THE APPLICATION OF THE SUBDIVISION ALGORITHM FOR SURFACE MODELING

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ABSTRACT: *Surface Modeling (SM) poses many challenges in Computer Graphics (CG) and Image Processing (IP). Conventionally, these shapes are modeled with NURBS surfaces despite of its topological restrictions. In this research we try to resolve these restrictions by using a Subdivision Algorithm (SA). Although the selected algorithm has its own limitations to produce an image or model but it performs well on Surface Modeling (SM). We compare the behavior of limited curves and propose and analyze a subdivision scheme which unifies all approximating subdivision schemes in its compact form and produce complex geometrical structures in a short computational time with high smoothness. Furthermore, the application of SA is demonstrated through different examples.*

Key-words: Surface Modeling (SM); Computer Graphic (CG); Image Processing (IP); Subdivision Algorithm (SA)

1. INTRODUCTION

Due to the development of computer graphics, subdivision schemes are actively studied in 1970, which are initially studied by G. de Rham in 1940. The rise of multi-resolution analysis (MRA) gave birth to significant advances in a wide range of domains during the last three decades. Wavelet decomposition of signals or images is a very important tool for the building efficient algorithms dedicated to 3D models represented by discrete polygonal surfaces, along with the growth of computing power and the increase of network applications make discrete surfaces an attractive field of study. Which is actually one of the most obvious and vastly used applications of MRA. In the field of computer graphics and approximation theory geometric modeling of surfaces of arbitrary topology is an important and interesting area of research. For the construction of such surfaces subdivision algorithms are powerful paradigm. Beginning with a input mesh a sequence of meshes is defined new vertices are inserted as, preferably, simple local affine combinations of neighboring vertices. An attractive feature of these schemes is that they are local, i.e., no global sys-tem of equations needs to be solved. Although the mathematical analysis of the surfaces resulting from subdivision algorithms are not always straightforward.

However, the clarity and simplicity of the schemes and associated data structures make them interactive and attractive uses where speed is of the essence. In the field of Computer Aided Geometric Design (CAGD), the de-facto approach for shape modeling is at present Non-Uniform Rational B-Splines (NURBS). NURBS representation, however, uses a rectangular grid of control points and has in-adequacy in the construction of shapes of general topology. Subdivision is a technique for construction of smooth curves/surfaces, which first applied as an extension of splines to arbitrary topology control nets. Simplicity and flexibility of subdivision algorithms, make them suitable for many interactive computer graphics applications. Although subdivision was introduced as a generalization of knot insertion algorithms for the splines, however it is much more general and allows considerable change in the selection of subdivision rules. Purity of the subdivision lies in the construction of smooth curves/surfaces. However, the uses such as special aspects and animation need generation and construction of composite geometric shapes, which, like real world geometry, carry detail at many scales. Manipulating such as refine meshes can be considered difficult task, particularly when they are to be animated or edited. Interactively, which is burning issue in these cases, is challenging to achieve.

Computer Aided Geometric Design (CAGD) is a computational geometry related with algorithms for the designing of smooth curves/surfaces and efficient tool for mathematical representation. This representation is used for the generation of the curves/surfaces, as well as geometrical measures of importance such as curvatures. One common approach to the design of curves/surfaces which related to CAGD is the subdivision schemes. Subdivision schemes have be-come important in recent years because they provide a precise way to describe smooth curves/surfaces. It is an algorithmic technique to generate smooth surfaces as a sequence of successively refined polyhedral meshes.

