OPTIMIZATION OF VEHICLE ROUTING PROBLEM WITH TIMES WINDOWS, STOCHASTIC DEMAND AND CONSTRAINED CAPACITY (VRPTW-SDCC) USING CROSS ENTROPY

Pusparani Asri Sugiyarto Putri, Iwan Aang Soenandi, Budi Marpaung

Department of Industrial Engineering, Krida Wacana Christian University, Jakarta, Indonesia

ABSTRACT: *The purpose of this study is to determine the optimal number of vehicles distribute goods from warehouse to all customers, consider with constraint and stochastic problem. Delivery of goods from the warehouse to some customers with limited capacity is an optimization problem often faced by the company. Limited vehicles capacity, lead-time, and changing customer requests make a company concern to determining the optimal vehicle route. The characteristics of the problem, it called Vehicle Routing Problem with Times Window, Stochastic Demand Constrained Capacity (VRPTW-SDCC). Using Matlab R2012a, Cross Entropy is the method to overcome VRP and can perform stochastic perform. Using Cross Entropy, all shipments none exceeds limit set of lead time customer and driver back to depot none exceeds working hours. The optimal total vehicle required to deliver the goods from producer to customers was found quickly.*

Keywords: VRPTW-SDCC, Cross Entropy, optimization

1. INTRODUCTION

Delivery of the goods from the warehouse to some customers using vehicles which that limited capacity is an optimization problem often faced by the manufacturing industry. Every day companies must deliver the goods to each customer with stochastic of demand. The distance and time of delivery as well as the capacity of the vehicle becomes an obstacle to finding the optimal delivery sequences.

Similarly, what happens to a manufacturing company faced with a problem sending some goods to a few customers a day? Very limited vehicle makes the company had to hire a vehicle outside the vehicle owned by the company to meet delivery targets. In one day, the number of vehicle with available capacity should be shipping goods within eight working hours. Every day the amount of goods and customers will change to adjust to the delivery schedule. Lead time customer making company more closely in terms of determining the route of each vehicle.

Until now, the company often gets complaints from customers. To meet customer demand for the company not only rely on its own fleet, but also hired a number of trucks with the same capacity. This problem has not been resolved until now. Based on the problems that occurs late delivery makes, the company wanted to find the optimal solution in terms of determining the delivery route.

Vehicle with limited capacity can be used for delivery of goods to multiple customer at one time with the minimum distance is a problem known as the Vehicle Routing Problem (VRP). The delivery time is limited so called times windows. Requests are always changing; making the company must calculate the probability so called stochastic demand. Their limited in terms of capacity and number of vehicles was referred to as capacity constrained. Having regard to the characteristic of the problems facing the company then known as the Vehicle Routing Problem with Times Windows, Stochastic Demand Constrained Capacity (VRPTW-SCDD).

2. BACKGROUND

VRP is a formulation of vehicle with a limited capacity to visit a number of customers at minimum cost [1]. Dynamic Routing Problem (DVRP) was referred to as on-line vehicle. Delivery of the products from depot to customer or a particular warehouse with a different purpose is an optimization problem often by companies. These vehicles must be determined to achieve efficiency in accordance with customer demand. This problem is often referred to Capacitated Vehicle Routing Problem (CVRP) [2]. Joyo [3] referred to problems occurring on the positioning under disturbance.

Vehicle Routing (VR) is very important for the logistic distribution. However, customer demand is stochastic generally not considered in this study [4]. Another research is using two loading constraints vehicle routing problem (2L-CVRP) to solve this issue with the capacity of two dimensional [5]. Heterogeneous Fixed Fleet Vehicle Routing Problem (HFFVRP) as a variation from VRP classic [6].

Vehicle Routing Problem with Multiple Trips and Time Windows (VRPMTW) developed as a variant of VRP [7]. Vehicle Routing Problem with Soft Time Windows (VRPSTW) also developed for the special case VRP [8]. This research was considered total distance, lead-time, and total vehicle. Vehicle Routing Problem with Stochastic Demands (VRPSD) is an alternative classic formulation VRP [9]. Multi depot Vehicle Routing with Time Windows (MD-VRPTW) was developed to determine routing with constrained capacity from customer to customer with lead-time consideration [10].

Hybrid methods, which compare Genetic Algorithms, Simulated Annealing, and Hill Climbing, developed to solve Pickup and Delivery Problem with Time Windows (PDPTW). Disruption metaheuristic was developed to solve Vehicle Routing Problem with Private Fleet and Common Carrier (VRPPC) [10]. Algorithm CODEQ was developed as variant of VRP to solve Vehicle Routing Problem (VRP) [11].

