INDUCTANCE CALCULATION OF INFINITE NETWORKS USING GREENS FUNCTION: PERFECT AND PERTURBED TRIANGULAR LATTICES

Lama O. AL-Sohimat, Amer D. Al-Oqali, Emad K. Jaradat, Marwan Y. Ajoor, Ra'ed S. Hijjawii
Department of Physics, Mu'tah University, Al-Karak, Jordan
Contact: ameralaqali@yahoo.com

ABSTRACT: *The Inductance of an infinite triangular network of identical inductor is investigated within the frame work of the lattice Green's function. This work deals with two cases: perfect and perturbed lattices. In this paper a connection is made between the inductance and the lattice Green's function of the perturbed network. The lattice Green's function and the inductance of the perturbed lattice are presented in terms of unperturbed (perfect) lattice by solving Dyson's equation.*

Keywords: Inductance; Lattice Green's function; Infinite triangular network.

1. INTRODUCTION

The lattice Green's Function plays an important role in the study of solid state physics and condensed matter physics, especially when impure solids are studied [1]. Green's Function is used in classical field theories, quantum mechanics, as well as, quantum field theories. It is an example of a mathematical problem in quantum theory whose essence is solving linear operator equation with given boundary conditions. Green was the first physicist who built the basic concepts of Green's function in potential theory. Green's work was set to solve Laplacian's and Poisson's equations with variant boundary conditions [2]. Lattice Green's function also arises in the study of statistical mechanics of spherical model [3].

Maradodin [4] showed that the lattice Green's function for the body centered cubic at the origin can be expressed as a product of complete elliptic integrals of the first kind. Horiguchi and Morita [5] presented a recurrence relation for a simple cubic lattice connecting the lattice greens function and its first derivative in term of energy band.

Complex temperature singularities of this system were studied by Barton [6]. Recently Greens function is one of the most important concepts in many aspects of physics, as many quantities in solid state physics can be expressed in terms of the lattice green's function and is widely used in the literature [7].

A great deal of research has been done on lattice Green's function over the last fifty years or so.

The lattice Green's function for a body centered cubic lattice with isotropic nearest neighbor interactions can be evaluated exactly in terms of complete elliptic integrals [8]. Also, the lattice Green's function is used to find the inductance of an inductor network [9, 10] and it was used to find the capacitance of a network of capacitors [11]. Cserti [9] introduced an alternative method based on lattice Green's function rather than using the superposition distribution of current distribution. The Green's function given in Morita's [12] obtained the inductance formulas for the inductance of the square and simple cubic lattices. The inductance between arbitrary nodes in an infinite network of inductors is studied when the network is perturbed by removing one bond (inductor) from the perfect lattice [10], where the inductance in a perturbed lattice is expressed in terms of the inductance in a perfect lattice. Wu [13] presented a new formulation of resistors network which leaded to an expression of the effective resistance between any two nodes in any network, which can be either finite or infinite in terms of the eigenvalues and eigenfunctions of the Laplacian matrix associated with the network. He obtained explicit formulae for the resistance in one, two and three-dimensions under various boundary conditions. Asad, *et al*., [11] investigated many perfect infinite lattices of identical capacitors using the lattice Green's function

Hijjawi *et al.*, [14] calculating the capacitance of infinite networks when the network is perturbed by removing one bond from the perfect lattice. More recently, Asad, *et al.*, [15] investigated infinite networks of identical capacitors using the superposition principle and charge distribution. Finally, Cserti, *et al.*, [7] obtained the electric resistance between two arbitrary nodes on any infinite lattice structure of resistors that is periodic tilling of space.

This paper presents a formalism of the lattice Green's function and the inductance of the perturbed infinite triangular network. Two perturbed cases are considered: substitutional single inductance, and single broken bond (missing bond) inductance.

The remainder of this paper is organized as follows: Section two is devoted the formalism, which includes the derivation of the formulae that relate the inductance in perfect infinite triangular networks of identical inductors to the lattice Green's function. In section three we presented the formalism of the lattice Green's function and the inductance of the perturbed infinite triangular network. In section four, the results of this work are discussed .Finally; the conclusions are presented in section five.

2. Perfect Triangular Lattice Using Dirac Vector Notation

Consider an infinite triangular network of identical inductors L. We wish to calculate the inductance between two arbitrary lattice points of infinite triangular lattice.