The beauty lies in the elegant mathematical formulation and simple implementation. Computer Aided Geometric Design is widely used in providing methods and algorithms for mathematical description of shapes. CAGD has its influence in the field of geology and medical as it plays an important role in geographic information systems and image processing respectively. CAGD is the mostly used in applied technical fields such as aerospace, automotive design, industrial design, Computer Aided Design (CAD), Computer Aided Manufacturing (CAM) in numerical computing, architecture and mechanical engineering. In the emerging era of computer science, it helps in animation, simulation of shape behavior, graphical representation of large data sheets, reconstruction of 3D models form images and also fitting surfaces to scanned 3D-prints. Nevertheless, CAGD provides main ingredient in Isogeometric analysis, which is used in numerical treatment of PDE's. The most commonly used applications of CAGD are: Modeling the surface appearing in the shapes of airplanes, ships and cars, Controlling and planning surgery, Relief maps in cartography and drawing machine charts, Constructing images in the film industries, television and advertising, Automatically creating sectional drawing, Production and quality control, Representation products and Visualizing of large data sets.

In future, there will be many new applications of CAGD. Subdivision is the commonly used and most attractive field of CAGD for the designing of curves/surfaces. Subdivision scheme are used to construct smooth curves and surfaces from a given set of control points through iterative refinements. Due to its clarity and simplicity, subdivision schemes have been esteemed in many fields such as image processing, computer graphics and computer animation. Subdivision schemes are easy to implement and suitable for computer applications. In general, we can classify subdivision scheme according to the following standards:

i. By the number of edges constructing control grid just like triangle grid, quadrangle grid, hexagon grid etc.
ii. By the splitting style of topological grid; for example, point split and face split
iii. By the relationship of limit surface, limit curve and control polygon such as approximation subdivision and interpolator subdivision.(At each refinement level, if new points are inserted into existing control polygon and original points remain, the member of all the subsequent sequences becoming point of limiting curve itself, the scheme is called interpolating subdivision scheme. Approximating scheme is not required matching the original position of the vertices in the original mesh, they can and will ad-just this position of vertices as needed. In general approximating scheme has greater smoothness. Approximating means that limit surfaces approximate the initial meshes after the subdivision, the newly generated control points are not in the limit surfaces. Subdivision surfaces are also making fundamental contributions to new applications areas in geometric modeling)
iv. By the continuity and smoothness of limit surface for instance C^0 and up to C^m
v. By the characteristic of subdivision in the same layer, such as uniform subdivision and non-uniform subdivision.
vi. By the relationship of geometric rule with subdivision layer, just like stationary subdivision and dynamic subdivision.
vii. By the number of control points inserted at the level k + 1 between two consecutive points such as; binary, ternary,...,n-array.

Modeling the geometry of surface is an important area of research in computer graphics. A significant standard for the construction of surfaces is sub-division. We begin with an initial mesh and insert new vertices in it. As a consequence, we have a sequence of new meshes which is linear combination of previous ones. The field of CAGD compiles with the visual demonstration. Subdivision deals with representation, construction, interpolation and approximation of curves, surfaces and volumes. CAGD studies especially the construction and handling of curves and surfaces given by a set of data points. Constructing surfaces through subdivision elegantly addresses many issues that computer graphics practitioners are confronted with:

Arbitrary Topology: Subdivision algorithm unifies and generalizes classical spline patch moves toward to arbitrary topology. Means that there is no need for trimming the curves/surfaces or awkward constraint management between patches.

Scalability: Due its recursive arrangement, subdivision algorithm logically provides level-of-detail rendering and adaptive approximation with error bounds. The result are algorithms which can make the best of limited hardware resources, such as those found on low end PCs.

Uniformity of Representation: Polygonal meshes or spline patches are usually used for traditional modeling. Subdivision algorithm spans the spectrum between these two extremes. Surfaces can behave as if they are made of patches, or they can be treated as if consisting of many small polygons.

Numerical Stability: The meshes produced by subdivision have many of the nice properties finite element solvers require. As a result subdivision representations are also highly suitable for many numerical simulation tasks which are of importance in engineering and computer animation settings.

Code Simplicity: Last but not least the basic ideas behind subdivision are simple to implement and execute very efficiently. While some of the deeper mathematical analyses can get quite involved this is of little concern for the final implementation and runtime performance.

