DEVELOPING AN EFFICIENT ALGORITHM FOR THE POSE ESTIMATION PROBLEM WITH SOME APPLICATIONS IN COMPUTER VISION AND ROBOTICS

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***:** In Computer vision and robotics, a typical task is to identify special objects of an image to determine each object's position and orientation relative to some coordination systems. The pose of an object is the combination of its position and orientation. This information can be used, for example, to allow a robot to manipulate an object or to avoid moving into the object. Post estimation, also known as the Perspective-n-Point Problem (PnP), is to estimate the pose of the camera based on the given 3D reference points and their associated 2D images. It is one of the important problems in computer vision, photogrammetry and robotics. In this paper, we design more effective, fast and efficient branch and bound algorithm for the pose estimation problem which will help us to get an optimal solution as well as a number of local optimal solutions. As applications, we design a tighter convex relaxation algorithm for the graph matching problem and establish the global model for the fundamental matrix recovery problem of candidate corresponding points in stereo vision.*

***Keywords*:** Pose Estimation, Perspective-n-Point Problem, Robotics, Branch- and- Bound.

1. INTRODUCTION

Pose estimation, also known as the perspective endpoint problem (PnP), is to estimate the pose of the camera based on the given 3D reference points and their associated 2D images [7,17,18]. Pose estimation problem is considered as one of the most important problems in computer vision, photogrammetry and robotics. The solution methods for solving pose estimation problem can be divided into the following three groups: The first group is composed of the iterative local search method ([2, 3, 8-10, 12, 22]). The orthogonal iteration (OI) Algorithm [8, 19, 23], may be the most efficient. The basic idea of OI algorithm is to minimize object space error by alternatively minimizing the estimation of the rotation matrix and the translational vector. Starting from a proper initialization, the OI algorithm often-fast coverage to a high-accuracy-global minimizer. But if it is poorly initialized, OI could get trapped in a local minimizer. The second group is made up of the iterative global optimization methods. [1, 28, 29] proposed a branch-and-bound algorithm to solve the triangulation and camera pose estimation, where the objective function is fractional. The lower bounding approach is to solve the second order cone programming (SOCP), relaxation, by noting that a single fraction t/s with bounded s and t can be rewritten as an SOCP [16, 20, 21]. This algorithm was further employed by [11] to minimize the image space error, where the rotation matrix was parameterized by quaternion. [5] developed a branch-and-bound method to minimize the ℓ_∞ norm of the tangent of the angle error, based on SOCP relaxation. Through providing the solution of proven global optimality, the branch-and-bound methods are of limited application in practice because of their high computational complexity. For example the average running time reported in [5] is 1.5 minutes for 10 reference points.

The third group consists of the non-iterative methods ([4, 6, 24-27]). The pose estimation problem is first reformulated to a single (large) equation system, then the system is approximately solved in order to gain the speed. Recently, [12-14] proposed the semi definite relaxation (SDR) approach by lifting the quaternion model of the pose estimation problem, moreover, to perform better, the standard SDR often gives a solution close to the global minimize, even for small number of points and large noise. The limitation is that the accuracy of the solution obtained by SDR is lower than that of OI.

To our knowledge, the branch-and-bond algorithm for minimizing the object space error (which is the same cost as in OI and SDR) has not been studied in literature. Suppose now we directly employ the branch-and-bound algorithm developed in [1] to minimize the object space error, at first we have to introduce how much more additional variables to linearize the cost function, which certainly is far from efficient. In this paper, we observe that the object space is already a convex quadratic function. It motivates us to develop a new branch-and-bound, method based on quadratic programming (QP) relaxation. To improve the efficiency, we establish a tighter Lagrangian reformulation of the quadratic object space error.

2. RESEARCH PROBLEM: FORMULATIONS AND RELAXATIONS

The main research problem is how to branch and construct a compact lower bound for the branch and bound algorithm of the pose estimation problem.

There are some branch and bound methods for nonlinear objectives in literatures. While the drawback is the loose relaxed lower bound causing excessive branching, the algorithms take a long time. In our paper, we will design branch and bound for linear target directly, but how to construct a tighter lower bound is a similar problem. Noting the secondary objective function has many variations, happens to anyone of Lagrangian function under orthogonal constraints is equal to the original target, which gives us the selected source of compact lower bound. In addition, how to branch is also the key to determine the efficiency of algorithm, the literature of traditional branch and bound is standard without considering the structure of the problem. The paper attempts to develop efficient branches with utilization of the structure.