We denote the current that can enter at site $\vec{r}_i$ by $\frac{dI}{dt}(\vec{r}_i)$ and the potential at site $\vec{r}_i$ will be denoted by $V(\vec{r}_i)$, we have, by combination of Ohm's and Kirchhoff's law's,

$$\sum_{i=1}^{3} V(\vec{r}_l+\vec{a}_i)-2V(\vec{r}_l)+V(\vec{r}_l-\vec{a}_i)=-\frac{dI}{dt}(\vec{r}_l)L \quad (2.1)$$

If we let $|l\rangle$ basis vector associated with the lattice point $\vec{r}_l$,then

$$\frac{dI}{dt}(\vec{r}_l)=\left\langle l\left|\frac{dI}{dt}\right.\right\rangle V(\vec{r}_l)=\langle l|V\rangle \quad (2.2)$$

forms a complete orthonormal set We assume that $|l\rangle$ and $\sum_l|l\rangle\langle l|=1$ i.e. $I\langle n|l\rangle=\delta(\vec{r}_n,\vec{r}_l)=\delta(n,l)$. In the lattice basis ,the vector $|V\rangle$ and $\left|\frac{dI}{dt}\right\rangle$ are

$|V\rangle = \sum_l |l\rangle\langle l|V\rangle = \sum_l |l\rangle\, V(\vec{r}_l)$ (2.3)

$\left|\frac{dI}{dt}\right\rangle = \sum_l \left|l\right\rangle \left\langle l\left|\frac{dI}{dt}\right.\right\rangle = \sum_l \frac{dI}{dt}(\vec{r}_l)$ (2.4)

Eq.(2.1) becomes

$$\sum_i^3 \langle l+i|V\rangle - 2\langle l|V\rangle + \langle l-i|V\rangle = -\left\langle l\left|\frac{dI}{dt}\right.\right\rangle L \quad (2.5)$$

or

$$\sum_n \sum_i^3 \left[\langle l+i|n\rangle\langle n|V\rangle - 2\langle l|n\rangle\langle n|V\rangle + \langle l-i|n\rangle\langle n|V\rangle\right] = -\left\langle l\left|\frac{dI}{dt}\right.\right\rangle L\Big] \quad (2.6)$$

Eq.(2.6) can be written as

$$\sum_n \sum_{i-1}^3 \left[\delta_{l+i,n} - 2\delta_{l,n} + \delta_{l-i,n} = -\left\langle l\left|\frac{dI}{dt}\right.\right\rangle L\right] \quad (2.7)$$

where we have used

$$V(\vec{r}_l) = \langle l|V\rangle = \sum_n \langle l|n\rangle\langle n|V\rangle = \sum_n \delta_{l,n} V(\vec{r}_n) \quad (2.8)$$

and

$$V(\vec{r}_l + \vec{a}_i) = \sum_n \delta_{l\pm i,n} V(\vec{r}_n) \quad (2.9)$$

Multiply both sides of Eq (2.7) by $\sum_l |l\rangle$, we have

$$\sum_l \sum_n \sum_{i=1}^3 \left[|l\rangle\delta_{l+i,n} - 2\delta_{l,n} + \delta_{l-i,n}\right]\langle n|V\rangle = -L\left|\frac{dI}{dt}\right\rangle \quad (2.10)$$

In the Dirac vector space notation Eq . (2.10) ,is written as

$$l_0|V\rangle = -L\left|\frac{dI}{dt}\right\rangle \quad (2.11)$$

Where $l_0$is called Laplacian operator of the perfect lattice [9, 10, 17].

$$l_0 = \sum_l \sum_n \sum_{i=1}^3 |l\rangle\left[\delta_{l+i,n} - 2\delta_{l,n} + \delta_{l-i,n}\right]\langle n| \quad (2.12)$$

$$L_0 = \begin{cases} -4 & l = n \\ 1 & if\ |n-l| \\ 0 & othewise \end{cases} \quad (2.13)$$

Now, one can solve Eq.(2.11), formally as

$$|V\rangle = -L l_0^{-1}\left|\frac{dI}{dt}\right\rangle \quad (2.14)$$

The lattice Green's function is defined by (Economou,2006)

$$l_0 G_0 = -1 \quad (2.15)$$

Eq(2.14)becomes

$$|V\rangle = L G_0 \left|\frac{dI}{dt}\right\rangle \quad (2.16)$$

We need to calculate the inductance between the sites $\vec{r}_i$ and $\vec{r}_j$ we assume that the current $\frac{dI}{dt}$ enters at site $\vec{r}_i$ and -$\frac{dI}{dt}$ exits at site $\vec{r}_j$ and the current is zero at all other sites. Hence, the current at lattice point $\vec{r}_n$ is written as

$$\frac{dI(\vec{r}_n)}{dt} = \frac{dI}{dt}(\delta_{n,i} - \delta_{n,j}) \quad (2.17)$$