We apply the tensor product to generate the mask of proposed schemes and Laurent Polynomial Formulism for the analysis of the subdivision schemes and result are obtained by using Mat Lab software. The rest of the paper is organized as follows: Section 2 is about the Literature Review, in Section 3 we introduce a subdivision algorithm for generating the mask of binary subdivision schemes, Section 4 is about the application of the algorithm, Results are discussed in Section 5and finally the conclusion is drawn in Section 6.

2. Literature Review

Geometric modelling developed its roots from the Era of Euclid and Descartes [1]. The manufacturing of curves had started back to Roman times. It started with the purpose of shipbuilding such as ship's rib. This technique of shipbuilding was modernized by the Venetians from 13^{th} to 16^{th} century. The ship hull was constructed by changing the rib's shapes along with the keel (keel is a large beam around which the hull of a ship is built). Earlier, there was no definite sketch of a ship hull. The spline is a wooden beam used to make smooth curves, which lately known as *Classical Spline*.

In 1944, Liming came out with a book entitled *Analytical Geometry with Applications to Air Craft*. In this book, drafting methods were purposed and analyzed with computational techniques. These methods help in designing and manufacturing of air craft in World War II. In 1950, the influence of computational geometry was on the numerical control (NC). Early computers were capable of generating numerical instructions which drove milling machines used for the production of dies and stamps for sheet metal. In this regard, computer assisted blue prints of dies and stamps were the need of time. In 1959, citroën, an auto-mobile company in Paris, hired Paul de Casteljau (a mathematician) to create a link between 2D blue prints and milling machines. de Cateljau initiated with *ab initio* design of curves. He employed Bernstein polynomial for designing curves and surfaces. On the other hand, Pierre Bézier in 1960, also recognized the need of computers for representation of mechanical parts at Renault. Bézier intro-duced a special parametric curve, based on control polygon and used Bernstein polynomial as a blending function. Since then Bézier curves led the basis of CAGD. Schoenberg coined the term B-spline. But in 1960 Carl de Boor started to work with the General Motors Lab and use B-spline as a tool to make the geometrical presentation.

The basic ideas behind subdivision are very old indeed and initial work on sub-division schemes was started in 1940, when G. de Rham used “corner cutting” to describe smooth curves. Rham algorithm paved the way to initiate work on subdivision schemes, but the relevance to the modeling of shape started with the proposal of Chaikin [3], who devised a method of generating smooth curves for plotting, in 1974, at the CAGD conference in the university of Utah, this was soon analyzed by Forrest [4] and by Riesenfeld [5] and linked with the burgeoning theory of B-spline curves. It became clear that equal-interval B-spline curves of any degree would have such a subdivision construction. Both Riesenfeld and Sabin argued that Chaikin had invented an iterative method to generate a uniform quadratic B-spline curve. The extension to surface took just a few years, until 1978, when Catmull and Clark [6] published their descrip-tions of both quadratic and cubic subdivision surfaces, the exciting new point being that the surface could be described which was not forced to have a regular rectangular grid in the way that the tensor product B-spline surfaces were, the definition of a specific surface in terms of control mesh could follow the needs of the boundaries and the curvature of the surface. This was made possible by the extension of the subdivision rules to allow for "extraordinary points", being either "extraordinary vertices", where either other than four faces came together at a vertex, or else" extraordinary faces", where a face had other than four sides.

At about the same time Doo [7] and Sabin, who had also been working on quadratic subdivision, showed a way of analyzing the behavior of these schemes at the extraordinary points, treating the refinement process in terms of matrix multiplication, and using Eigen analysis of the spectrum of this matrix [8]. This aspect was followed up by Ball and Storry [9, 10] who made this analysis process more formal and succeeded in making some improvements to the coefficients used around the extraordinary points in the Catmull-Clark scheme. Storry identified that in the limit, the configuration around and extraordinary point was always an affine transform (dependent on the original polyhedron) of a point distribution which was completely defined by the eigenvectors of the subdivision matrix. He called this the natural configuration.