The Cross Entropy (CE) method was conceived by Rubenstein [12] as a way of adaptively estimating probabilities of rare event in complex stochastic networks [13]. Optimization scheduling using algorithm method Cross Entropy-Genetic Algorithm (CEGA) [14]. Cross Entropy was developed for Buffer Allocation problem (BAP) [13]. Minimum Cross Entropy (MCE) used to approximate the optimal solution NP-Hard on combinatorial problem and the estimated probability rare-event [14].

Survivability is important criteria considered in the design and planning of the network [15]. Cross Entropy was used to generate Entangled Network [16]. An adaptation of Cross Entropy method was called Projection Adapted Cross Entropy (PACE) developed to solve the transmission expansion problem arising from the management of national and provincial electricity grid [17]. Generalized Cross Entropy (GCE) successfully used to solve probability density estimation [18].

Vehicle problem distributing these materials was developed that compares four types of Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO) and Cross Entropy (CE). In the study revealed that GA, PSO, and ACO has a value of optimization better than CE and requires more resources for computing time. However, CE is able to provide the fastest computing time, while the slowest GA [19].

On previous research, some study did not consider with stochastic aspect, lead-time customer, and capacity vehicle. In this research will combine all the constraint. From this characteristics of the problem, it called be Vehicle Routing Problem with Times Window, Stochastic Demand Constrained Capacity (VRPTW-SDCC). Cross Entropy is the faster method to solve VRP. Previous Cross Entropy has never been used to solve complex problems that are stochastic, lead-time and capacity together. Thus, this study is the first research in the optimization of VRP with Times Windows, and Stochastic Demand Constrained Capacity.

3. RESEARCH METHODS

A. Initial Research

At this stage includes the review some of literature such us books and journals which related with VRP and Cross Entropy. Further problems were found through observation. So that the formulation of the problem is found and acquired research purposes.

B. Data Collection

At this stage conducted through interviews, data analysis, and observations. The data required are total customers demand, the number of vehicles, vehicle capacity, total customers, distance, and time from producers to customers, data delay delivery, and lead time customers.

C. Data Processing using Cross Entropy

1. Initial Phase

At this stage set the parameters used in the calculation. These parameters are:

ρ = 0.3

α = 0.8

Tolerance (β) = 0.005

Total sample = 16

2. Generate Early Transition Matrix

At this stage raised matrix transition n x n. Where n is the number of nodes contained in the problem. The mechanism of generation transition matrix is as follow:

$$P_{ij} = \sum_i^j \frac{1}{n} - 1, \ldots\ldots\ldots \forall i = j \quad \ldots\ldots\ldots\ldots(1)$$

$$P_{ij} = \sum_i^j \frac{1}{n-1}, \ldots\ldots\ldots \forall i \neq j \quad \ldots\ldots\ldots\ldots(2)$$

3. Generate N Route

At this stage, raised a number of N route as the initial sample.

4. Objective Calculation

Each route that has been raised, the value calculated for each trajectory.

$$T_r = \sum_i^j p_i * L_{(r_i, r_{i+1})} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

5. Sample Elite Selection

After getting value for each route was raised then the next step is to take a sample of the elite, which is the path that has a value T (i) minimum. With reference to values that count or the previous stages at this stage taken as ρ*N best.

6. Update Parameter

After obtained a set of best sample, next stage is update parameter. Renewed parameters to generate a new input sample trajectory better. This parameter is update using the formula:

$$P^{k+1} = \alpha * \omega + (1-\alpha) P^k \quad \ldots\ldots\ldots\ldots\ldots(4)$$

D. Conclusions

Conclusions are the final stage of this research.

4. DISCUSSION

There are 16 customers should be goods shipped in every day. The 16 customers expressed in N_1, N_2,……..,N_{16}. Total shipment depend on customers demand. In a week, total customers demand is not always same. The vehicles used are box type with a capacity of 2000 kg. Delay delivery occur almost every day Therefore these vehicles and the number of vehicle must be established to obtain optimal delivery.