The potential at lattice site $\vec{r}_m$ can be found by substituting Eq.(2.17), into Eq.(2.16), we get

$$V(\vec{r}_m) = \langle m|V\rangle = \left\langle m\left|G_0\right|\frac{dI}{dt}\right\rangle$$

$$= L\sum_n \langle m|G_0|n\rangle \left\langle n\left|\frac{dI(\vec{r}_n)}{dt}\right.\right\rangle$$

$$= L\frac{dI}{dt}\left[G_0(m,i) - G_0(m,j)\right] \quad (2.18)$$

Where $G_0(m,n) = \langle m|G_0|n\rangle = G_0(\vec{r}_n - \vec{r}_m)$

The inductance between the sites $\vec{r}_i$ and $\vec{r}_j$ is

$$L_0(i,j) = \frac{V(\vec{r}_i) - V(\vec{r}_j)}{\frac{dI}{dt}} \quad (2.19)$$

Substituting Eq.(2.18), into (2.19), we get

$$L_0(i,j) = L\left[G_0(i,i) - G_0(i,j) - G_0(j,i) + G_0(j,j)\right] \quad (2.20)$$

But $G_0(i,j) = G_0(j,j)$ from diagonal, and $G_0(i,j) = G_0(j,i)$ from symmetry

In general, for a prefect lattice the inductance in terms of G_0between the sites $\vec{r}_i$ and $\vec{r}_j$ can be written as

$$L_0(\vec{r}_i, \vec{r}_j) = 2L\left[G_0(\vec{r}_i, \vec{r}_i) - G_0(\vec{r}_i, \vec{r}_j)\right] \quad (2.21)$$

$$L_0(i,j) = 2L\left[G_0(i,i) - G_0(i,j)\right] \quad (2.22)$$

The final expression of the inductance between the origin and lattice point $\vec{r}_l = (l,m)$ in the triangular infinite lattice is

$$L_0(i,j) = L\int_{-\pi}^{\pi}\frac{dx_1}{d\pi}\int_{-\pi}^{\pi}\frac{dx_2}{d2\pi}\frac{1 - e^{i(lx_1 + mx_2)}}{\sum_{i=1}^3(1 - \cos x_i)} \quad (2.23)$$

3. Substitutional single inductance in a perfect Lattice

In this section, we use Green's function technique to determine the inductance for the so-called perturbed lattice that is obtained by replacing one inductance by another (substitutional inductance) in the perfect lattice. As an example (see figure1), consider the infinite triangular arrays of identical inductance L.

Replacing the inductance between sites $\vec{r}_{i_0}$ and $\vec{r}_{j_0}$in a perfect lattice by the substitutional inductance $L' = \propto L$ results in a perturbed lattice, where α is a positive number. Then the question is how one can find the effective inductance between two arbitrary sites. It is simple to find the effective inductance across the substitutional inductance. However, this inductance equals the parallel resultant of the substitutional inductance and the effective inductance across the missing bond$(\vec{r}_{i_0}, \vec{r}_{j_0})$. The current contribution $\delta I(\vec{r}_{i_0})$ and $\delta I'(\vec{r}_{i_0})$ at site $\vec{r}_{i_0}$ due to the bond $(\vec{r}_{i_0}, \vec{r}_{j_0})$ before and after the inductor replacement respectively, are given

$$\delta\frac{dI(\vec{r}_i)}{dt}L = \delta(\vec{r}_i, \vec{r}_{i_0})\left(V(\vec{r}_{i_0}) - V(\vec{r}_{j_0})\right) + \delta(\vec{r}_i, \vec{r}_{j_0})(V(\vec{r}_{j_0}) - V(\vec{r}_{i_0})) \quad (3.1)$$

And

$$\delta\frac{dI'}{dt}(\vec{r}_i)L = \frac{\delta\frac{dI(\vec{r}_i)}{dt}}{L'}L = \frac{\delta\frac{dI(\vec{r}_i)}{dt}}{\alpha}L$$

$$= \frac{1}{\alpha}\delta(\vec{r}_i, \vec{r}_{i_0})\left[V(\vec{r}_{i_0}) - V(\vec{r}_{j_0})\right] + \frac{1}{\alpha}\delta(\vec{r}_i, \vec{r}_{j_0})\left[V(\vec{r}_{j_0}) - V(\vec{r}_{i_0})\right] \quad (3.2)$$

due to replacing L by $L' = \alpha L$. The net contribution of the current at site $\vec{r}_i$ is