Given a set of 3D reference points $p_{i,} i = 1,2,...,n (n \geq 3)$ in the object coordinate system and the associated normalized

2D image projections v_i in the camera coordinate system, we minimize the following object space error [12]:

$$\min_{R\in S(3),t}\{E(R,t)=\sum_{i=1}^{n}\|(I-\hat{v}_i)(Rp_i+t)\|^2\} \quad (1)$$

where S(3) is the set of 3 × 3 orthogonal matrices, I is the 3 × 3 identity matrix, ‖ · ‖ is standard l_2-norm, and:

$$\hat{V}_i=\frac{\hat{v}_i\hat{v}_i^T}{\hat{v}_i^T\hat{v}_i} \ ,$$

Since (1) is an unconstrained quadratic program in terms of t, by setting the partial gradient of (1) with respect to t equal to zero:

$$\frac{\partial}{\partial t}E(R,t)=\sum_{i=1}^{n}2\{(I-\hat{V}_i)(Rp_i+t)\}=0 \ ,$$

we can get the optimal translation vector [15, 23]:

$$t_{opt}=-\left(\sum_{i=1}^{n}Q_i\right)^{-1}\sum_{i=1}^{n}(Q_iRp_i), \quad (2)$$

Where

$$Q_i=(I-\hat{V}_i)^T(I-\hat{V}_i)=I-\hat{V}_i.$$

As in [23], define the following operators for the 3D vector p and 3 × 3 matrix R, receptively,

$$C(p)=\begin{bmatrix} p^T & 0_{1\times3} & 0_{1\times3}\\ 0_{1\times3} & p^T & 0_{1\times3}\\ 0_{1\times3} & 0_{1\times3} & p^T\end{bmatrix}.\ r(R)=\begin{bmatrix} r_1^T\\ r_2^T\\ r_3^T\end{bmatrix}$$

Where $0_{1\times3}$ is a zero matrix of size 1 × 3 and $R=[r_1^T r_2^T r_3^T]^T$. Now we can rewrite (2) as:

$$t_{opt}=T_{3\times9}\ .r, \quad (3)$$

Where

$$T_{3\times9}=-\left(\sum_{i=1}^{n}Q_i\right)^{-1}\sum_{i=1}^{n}(Q_iC(p_i)).$$

Substituting (3) into (1) and rearranging the formulation yields the following simple model:

$$\min_{R\in S(3)}\left\{f_1(R):=r(R)^TMr(R)\right\} \quad (4)$$

Where

$$M=\sum_{i=1}^{n}\left((C(p_i)+T_{3\times9})^TQ_i(C(p_i)+T_{3\times9})\right).$$

It is easy to verify that

$$R\in S(3)\ \Leftrightarrow\ R^TR=I\ \Leftrightarrow\ RR^T=I.$$

Then (4) has the following three quadratic constrained quadratic programming (QCQP) reformulations:

$$\min_{R^TR=I} r(R)^TMr(R)\ , \quad (5)$$

$$\min_{RR^T=I} r(R)^TMr(R)\ , \quad (6)$$

$$\min_{R^TR=I,RR^T=I} r(R)^TMr(R)\ , \quad (7)$$

Where the idea to add two redundant constraints in (7) is not new, see for example, [3, 26].

For QCQP, Lagrangian dual often provides a high-quality lower bound for the primal problem. We first present the Lagrangian dual of (7). Let S, T be two symmetric matrices of size 3 × 3, respectively. The Lagrangian function of (7) is:

$$\begin{aligned} &L(r(R).S.T)\\ &=r(R)^TMr(R)-tr((R^TR-I)S)-tr((RR^T-I)T)\\ &=r(R)^T(M-I\otimes S-T\otimes I)r(R)+tr(S+T) \quad (8)\end{aligned}$$

Where tr(A) denotes the trace of the matrix A (i.e., the sum of all the diagonal entries of A), A ⊗ B denotes the Kronecker product of A and B. Then the dual function reads:

$$d(S,T)=\min_{r(R)}L(r(R),S,T)$$

$$=\begin{cases} tr(S+T), if \ \ M-I\otimes S-T\otimes I\geq 0\\ -\infty, \qquad otherwise,\end{cases}$$