Boor discovered the generalization of Chaikin algorithm [11]. After that Dubuc [12] brought 4-point C^1 interpolating subdivision scheme. Before that, the schemes were approximating quadratic B-spline having the same weigh-tage. Dyn *et al.* [13] worked simultaneously and independently having no breaks and introduced 4-point C^1 interpolatory subdivision scheme with one shape parameter. For fix value of parameter it became Dubuc’s 4-point scheme. Dyn *et al.* [14] proved 4-point subdivision scheme and found the continuity of the scheme through Eigen analysis. Further it is preceded by Tang *et al.* [15]. They also introduced 4-point interpolatroy subdivision scheme and found the continuity by using Laurent polynomial. Hassan et al. [16] followed the same idea but with difference of arity and they proved that scheme gives continuity.

An interpolatory subdivision curve with local shape control parameter was p-resented by Beccari *et al.* [17]. The scheme reproduces conic section, arcs of arbitrary length and creates variety of shape effects and mixes them in the un-restricted form. Cai [18] introduced 4-point interpolatory subdivision scheme and discussed the convergence of scheme in non-uniform control point and the error estimation. Hormann and Sabin [19] introduced the family of subdivision scheme with cubic precision and determine how the support, the Hölder regularity, the precision set, the degree of polynomials spanned by the limit curves, and the artifact behavior vary with the integer parameter that identifies the members of the family. Dyn et al [20] presented necessary and sufficient conditions for a linear, binary, uniform, and stationary subdivision scheme to have polynomial reproduction of degreed. Their conditions were partly algebraic and easy to check by considering the symbol of a subdivision scheme, but also relate to the parameterizations of the scheme. After discussing some special properties that hold for symmetric schemes, they used conditions to de-rive the maximum degree of polynomial reproduction for two families of symmetric schemes, the family of pseudo-splines and a new family of dual pseudo splines. Mustafa et al [21] proposed the m-point approximating subdivision scheme with one parameter and analyzed for $m > 1$. Their scheme is a generalized form of existing subdivision schemes. The smoothness of their scheme was checked by Laurent polynomial.

Mustafa and Najma [22] introduced General formula for the mask of (2*b* + 4)-point *n*-array subdivision schemes. Their formulas corresponding to the mask not only generalized well known existing schemes but as well as provide the mask of high rarity schemes. Some families of odd-point interpolating subdivision schemes [23] have been presented in the literature but the study of families of odd-point approximating subdivision scheme for curve smoothing of noisy random data is still an open research area. In this year some fabulous work has been done by Mustafa et al. [22, 24] in the field of subdivision schemes. In coming sections, we present the tensor product of Hassan [16] subdivision scheme, subdivision algorithm for surface modeling and the applications of these algorithms.

3. Subdivision Algorithm

The developments of recent years have convinced us of the importance of understanding the mathematical foundations of subdivision algorithm. A Computer Graphics professional who wishes to use subdivision algorithm, probably is not interested in the subtle points of a theoretical argument. However, under-standing the general concepts that are used to construct and analyze subdivision schemes allows one to choose the most appropriate subdivision algorithm or customize one for a specific application.

Subdivision surfaces are polygon mesh surfaces generated from a given mesh through a refinement process makes the mesh smooth while increasing its density. Complex smooth surfaces can be derived in a reasonably predictable way from relatively simple meshes. These are: stationary or non-stationary, binary or ternary, type of mesh (triangle or quadrilateral), approximating or interpolating and linear or non-linear. The other are: Vertex insertion (primal): Insert a vertex on the interior of each edge and one on the interior of each face. Loop, Kobbelt, Catmull-Clark, Modified which are called as Subdivision Zoo.