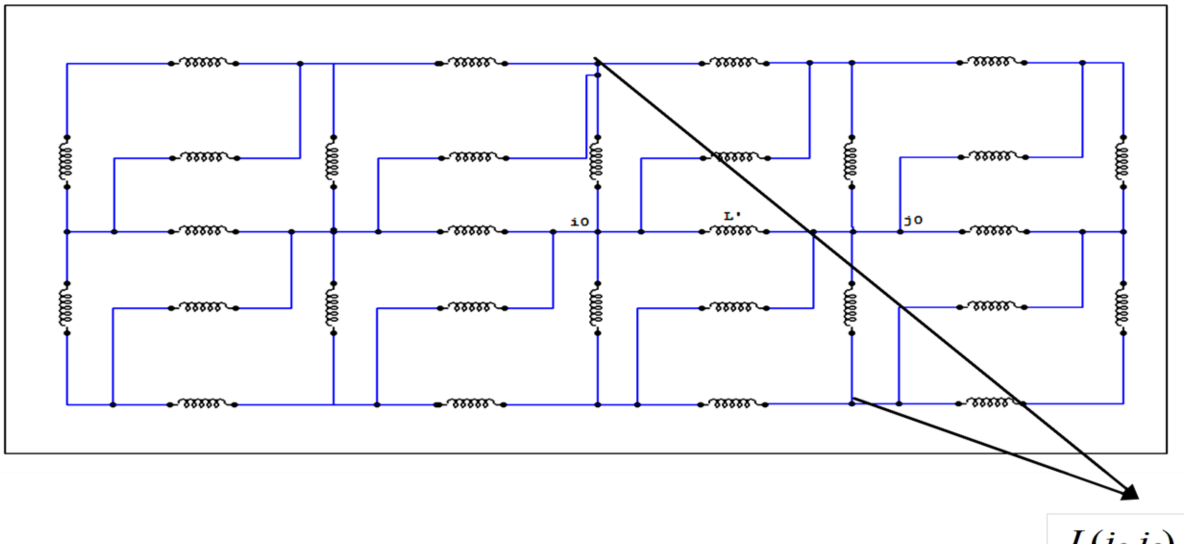

Figure 1:. Perturbation of a triangular lattice of identical inductors L by replacing the inductance between $\vec{r}_{i_0}$ and $\vec{r}_{j_0}$ by inductance L'.

$$\Delta\delta\frac{dI(\vec{r}_i)}{dt}L = \delta\frac{dI'(\vec{r}_i)}{dt}L - \delta\frac{dI(\vec{r}_i)}{dt}L$$
$$=\frac{(1-\alpha)}{\alpha}\left[\delta(\vec{r}_i,\vec{r}_{i_0})[V(\vec{r}_{i_0}) - V(\vec{r}_{j_0})]\delta(\vec{r}_i,\vec{r}_{j_0})[V(\vec{r}_{j_0}) - V(\vec{r}_{i_0})]\right] \quad (3.3)$$

Using $\delta(\vec{r}_i,\vec{r}_{i_0}) = \langle i|j\rangle$ and $V(\vec{r}_{i_0}) = \langle i|V\rangle$ we obtain

$$\Delta\frac{dI(\vec{r}_i)}{dt}L = \frac{(1-\alpha)}{\alpha}[\langle i|i_0\rangle(\langle i_0|V\rangle - \langle j_0|V\rangle) + \langle i|j_0\rangle(\langle j_0|V\rangle - \langle i_0|V\rangle)] \quad (3.4)$$

Where the operator l_1 is the perturbation part arising from the substitutional inductance. In fact l_1 is given by

$$l_1 = \frac{(1-\alpha)}{\alpha}(|i_0\rangle - |j_0\rangle)(\langle i_0| - \langle j_0|) = -a|x\rangle\langle x| \quad (3.5)$$

Where

$$|x\rangle = |i_0\rangle - |j_0\rangle \quad ,\langle x| = \langle i_0| - \langle j_0| \quad \text{and } a = \frac{(\alpha-1)}{\alpha} \quad (3.6)$$

is the projection operator and $(|i_0\rangle - |j_0\rangle)(\langle i_0| - \langle j_0|)$ for the bond (i_0, j_0) and $\vec{r}_{j_0}$, now replace the inductance between$\vec{r}_{i_0}$, then the current in the perfect lattice by L'.

Now, replace the inductance between $\vec{r}_{i0}$and $\vec{r}_{j0}$ in the perfect lattice byL', then the current $\frac{dI}{dt}(\vec{r}_i)$ at site $\vec{r}_i$ is given by

$$(-l_0V)(\vec{r}_i) + \Delta\frac{dI(\vec{r}_i)}{dt}L = L\frac{dI(\vec{r}_i)}{dt} \quad (3.7)$$

Substituting of Eq.(3.4) in Eq.(3.7) together with Eq.(2.2) gives

$$-\langle i|l_0|V\rangle + \langle i|l_1|V\rangle = \left\langle i\middle|\frac{dI}{dt}\right\rangle L \quad (3.8)$$

We may write

$$(l_0 - l_1)|V\rangle = -L\left|\frac{dI}{dt}\right\rangle \quad (3.9)$$

Similarly to the perfect lattice. One can write Ohm's and Kirchhoff's law's for perturbed lattice as

$$l|V\rangle = -L\left|\frac{dI}{dt}\right\rangle \quad (3.10)$$

Where l is the lattice Laplacian operator for the perturbed lattice.

$$l = l_0 - l_1 \quad (3.11)$$

One can see that the operator l is decomposed into two parts l_0 associated with the perfect lattice and l_1 corresponding to the perturbation [16].