Where A ≽ 0 denotes that A is positive semi definite. Now, the Lagrangian dual problem is:

$$\max_{S=S^T,T=T^T}\{d(S,T)\}.$$

$$=\max_{M-I\otimes S-T\otimes I\geq0,S=S^T,T=T^T} tr(S+T). \quad (9)$$

We similarly write the Lagrangian dual problems of (5) and (6) as follows:

$$\max_{M-I\otimes S\geq0,S=S^T} tr(S). \quad (10)$$

$$\max_{M-T\otimes I\geq0,T=T^T} tr(T). \quad (11)$$

The above three dual problems are all semi-definite programming (SDP) problems. They can be globally solved by the publicly available optimization tools SeDuMi [15].

3. A NEW BRANCH-AND-BOUND METHOD

The branch-and-bound algorithm plays a great role in globally minimizing the nonconvex problems, see for example, [22]. It terminates with a certificate proving that the obtained solution is ϵ-suboptimal, by iteratively updating the upper and lower bounds on the optimal objective value. However, in general, the worst-case complexity of the branch- and-bound method grows exponentially with the problem size. For this purpose, we rewrite the pose estimation problem (4) as

$$\min_{(\alpha,\beta,\gamma)\in[0,2\pi]\times[0,\pi]\times[0,2\pi]} f_1(R(\alpha,\beta,\gamma)) \quad (12)$$

by observing there is a one-to-one mapping between the rotation matrices and the Euler angles:

$$R(\alpha,\beta,\gamma)=\begin{bmatrix} R_{11} & R_{12} & \sin(\alpha)\sin(\beta) \\ R_{21} & R_{22} & -\cos(\alpha)\sin(\beta) \\ R_{31} & R_{32} & \cos(\beta) \end{bmatrix} \quad (13)$$

Where $R_{11}=\cos(\alpha)\cos(\gamma)-\cos(\beta)\sin(\alpha)\sin(\gamma)$,

$R_{12}=-\cos(\beta)\cos(\gamma)\sin(\alpha)-\cos(\alpha)\sin(\gamma)$,

$R_{21}=\cos(\gamma)\sin(\alpha)+\cos(\alpha)\cos(\beta)\sin(\gamma)$,

$R_{22}=\cos(\alpha)\cos(\beta)\cos(\gamma)-\sin(\alpha)\sin(\gamma)$,

$R_{31}=\sin(\beta)\sin(\gamma)$ and $R_{32}=\cos(\gamma)\sin(\beta)$.

Denote by $f_{lb}(Q)$ and $f_{ub}(Q)$ the lower and upper bounds of the objective function over Q, respectively. The following general branch-and-bound algorithm presented in [6] is employed to solve (20):

Algorithm

Step 0: *Set* $\in>0$. Initialize k=0, $S_0=\{Q_0\}$, $L_0=f_{ib}(Q_0)$ and $U_0=f_{ub}(Q_0)$.

Step 1: If $U_k-L_k<\in$, stop and return an $\in-suboptimal$ solution R^* such that $f(R^*)=U_k$ otherwise, goto step2.

Step 2: (Branching) Select $Q\in S_k$ such that $f_{lb}(Q)=L_k$ and then split Q along one of its longest edges into Q_l and Q_r. More precisely, suppose $Q=[\underline{q}_1,\bar{q}_1]\times[\underline{q}_2,\bar{q}_2]\times[\underline{q}_3,\bar{q}_3]$. Let $j=\arg\max_{i=1,2,3}(\bar{q}_i-\underline{q}_i).q_j=(\underline{q}_j+\bar{q}_j)/2$. If $j=1$, $Q_1=[\underline{q}_j,q_j]\times[\underline{q}_2,\bar{q}_2]\times[\underline{q}_3,\bar{q}_3]$, $Q_r=[q_j,\bar{q}_j]\times[\underline{q}_2,\bar{q}_2]\times[\underline{q}_3,\bar{q}_3]$. When $j=2,3,Q_l$ and Q_r are similarly defined. Let: $S_{k+1}=S_k\cup Q_1\cup Q_r\setminus Q$. Goto step 3.