Insert a face in the middle of each old face and connect faces in adjacent old faces. Interpolating-Control the limit surface in a more intuitive manner. Simplify algorithms. Approximating- Higher quality surfaces. Faster convergence. We briefly describe some considerations that are useful when choosing appropriate data structures for implementing subdivision surfaces. We restrict ourselves to binary case. The simplest way to extend univariate scheme to bivariate scheme is tensor-product scheme. Laurent polynomial of tensor product scheme can be obtained by the rule given in eq. (1).

$$a(\mathbf{z}) = a(z_1, z_2) = a(z_1)a(z_2) \quad (1)$$

Where $a(z_1)$ and $a(z_2)$ are the Laurent polynomials of univariate schemes.

A general compact form of binary subdivision scheme S which maps a polygon $q^k = \{q_{i,j}\}_{i,j\in Z}$ to a refined polygon $q^{k+1} = \{q_{i,j}\}_{i,j\in Z}$ is defined in eq. (2)

$$q_\alpha^{k+1} = \sum_{\beta\in\mathbb{Z}^s} a_{\alpha-2\beta} q_\beta^k, \quad \alpha\in\mathbb{Z}^s \quad (2)$$

Where s = 1 for curve and s = 2 for surface.

In case of bivariate subdivision scheme there are four rules depending on the parity of each component in the multi-index $i = (i_1, i_2)$. Writing all the multi-indices by components, we have four rules given in eq. (3).

$$q_{2i_1,2i_2}^{k+1} = \sum_{v_1,v_2\in\mathbb{Z}} a_{2v_1,2v_2} q_{i_1-v_1,i_2-v_2}^k,$$

$$q_{2i_1+1,2i_2}^{k+1} = \sum_{v_1,v_2\in\mathbb{Z}} a_{2v_1+1,2v_2} q_{i_1-v_1,i_2-v_2}^k,$$

$$q_{2i_1,2i_2+1}^{k+1} = \sum_{v_1,v_2\in\mathbb{Z}} a_{2v_1,2v_2+1} q_{i_1-v_1,i_2-v_2}^k,$$

$$q_{2i_1+1,2i_2+1}^{k+1} = \sum_{v_1,v_2\in\mathbb{Z}} a_{2v_1+1,2v_2+1} q_{i_1-v_1,i_2-v_2}^k \quad (3)$$

3.1 Construction of Subdivision Algorithm

Proposed algorithm comes from the regular tensor product of [16].

Problem: Sub- division for tensor product 3-point subdivision surface has rigid restrictions on the topology. Each vertex must have order 4. This restriction makes the design of many surfaces difficult. Our presented an algorithm that eliminated this restriction by generalizing the 3-point subdivision surface subdivision rules to include arbitrary topology. And the behavior of the limit surface defined by a recursive division construction can be analyzed in terms Laurent polynomial. For this we consider 3-point binary univariate subdivision scheme of [16].

$$q_{2i}^{k+1} = \frac{9}{32}q_{i-1}^k + \frac{22}{32}q_i^k + \frac{1}{32}q_{i+1}^k,$$
$$q_{2i+1}^{k+1} = \frac{1}{32}q_{i-1}^k + \frac{22}{32}q_i^k + \frac{9}{32}q_{i+1}^k. \quad (4)$$

and its Laurent polynomial is given as

$$a(z) = z^{-3}\left\{\frac{1}{32}z^0 + \frac{9}{32}z + \frac{22}{32}z^2 + \frac{22}{32}z^3 + \frac{9}{32}z^4 + \frac{1}{32}z^5\right\} \quad (5)$$

$$a(z_1,z_2) = z^{-6}\left\{\frac{1}{32}z^0 + \frac{9}{32}z + \frac{22}{32}z^2 + \frac{22}{32}z^3 + \frac{9}{32}z^4 + \frac{1}{32}z^5\right\}$$
$$\left\{\frac{1}{32}z^0 + \frac{9}{32}z + \frac{22}{32}z^2 + \frac{22}{32}z^3 + \frac{9}{32}z^4 + \frac{1}{32}z^5\right\}. \quad (6)$$