The Green's function $G$for perturbed lattice is defined by

$$lG = -1 \quad (3.12)$$

Using Eqs.(3.10) and (3.12), one obtains

$$|V\rangle = LG\left|\frac{dI}{dt}\right\rangle \quad (3.13)$$

To calculate the inductance between sites $\vec{r}_i$ and $\vec{r}_j$ we assume that a current $\frac{dI}{dt}$ enter at site $\vec{r}_i$ and $-\frac{dI}{dt}$ exists at site $\vec{r}_i$.The currents are zero at all other lattice points.

Hence, the current at lattice points $\vec{r}_n$ is written as

$$\frac{dI(\vec{r}_n)}{dt} = \frac{dI}{dt}(\delta_{n,i} - \delta_{n,j}) \quad (3.14)$$

And the potential at lattice point $\vec{r}_m$ is

$$V(\vec{r}_m) = \langle m|V\rangle = L\left\langle m\middle|G\middle|\frac{dI}{dt}\right\rangle = L\sum_n\langle m|G|n\rangle\left\langle n\middle|\frac{dI}{dt}\right\rangle$$

$$V(\vec{r}_m) = L\sum_n G(m,n)\frac{dI(\vec{r}_n)}{dt} \quad (3.15)$$

The substitution of Eq. (3.14) into Eq. (3.15) yields

$$V(\vec{r}_m) = L\frac{dI}{dt}\sum_n G(m,n)\left(\delta_{n,i} - \delta_{n,j}\right)$$

$$= L\frac{dI}{dt}\left(G(m,i) - G(m,j)\ \ \right) \quad (3.16)$$

The inductance between sites $\vec{r}_i$ and $\vec{r}_j$ is given by

$$L(i,j) = \frac{V(\vec{r}_i) - V(\vec{r}_j)}{\frac{dI}{dt}}$$

$$L(i,j) = L[G(i,i) - G(i,j) - G(j,i) + Gj,j)] \quad (3.17)$$

Notice that the perturbation destroyed the translational symmetry in the perturbed lattice (i.e. $G(i,i) \neq G(j,j)$, but $G(i,j)$is still a symmetric matrix (i.e. $G(i,j) = G(j,i)$).

So Eq.(3.17), becomes

$$L(i,j) = L[G(i,i) + G(j,j) - 2G(i,j)] \quad (3.18)$$

Now, our aim is to calculate Green's function G. using Eq's.(2.15), (3.11) and (3.12) for Green's function G becomes

$$G = (1 + G_0 l_1)^{-1} G_0 \quad (3.19)$$

Rewriting the above equation we obtain the following Dyson's equation

$$G = G_0 - G_0 l_1 G \quad (3.20)$$

Green's function G can be calculated by expanding $(1 + G_0 l_1)^{-1}$ in Eq.(3.19) or solving Dyson's equation (3.20) by iteration to obtain an infinite geometric series:

$$G = G_0 + G_0 l_1 G_0 + G_0 l_1 G_0 l_1 G_0 + G_0 l_1 G_0 l_1 G_0 l_1 G_0 + \cdots \quad (3.21)$$

Because of simple from of l_1, the summation in the above equation can be performed exactly.

By substituting Eq.(3.5) in (3.21), we obtain

$$G = G_0 + aG_0|x\rangle\langle x|G_0 + a^2 G_0|x\rangle\langle x|G_0|x\rangle\langle x|G_0 + \cdots$$

$$G = G_0 + aG_0|x\rangle[1 + a\langle x|G_0|x\rangle + (a\langle x|G_0|x\rangle)^2 + \cdots]\left\langle x\middle|G_0\right.$$

$$G = G_0 + aG_0|x\rangle\left[\frac{1}{1 - a\langle x|G_0|x\rangle}\right]\langle x\backslash G_0 \quad (3.22)$$

Where we have assumed that $G_0(l_0^{-1})$ exists and the denominator in the above equation is never equal to zero.