Step 3 :(Bounding) Compute $f_{ub}(Q_l)$ and $f_{ub}(Q_r)$. Update the upper bound $U_{k+1}=\min\{U_k\, f_{ub}(Q_l),\, f_{ub}(Q_r)\}$, and the lower bound $L_{k+1}=\min Q\in S_{k+1} f_{lb}(Q)$. Update the candidate optimal solution R^* as the feasible solution corresponding to U_{k+1}. Prune: $\{Q: f_{lb}(Q)>U_{k+1}\}$ from S_{k+1}. Let *k:=k+1* and goto step1.

Now, we discuss in detail the estimation of lower and upper bounds, $f_{lb}(Q)$ and $f_{ub}(Q)$, which are critical for the efficiency of the branch-and-bound algorithm.

Suppose the cuboid $Q=[\underline{q}_1,\bar{q}_1]\times[\underline{q}_2,\bar{q}_2]\times[\underline{q}_3,\bar{q}_3]$. It follows from the representation (13) that we can easily calculate the element-wise lower and upper bounds of *R*, denoted by *L and* U, respectively. For example, if $\max(q_2, q_3)\le\pi/2$, $L(3,1)=\sin(\underline{q}_2)\sin(\underline{q}_3)$ and $U(3,1)=\sin(q_2)\sin(q_3)$. Define:

$$R(Q)=\{R(\alpha,\beta,\gamma):\ (\alpha,\beta,\gamma)\in Q\}$$

Then, according to (13), we have:

$$R(Q)=\{R\in S(3):\ L\le R\le U\}. \quad (14)$$

Removing the constraint $R\in S(3)$ yields a lower relaxation of (12):

$$f_{lb}(Q):=\min_{L\le R\le U}\{f_1(R)=r(R)^T Mr(R)\}. \quad (15)$$

which is a box-constrained convex quadratic programming (QP) problem and hence globally solved in polynomial time. In particular, at the root note (i.e., $Q=Q_0$), all the entries of L are -1 and $U=-L$. Then, we have $f_{lb}(Q_0)=0$ without any need to solve (15). The upper bound $f_{ub}(Q)$ is set as $f_1(R^*)$, where $R^*\in R(Q)$ is obtained by some heuristic. Solving (15), we obtain a solution matrix, denoted by $\tilde{R}$ If $\tilde{R}\in S_3$, then according to (14), we have $\tilde{R}\in R(Q)$ and then set $R^*=\tilde{R}$, $f_{ub}(Q)=f_1(\tilde{R})$. Otherwise, let $\tilde{R}^*$ be the closest point to $\tilde{R}$ in *S*(3), i.e.,

$$\tilde{R}^*=\arg\min_{R\in S(3)}\{\|R-\tilde{R}\|_F^2=tr((R-\tilde{R})^T(R-\tilde{R}))$$

$$=tr(I)-2tr(\tilde{R}^T R)+tr(\tilde{R}^T\tilde{R})\}.$$

where $\|\cdot\|_F$ is the Frobenius norm. Or equivalently,

$$\tilde{R}^*=\arg\max_{R\in S(3)} tr(\tilde{R}^T R). \quad (16)$$

Let $\tilde{R}=USV^T$ be the singular value decomposition (SVD) of $\tilde{R}$, where *U* and *V* are orthogonal matrices and *S* is a diagonal matrix. As first given in [7], the solution of (16) is $\tilde{R}^*=UV^T$

Finally, we notice that the above branch-and-bound algorithm for solving (4) can be similarly employed to solve (12)-(14). Denote these four algorithms by B&B1, B&B2, B&B3 and B&B4, respectively.