k

$$
\begin{aligned}
q^{k+1}_{2i,2j} &= \frac{81}{1024}q^k_{i-1,j-1} + \frac{99}{512}q^k_{i,j-1} + \frac{9}{1024}q^k_{i+1,j-1} + \frac{99}{512}q^k_{i-1,j} \\
&+\frac{121}{256}q^k_{i,j} + \frac{11}{512}q^k_{i+1,j} + \frac{9}{1024}q^k_{i-1,j+1} + \frac{11}{512}q^k_{i,j+1} \\
&+\frac{1}{1024}q^k_{i+1,j+1}, \\
q^{k+1}_{2i+1,2j} &= \frac{9}{1024}q^k_{i-1,j-1} + \frac{99}{512}q^k_{i,j-1} + \frac{81}{1024}q^k_{i+1,j-1} + \frac{11}{512}q^k_{i-1,j} \\
&+\frac{121}{256}q^k_{i,j} + \frac{99}{512}q^k_{i+1,j} + \frac{1}{1024}q^k_{i-1,j+1} + \frac{11}{512}q^k_{i,j+1} \\
&+\frac{9}{1024}q^k_{i+1,j+1},
\end{aligned}
$$

$$
\begin{aligned}
q^{k+1}_{2i,2j+1} &= \frac{9}{1024}q^k_{i-1,j-1} + \frac{11}{512}q^k_{i,j-1} + \frac{1}{1024}q^k_{i+1,j-1} + \frac{99}{512}q^k_{i-1,j} \\
&+\frac{121}{256}q^k_{i,j} + \frac{11}{512}q^k_{i+1,j} + \frac{81}{1024}q^k_{i-1,j+1} + \frac{99}{512}q^k_{i,j+1} \\
&+\frac{9}{1024}q^k_{i+1,j+1}, \\
q^{k+1}_{2i+1,2j+1} &= \frac{1}{1024}q^k_{i-1,j-1} + \frac{11}{512}q^k_{i,j-1} + \frac{9}{1024}q^k_{i+1,j-1} + \frac{11}{512}q^k_{i-1,j} \\
&+\frac{121}{256}q^k_{i,j} + \frac{99}{512}q^k_{i+1,j} + \frac{9}{1024}q^k_{i-1,j+1} + \frac{99}{512}q^k_{i,j+1} \\
&+\frac{81}{1024}q^k_{i+1,j+1}.
\end{aligned} \tag{7}
$$

Geometric view of proposed subdivision algorithm: the new points are centroid of the sub-face formed by the face centroid, a corner vertex and the two mid-edge points next to the corner. The new points then are connected. There will be two vertices along each side of each edge in the old mesh, by construction. These pairs are connected, forming quadrilaterals across the old edges. The new mesh, therefore, will create quadrilaterals for each edge in the old mesh, will create a smaller n-sided polygon for each n-sided polygon in the old mesh, and will create an n-sided polygon for each n-valence vertex (valence being the number of edges that share the vertex). After one application of the scheme all vertices will have a valence of four, so subsequent applications will create quadrilaterals for the vertices. (The original n-sided polygons are retained, however, and shrink to extraordinary points where the mesh is not as smooth, as the scheme is repeatedly applied.)

4. Application of Algorithm on Subdivision Surfaces

Subdivision surfaces are polygon mesh surfaces constructed from a base mesh through an iterative process that smooth the mesh while increasing its density. Complex smooth surfaces can be achieved in a logically provable way from comparatively simple meshes. An example, comparing the proposed algorithm, shows how the basic process can work.

The base mesh consists of a mere 36 polygons (16 vertices, 16 faces and 32 edges) in Fig. 1. Each polygon was designed by hand separately, then all were merged into one figure.