Inserting Eq.(3.6) into Eq.(3.22) leads to the Green's function

$$G = G_0 + \frac{(\alpha-1)G_0(|i_0\rangle - |j_0\rangle)(\langle i_0| - \langle j_0|)G_0}{\alpha - (\alpha-1)(\langle i_0| - \langle j_0|)G_0(|i_0\rangle - |j_0\rangle)} \quad (3.23)$$

The matrix element of $G(i,j)$can be written in terms of the matrix elements of G_0

$$G(i,j) = G_0(i,j) + \frac{(\alpha-1)[G_0(i,i_0) - G_0(i,j_0)][G_0(i_0,j) - G_0(j_0,j)]}{\alpha - 2(\alpha-1)[G_0(i_0,i_0) - G_0(i_0,j_0)]} \quad (3.24)$$

Where we have used the symmetry properties of lattice Green's function for a pure lattice:

$G_0(i,j) = G_0(j,i)$ and $G_0(i,i) = G_0(j,j)$

From Eqs.(3.18) and (3.24),the inductance between sites $\vec{r}_i$ and $\vec{r}_j$ can be obtained in terms of G_0 as

$$L(i,j)2L[G_0(i,i) - G_0(j,i)] + \frac{L(\alpha-1)[G_0(i,i_0) - G_0(i,j_0) - G_0(j,i_0) + G_0(j,j_0)]^2}{\alpha - 2(\alpha-1)[G_0(i_0,i_0) - G_0(i_0,j_0]} \quad (3.25)$$

Using Eq.(2.21),the above formula can be expressed in terms of the inductance of the perfect lattice L_0 as

$$L(i,j,\alpha) = L_0(i,j) + \frac{(\alpha-1)[L_0(i,j_0) - L_0(j,i_0) - L_0(i,i_0) + L_0(j,j_0)]^2}{4[L\alpha - (\alpha-1)L_0(i_0,j_0)]} \quad (3.26)$$

Now Eq.(3.26) is the final result for the inductance between two arbitrary lattice sites $\vec{r}_i$ and $\vec{r}_j$.

In any finite or infinite network, in which the inductance L between the sites $\vec{r}_{i_0}$ and $\vec{r}_{j_0}$ bond (i_0, j_0) is replaced by inductanceL',Eq.(3.26) is valid for any lattice structure in which each unit cell has only one lattice site.

It is worth mentioning that when $L' = \alpha L$, $(\alpha - 1)$, the problem reduced to the perfect case, we get

$$L(i,j) = L_0(i,j) \quad (3.27)$$

Now we compute the inductance between two sites $\vec{r}_{i_0}$ and $\vec{r}_{j_0}$ for a perturbed lattice. By Eq.(3.26), the inductance across the substitutional inductance is

$$L(i_0,j_0,\alpha) = L_0(i_0,j_0) + \frac{(\alpha-1)[L_0(i_0,j_0) + L_0(j_0,i_0) - 0 - 0]^2}{4[L\alpha - (\alpha-1)L_0(i_0,j_0)]}$$

$$= \frac{L\prime L_0(i_0,j_0)}{L' - (\alpha-1)L_0(i_0,j_0)} \quad (3.28)$$

4. Single Broken Inductance

As an example (see Fig.2), consider the infinite triangular arrays of identical inductors L, with a broken bond.

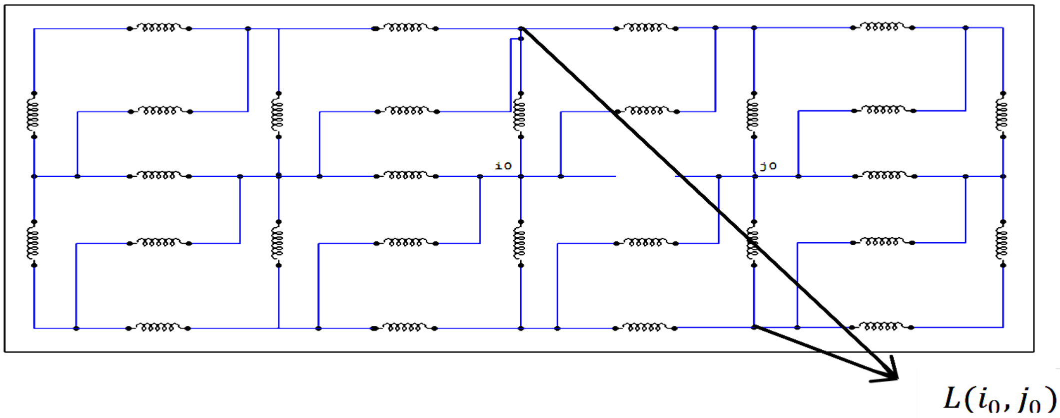

Figure 2 Perturbation of a triangular lattice of identical inductors L with a broken bond between sites $\vec{r}_{i_0}$ and $\vec{r}_{j_0}$ in a perfect lattice

