,j) = 0, \quad 15 < j.$$

$$x^v(i,0) = \frac{2}{i}, \quad 0 < i \le 15; \quad x_1^v(i,0) = 0, \quad 15 < i.$$

Solving the matrix inequalities in Theorem 2 with, $n_1 = 1$, $n_2 = 1$, gives rise to:

$$X_1 = \begin{bmatrix} 0.0576 & 0 \\ 0 & 0.0438 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 0.0674 & 0 \\ 0 & 0.0441 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -0.3850 & 0.1262 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} -0.3752 & 0.1207 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -0.2038 & 0.1072 \end{bmatrix}, \quad W_2 = \begin{bmatrix} -0.2179 & 0.1058 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -6.6800 & 2.8840 \end{bmatrix}, K_2 = \begin{bmatrix} -5.5667 & 2.7368 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} -3.5361 & 2.4502 \end{bmatrix}, H_2 = \begin{bmatrix} -3.2332 & 2.3991 \end{bmatrix}.$$

Figure 1 and Figure 2 depicts the trajectories of horizontal and vertical states $x^h(i,j)$ and $x^v(i,j)$ respectively. It can be concluded from Figures that closed-loop system (4) is asymptotically stable. The corresponding arbitrary sequence of switching is shown by Figure 3.

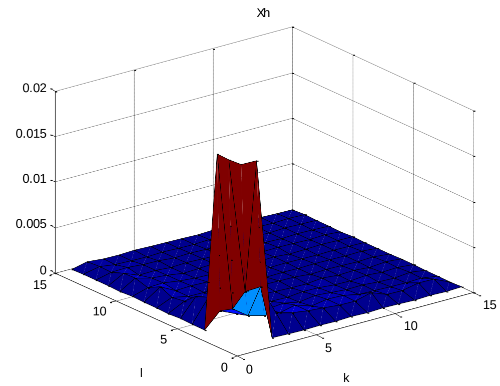

Figure 1 State trajectory of horizontal state $x^h(i,j)$.

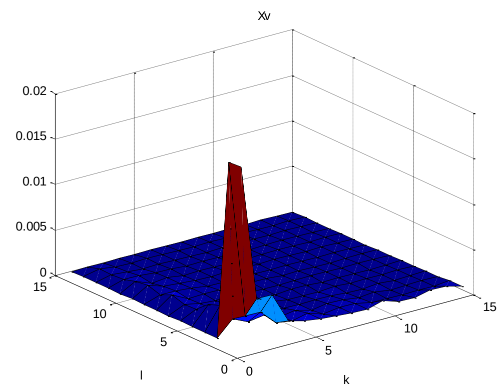

Figure 2 State trajectory of vertical state $x^v(i,j)$.

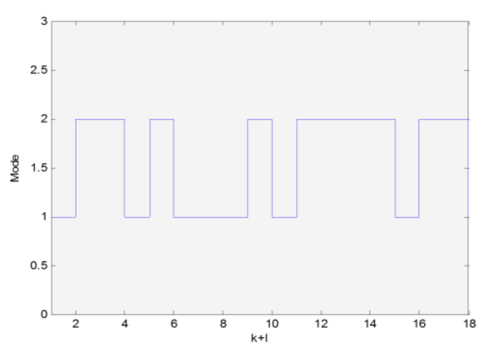

Figure 3 The switching signal.

5. CONCLUSION

State feedback H_∞ controller design problem for 2-D discrete switched systems with actuator saturation has been investigated in this paper. By utilizing the multiple Lyapunov functional approach, necessary and sufficient condition for asymptotical stability of corresponding closed-loop system with a prescribed H_∞ disturbance attenuation level γ is proposed. Our future work shall focus on the extension of the proposed control methodology for 2-D continuous-discrete switched systems which is still an open problem.

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