After one iteration of the prosed algorithm, the mesh has become 144 polygons (64 vertices, 64 faces and 128 edges) in Fig. 2. The sharpest points have been nicely rounded off. The lengths of the corners have become shorter and smoother, but the tips have become slightly pointed.

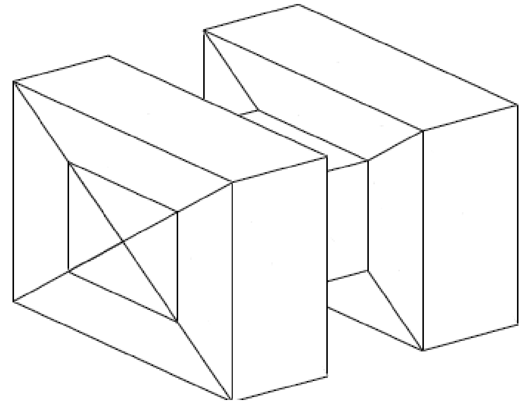

Fig. 1: Step 0 Initial Polygon

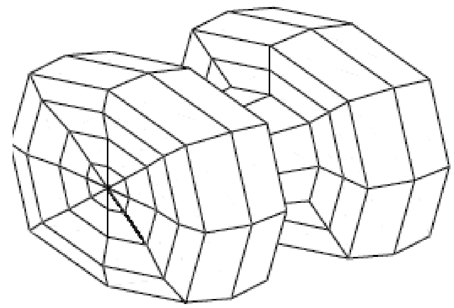

Fig. 2: Step 1 Subdivision Level

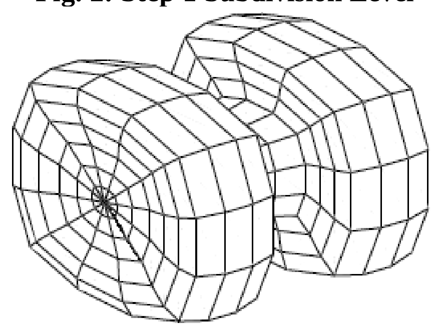

Fig. 3: Step 2 Subdivision Level

After one more iteration of the prosed algorithm, the mesh has become 576 polygons (256 vertices, 256 faces and 512 edges) in Fig. 3. The surface is already noticeably smooth. The lengths of the corners have become shorter and smoother.

After one more iteration of the prosed algorithm, the mesh has become 2304 polygons (1024 vertices, 1024 faces and 2048 edges) in Fig. 4. The sharpest points have been nicely rounded off.

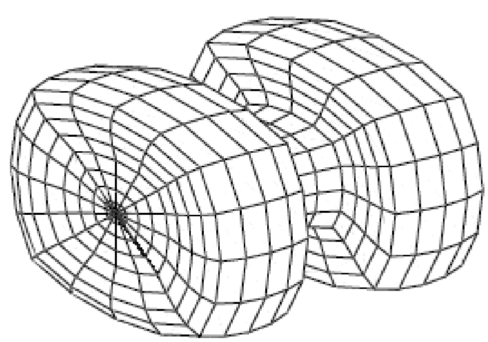

Fig. 4: Step 3 Subdivision Level

5. RESULTS AND DISCUSSION

New polygons are built from the old mesh in the following way. An edge point is formed from the in the ratio of [1: 10: 5]/16 of each edge. A face point is formed as the centroid of each polygon of the mesh. Finally, each vertex in the new mesh is formed as the average of a vertex in the old mesh, a face point for a polygon that touches that old vertex, and the edge points for the two edges that belong to that polygon and touch that old vertex. As an example, a square in the old mesh will create a new, smaller square centered in the old square.