(missing bond) between sites $\vec{r}_{i_0}$ and $\vec{r}_{j_0}$ in a perfect lattice From section (3), specifically Eq.(3.26) where α goes to infinity the problem is reduced to the broken bond case (Cserti, et al, 2002), and so we get

$$L(i,j) = L_0(i_0,j_0) + \frac{[L_0(i,j_0)+L_0(j,i_0)-L_0(i,i_0)-L_0(j,j_0)]^2}{4[L-L_0(i_0,j_0)]} \quad (4.1)$$

Using Eq.(3.46), the inductance across the broken bond is

$$L(i,j) = L_0(i_0,j_0) + \frac{[L_0(i,j_0)+L_0(i_0,j_0)-0-0]^2}{4[L-L_0(i_0,j_0)]}$$

$$= \frac{LL_0(i_0,j_0)}{L-L_0(i_0,j_0)} \quad (4.2)$$

As an example, consider the broken bond inductance between introduced between $i_0 = (0,0)$ and $j_0 = (1,0)$.

$$L(1,0;L) = L_0(1,0) + \frac{[L_0(1,0) + L_0(-1,0) - L_0(0,0) - L_0(0,0)]^2}{4[L - L_0(1,0)]} = 0.49993L$$

The results of calculation are listed in Table (3) for an infinite perturbed lattice.

5. RESULTS AND DISCUSSION

This section is devoted to discuss the result of the inductance for the triangular lattice. The theoretical results of the inductance perturbed triangular network are compared with those for the perfect network (Table1). Tables 2 and 3 show the calculated inductance between the sites r_i=(0,0) and r_j =(j_x,j_y) when the inductor between the sites r_{i0}=(0,0) and r_{j0}=(1,0) is replaced by L'=2L and L'=1/2L respectively. In the case of substitutional single inductance, Figures (3) and (4) show the theoretical inductance for the perfect and the perturbed (i.e. substitutional inductance case) infinite triangular lattice between the origin and the site $j = (j_x, 0)$ as function of j_x .

Figures (3) and (4) shows that the inductance of the perturbed lattice not symmetric in the direction of the perturbation. Also, it is divergent for large values of j_x. One can see from Figure (4) that the effective inductance L(i,j) in the perfect lattice when the substitutional inductance L; is larger than L. we see from Fig. (5) that the inductance L(i,j) is smaller in the perturbed lattice than L_0(i,j) in perfect lattice when the substitutional inductance L' is smaller than L. this is obvious from Eq,(3.26)

In the case of broken bond inductance, table 4 shows the calculated inductance when the bond is broken between the sites r_{i0}=(0,0) and r_{j0}=(1,0), Figure (5) illustrates the theoretical inductance for the perfect and perturbed (i.e. broken bond inductance case) infinite triangular lattices between the origin and the site $(j_x, 0)$ as a function of j_x . The Figure (5) shows that the inductance of the perturbed lattice is not symmetric in the direction of the perturbation. Also it is divergent for large values of j_x.

5. CONCLUSIONS

This work has aimed at calculating the inductance between two arbitrary points in an infinite network of identical inductors for a triangular network, theoretically, for both perfect and perturbed cases.

Derivation of analytical expressions for the lattice Green's function and the inductance of the perfect and perturbed infinite networks .A formula for the inductance for a perfect 2-D triangular lattice is derived by solving the integral of Eq.(2.23).some recurrence relations for a perfect triangular that allow the inductance calculation for arbitrary nodes are derived.

Expressions for the Green's function and the inductance for a perturbed lattice are given in terms of those for perfect lattice by solving Dyson's equation exactly .These expressions for

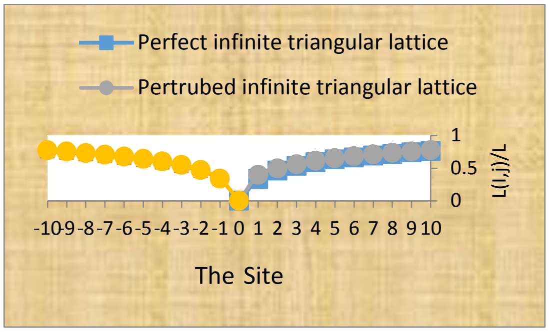

Figure (3)

The theoretical inductance in terms of L of the perturbed (circles) and the perfect (squares) infinite triangular lattice between the origin and the site $j = (j_x, 0)$ as function of j_x when substitutional inductance L'=2L is inserted between $i_0 = (0,0)$ and $j_0 = (1,0)$.