Table 4.1 Customer Demand

I	d_i (kg)	d_i (kg)	d_i (kg)	d_i (kg)	d_i (kg)
	Stoch1	Stoch2	Stoch3	Stoch4	Stoch5
0	0	0	0	0	0
1	2600	3000	2500	1925	2000
2	1300	1300	2500	1750	1500
3	1300	1250	1500	1525	1450
4	1200	1250	1000	1400	1200
5	1200	1100	950	1200	1000
6	900	775	900	1000	1000
7	800	760	710	680	1000
8	800	700	685	500	900
9	600	300	200	400	500
10	350	300	200	350	400
11	100	200	100	225	150
12	100	125	100	200	150
13	80	100	75	145	135
14	70	75	50	100	65
15	45	25	25	100	50
16	20	25	25	75	25
Total	11.465	11.285	11.520	11.575	1.525

Data obtained from SASD (Sales Administration and Shipping Department). This department takes care of all the administration and delivery of goods from producer to customers. The data required are total customers demand, the number of vehicles, vehicle capacity, total customers, distance, and time from producers to customers, data delay delivery, and lead time customers.

Each customer is symbolized by N_0, N_1,….,N_{16}. Customer demand is not always the same every day. Every day the produce deliver the goods according to the total customer demand. Requests that are not always referred to as stochastic demand. Data Table 4.1 as data, stochastic consumer demand every day.

Data required for Cross Entropy formulation are distance, time, and lead-time. Combine the data as a matrix for Cross Entropy calculation. Manual calculation cannot be performed because it requires lengthy formulations. Therefore, the data processing computed in software Matlab R2012a.

For running function using:

```
>> tspsnn(17,0.3,0.8,A)
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For running objective Z min using:

```
>> stsp(ans,A)
```

The optimal result Cross Entropy is:

0 – 10 – 15 – 8 – 7 – 4 – 16 – 5 – 12 – 3 – 13 – 6 – 2 – 14 – 9 – 1 – 11 – 0. It's mean that delivery goods start from depot and ends in depot. The optimal routed with Z min 6.239,85.

Routing analysis consider with data stochactic demand and lead time customer. The routing are shown from in Tabel 2.

Table 4.2 Optimal Vehicle Routing

k	Route	N	Time Arrived (hour)	Lead time
k_1	0 - 10 - 16 – 0	N_{10}	11:22am	02:00pm
		N_{16}	01:24pm	05:00pm
k_2	0 - 15 - 8 – 0	N_{15}	9:31pm	02:00pm
		N_8	11:34pm	05:00pm
k_3	0 - 7 - 0 – 11 or 0 - 7 - 11 - 0	N_7	10:14pm	12:00pm
		N_{11}	03:01pm/ 11:14am	05:00pm
k_4	0 - 4 - 0 - 5 - 0 or 0 - 4 - 5 - 0	N_4	9:57am	03:00pm
		N_5	02:31pm /12:34pm	03:00pm
k_5	0 - 12 - 3 – 0	N_{12}	10:46am	03:00pm
		N_3	02:15pm	03:00pm
k_6	0 - 13 – 0	N_{13}	02:12pm	04:00pm
k_7	0 - 6 - 2 – 0	N_6	11:05am	03:00pm
		N_2	01:11pm	05:00pm
k_8	0 - 14 – 0	N_{14}	11:18am	03:00pm
k_9	0 - 9 - 1 – 0	N_9	10:45am	02:30pm
		N_1	01:19pm	05:00pm

At the optimal route, the number of vehicle needed is 9. There are no vehicle that exceed the time limit specified delivery lead time customer.

The result of this study share to the company as a proposal for their routing. When the route was carried out in accordance with the calculation, of course will provide optimal result so do not delay in delivery. As a statement, that the routing tried for normal condition applied in the company with no request is too excessive or otherwise. Consider congestion, accident, vehicle constraints given the condition of roads in Jakarta are sometimes unpredictable. The calculation was done by giving leeway time not too close. Add a number of vehicles be considered and discussed further management. Additional vehicles means should perform the calculation of corporate assets.

5. CONCLUSION

From the experimental results we get total vehicle required to deliver the goods from producer to customers with optimal route with nine vehicle. Each vehicle carrying goods with a capacity of not more than 2000 kg. Each vehicle starting at 08:00 am from depot and back to depot not more than working hours at 05:00 pm. The vehicle k_6 and k_7 deliver goods to 1 customer every day. Vehicle k_1, k_2, k_5, k_7, and k_9 deliver goods to 2 customers every day. Vehicle k_3 and k_4 deliver goods to 2 customers every day. If sufficient capacity, the vehicle continue from customer 1 to customer 2. However, if the capacity is not sufficient, after delivered to consumer 1, the vehicle should be back to depot and next delivered to the customer 2.

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