The new points then are connected. There will be two vertices along each side of each edge in the old mesh, by construction. These pairs are connected, forming quadrilaterals across the old edges. Within each old polygon, there will be as many new vertices as there were vertices in the polygon. These are connected to form a new, smaller, inset polygon. And finally, around each old vertex there is a new vertex in the adjoining corner of each old polygon. These are connected to form a new polygon with as many edges as there were polygons around the old vertex. The new mesh, therefore, will create quadrilaterals for each edge in the old mesh, will create a smaller n-sided polygon for each n-sided polygon in the old mesh, and will create an n-sided polygon for each n-valence vertex (Va-lence being the number of edges that touch the vertex). After one application of the scheme all vertices will have a valence of four, so subsequent applications will create quadrilaterals for the vertices. (The original n-sided polygons are retained, however, and shrink to extraordinary points where the mesh is not as smooth, as the scheme is repeatedly applied.)

Observe in the images below that each face of the cube has been divided in-to four quadrilaterals. The vertex points, the corners of the cube, have been "pressed inward". The edge points are clearly derived from the midpoints of the original edges, but are "pressed in" as well. We present the Visual performance of 3-point tensor product binary scheme in Figs. 5 and 6.

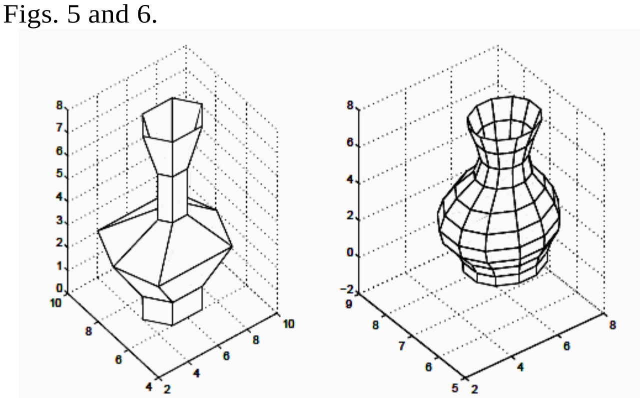

Fig. 5. Initial Polygon after Step 0 and Step 1respectively

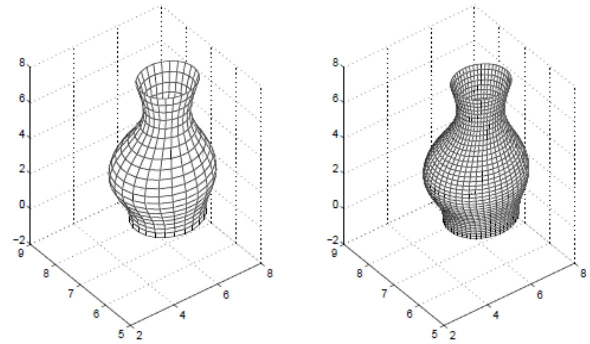

Fig. 6. Subdivision Surfaces after Step 2 and Step 3respectively

6. CONCLUSION

The performance of our 3-point tensor product binary approximating scheme is shown. The refinement algorithm of 3-point tensor product scheme involves computing a new vertex corresponding to each (vertex, face) pair of the original mesh. The new vertices are found as weighted averages of the points belonging to each face of the original mesh. For the 3-point tensor product case, these weights (going around a face) are:

$$\left\{\frac{81}{1024},\frac{99}{512},\frac{9}{1024},\frac{99}{512},\frac{121}{256},\frac{11}{512},\frac{9}{1024},\frac{11}{512},\frac{1}{1024}\right\}$$

The newly created vertices are then connected to form the faces of the refined control mesh. We conclude that the algorithm has the interesting features, first, Simplicity, algorithm iteratively updates the input mesh in a global manner based on a simple point-surface distance computation followed by translations of control vertices along the displacement vectors, second, Speed, algorithm runs at least six times faster than current state-of-the-art subdivision fitting methods, third, Scalability, for Subdivision Surface Approximation, we can progressively obtain a finer fit by performing more iterations and finally, Generality, since the algorithm is based on simple geometric procedures, it can be easily extended to curves and surfaces defined by control vertices. The Subdivision Algorithm for Surface Modeling will remain an avenue for further research.

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