4. DISCUSSION:

Note that we aim to construct the dual problem and design algorithm for the dual problem of orthogonal model compactness in the graph matching problem. In the paper, the directly dual Lagrangian relaxation of the orthogonal model doesn't been considered, since the relaxation is too loose and the result is not satisfactory. We consider how to construct a tighter dual problem, we are now introducing some rational redundant constraints can export tight dual relaxation depending on the empirical theory, and even can fill duality gap sometimes. While how to select the constraints and construct the dual problem in the paper is a key question. Furthermore, if we get the structure of the dual relaxation, how to restore the original rotation matrix is also a problem to be considered in depth. There are some branch and bound methods for nonlinear objectives in literatures. While the drawback is the loose, relaxed lower bound causing excessive branching, the algorithms take long time. In this paper, we design branch and bound for linear target directly, but how to construct a tighter lower bound is a similar problem. Noting the secondary objective function has many variations, happensto any one of the Lagrangian function under orthogonal constraints is equal to the original target, which gives us the selected source of compact lower bound. In addition, how to branch is also the key to determine the efficiency of the algorithm, the literature of traditional branch and bound is standard without considering the structure of the problem. The paper attempts to develop efficient branches with utilization of the structure. Tighter convex relaxation algorithm for graph matching problem. A relaxation by using a linear programming relaxation solution as Lagrangian multiplier in Lagrangian function is considered in the literature, noting that we can mine the upper and lower bounds of the variable of the linear programming relaxation deeper in order to improve the linear programming model and get a new multiplier, then we can obtain a new Lagrangian function and design convex relaxation algorithm upon the new function. Some details still need to demonstrate and research. Establish the model and design efficient algorithm for the fundamental matrix recovery problem of candidate corresponding points in stereo vision.

It is not difficult to establish the overall model, the key is how to rewrite the model, It is well known that different forms of the equivalent model determine the efficiency of algorithms , while the differences may even vary considerably. So how to carve an exquisite equivalent model is vital. In addition, how to design algorithm is also a crux, since there exist different algorithms for a same model, it is likely to lead to huge difference. It requires experience and numerical experiments repeatedly.

5. CONCLUSIONS:

In the area of pose estimation, the model of minimizing the object space error has been used in many heuristics including the well-known orthogonal iteration (OI) and the very recent semi-definite programming relaxation (SDR). In this paper, we devised a new approach also based on semi-definite programming (SDP). SDR method is to use quaternion first and then upgrade matrix deformation space for relaxation, SOS is a kind of relaxation theory raised to k-order polynomial-based space, which can guarantee equivalence with the original problem if k is sufficiently large. Literature [3,4] both derived SDP relaxations which can be solved by the well-known optimization tool SeDuMi, the coefficient scale are respectively 117×32 and 266×70. Is there any more effective small-scale SDP relaxation? We consider the Lagrangian dual problem on the form of orthogonal constraints, the dual problem is equivalent to a semi-definite programming problem, if we write the dual problem directly the relaxation will be very loose. The paper aims a new enhanced method to structure duality. The coefficient matrix of new model is much smaller in SeDuMi format under the new duality. The further research includes the study of theoretical details, how to restore orthogonal matrix from the solution of the SDP, and a large number of numerical simulations.

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REFERENCES

1. S. Agarwal, M. K. Chandraker, F. Kahl, D. Kriegman, S. Belongie, Practical Global Optimization for Multiview Geometry, European Conference on Computer Vision, 2006, pp, 592-605.
2. Ansar, K. Daniilidis, Linear Pose estimation from points or lines, IEEE European Conference on Computer Vision, 4(2002), pp. 282-296.
3. H. Araijo, R. J. Carceroni, C. M. Brown, A fully projective formulation to improve the accuracy of Lowe's pose-estimation algorithm, Computer Vision and Image Understanding, 71(1998), pp, 227-238.
4. P. D. Fiore, Efficient linear solution of exterior orientation, IEEE Transactions on PAMI, 23(2)(2001), pp. 140-148.
5. R. Hartley, F. Kahl, Global Optimization through rotation space search, International Journal of Computer Vision, 82(2009), pp. 64-79.
6. H. Hmam, K. Kim, Optional non-iterative pose estimation via convex relaxation, Image and Vision computing, 28(2010), pp. 1515-1523.
7. R. I. Hartley, A. Zisserman, Multiple view Geometry in Computer Vision (Second edition), Cambridge University Press, Cambridge, 2003.
8. C.P. Lu, G.D. Hanger, E. Mjolsness, Fast and globally convergent pose estimation from video and images, IEEE Transactions on PAMI, 22(6)(2000), pp.610-622.
9. F. Moreno-Noguer, V. Lepetit, P. Fua Accurate non-iterative o(n) solution to the pnp problem, IEEE 11th International Conference on Computer Vision, 2007, pp. 1-8.
10. D. Oberkampf, D. F. DeMenthon, L. S. Davis, Iterative pose estimation using coplanar feature points, Computers Vision and Image Understanding, 63(1996), pp. 495-511.
11. C. Olsson, F. Kahl, M. Oskarsson, Optional Estimation of Perspective Camera Pose, Proceedings of the 18th International Conference on Pattern Recognition, 2006, pp. 5-8.
12. G. Schweighofer, & A. Pinz, Robust pose estimation from a planer target, IEEE Transactions on PAMI, 28(2006), pp. 2024-2030.
13. G. Schweighofer, A. Pinz, Fast and globally convergent structure and motion estimation for general camera models, 17th British Machine Vision Conference, 2006, pp. 147-156.
14. G. Schweighofer, A. Pinz, Globally Optional O(n) Solutions to the PnP Problem for General Camera Models, Proceeding of the 19th British Machine Vision Conference, 2008, pp. 1-10.
15. J.F. Sturm, Using SeDuMi 1.02, a MATLAB toolbox for optimation over symmetric cones, Optimization Methods and Software, 1990. Pp. 625-653.
16. M. Tawarmalani, N. V. Sahinidis, Semidefinite relaxations of fractional programs via novel

convexification techniques, J. Glob, Opt. 20(2001), pp. 137-158.

17. Wajeb Gharibi, Y. Xia, “A Tight Linearization Strategy for Zero-One Quadratic Programming Problems”, International Journal of Computer Science Issues (IJCSI), Volume **9**, Issue 3, April 2012.
18. Wajeb Gharibi, Y. Xia, “New Heuristic Rounding Approaches to the Quadratic Assignment Problem,” Journal of Communication and Computer, USA, Vol. **7**, No.4, 2010.
19. Wajeb Gharibi, Y. Xia, "An Algorithm for Solving Mixed Linear-Nonlinear Minimization Problem". Accepted for publishing in the KKU Journal, March 2010.
20. Wajeb Gharibi ,Y. Xia , “A Study on the Quadratic Assignment Problem with Symmetric Rank-1 Input Matrices”, Umm Al-Qura Univ. J. App. Sci. Vol.**1**, No. 1, pp. 58-67, 2009.
21. Wajeb Gharibi, Y. Xia, "A Dual Approach for Solving Nonlinear Infinite-Norm Minimization Problems with Applications in Separable Cases". Numer. Math. J. Chinese Univ. (English Ser.), Issue 3, Vol. **16**, pp. 265-270, 2007.
22. Wajeb Gharibi, “Improved Balas and Mazzola Linearization for Quadratic 0-1 Programs with Application in a New Cutting Plane Algorithm”, Int. J. Communications, Network and System Sciences (IJCNS), Volume **5**, Number 4, 2012, USA.
23. Wajeb Gharibi, “A Note on Linear Complementarity Problem”, Umm Al-Qura University Journal of Science-Medicine- Engineering. Vol. **19**, No. 1, 1427, 2007.
24. Wajeb Gharibi, "Analytic Centers and Quadratic Programming", International Journal of Intelligent Computing & Information Sciences, Vol. **5**, No. 2, July 2005.
25. Wajeb Gharibi, “An Improved Lower Bound of the Spark with Application “, International Journal of Distributed and Parallel Systems (IJDPS), Vol.**3**, No.5, September 2012.
26. Wajeb Gharibi, “Developing Lower Bounder for the Spectral Radius of a Real Matrix”, Damascus University Journal for Basic Sciences and Agricultural Engineering. Vol. **12** No. 1, 2000.
27. Wajeb Gharibi, “Solving (0-1) Linear Integer Programs By Using Lifting Gomory’s Mixed Integer Cuts”, Damascus University Journal for Basic Sciences and Agricultural Engineering. Vol. **14** No. 1, 1998.
28. Y. Xia, Global Optimization of a Class of Nonconvex Quadratically Constrained Quadratic Programming Problems, ActaMethematica Sinica, English Series, 27(9)(2011), pp. 1803-1812.
29. Y. Xia, Wajeb Gharibi, "On Improving Convex Quadratic Programming Relaxation for the Quadratic Assignment Problem", Journal of Combinatorial Optimization (DOI: 10.1007/s10878-013-9655-3), 2013. Springer.