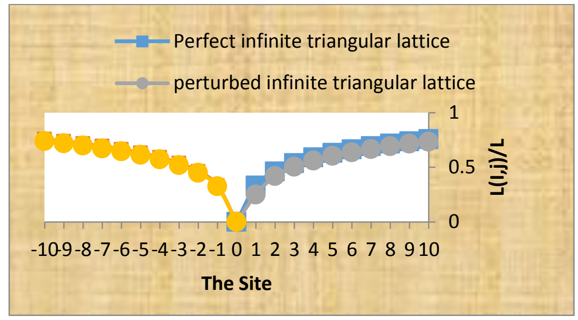

Figure (4)

The theoretical inductance in terms of L of the perturbed (circles) and the perfect (squares) infinite triangular lattice between the origin and the site $j = (j_x, 0)$ as function of j_x when substitutional inductance L'=1/2L is inserted between $i_0 = (0,0)$ and $j_0 = (1,0)$

Inductance is valid for any perturbed lattice structure either infinite or infinite in which each unit cell has only one lattice point .

We observe that the effective inductance between any two nodes in the perfect and perturbed finite networks is larger than that in the corresponding infinite network .In the case of substitutional inductance, the effective inductance between any two nodes in the perturbed lattice is larger (smaller) than that in the perfect lattice if the substitutional inductance is larger (smaller) than L . In the case of a broken bond the effective inductance between any two nodes in the perturbed lattice is always larger than that in the perfect case. As another example, the subtitutional is $L' = 0.5L$ introduced between nodes $i_0 = (1,0)$. the inductance between the origin and the sites $j = (j_x, j_y)$ are listed in table (2).

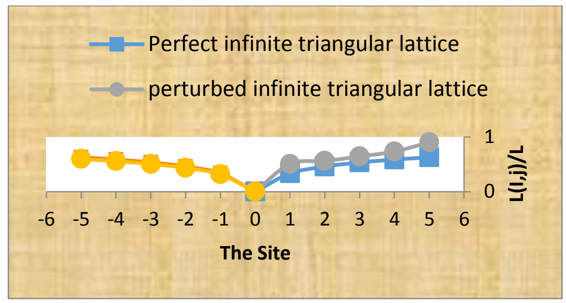

Figure (5)

The theoretical inductance in units of L of the perturbed (circles) and the perfect (squares) infinite triangular lattice are calculated between the sites $\vec{r}_i = (0,0)$ and $j = (j_x, 0)$ as a function of j_x when the broken bond is between $\vec{r}_{i0} = (0,0)$ and $\vec{r}_{j0} = (1,0)$

Table (1)

Numerical Values of the inductance in unit of L for an infinite perfect triangular lattice.

The site (i,j)	The value of $\frac{L(i,j)}{L}$ Infinite lattice	The site (i,j)	The value of $\frac{L(i,j)}{L}$ Infinite lattice
(0,0)	0	(0,0)	0
(1,0)	0.3333	(-1,0)	0.3333
(2,0)	0.4614	(-2,0)	0.4614
(3,0)	0.5362	(-3,0)	0.5362
(4,0)	0.5892	(-4,0)	0.5892
(5,0)	0.6302	(-5,0)	0.6302
(6,0)	0.6637	(-6,0)	0.6637
(7,0)	0.6920	(-7,0)	0.6920
(8,0)	0.7166	(-8,0)	0.7166
(9,0)	0.7382	(-9,0)	0.7382
(10,0)	0.7576	(-10,0)	0.7576

Table (2)

Calculated equivalent inductance in terms of L between the origin and the site $j = (j_x, j_y)$ for an infinite perturbed triangular lattice: substitutional inductance $L' = 2L$ is inserted between the site $i = (i_{0x}, i_{0y}) = (0,0)$ and $j_0 = (j_{0x}, j_{0y}) = (1,0)$

The site (i,j)	The value of $\frac{L(i,j)}{L}$ Infinite lattice	The site (i,j)	The value of $\frac{L(i,j)}{L}$ Infinite lattice
(0,0)	0	(0,0)	0
(1,0)	0.39995	(-1,0)	0.33962
(2,0)	0.49333	(-2,0)	0.47142
(3,0)	0.56118	(-3,0)	0.54798
(4,0)	0.61158	(-4,0)	0.60202
(5,0)	0.65121	(-5,0)	0.64368

(6,0)	**0.68388**	**(-6,0)**	**0.67765**
(7,0)	**0.71161**	**(-7,0)**	**0.70629**
(8,0)	**0.73581**	**(-8,0)**	**0.73117**
(9,0)	**0.75709**	**(-9,0)**	**0.75298**
(10,0)	**0.77626**	**(-10,0)**	**0.77256**

Table (3)

Calculated equivalent inductance in terms of L between the origin and the site $j = (j_x, j_y)$ for an infinite perturbed triangular lattice: substitutional inductance $L' = 0.5L$ is inserted between the site $i = (i_{0x}, i_{0y}) = (0,0)$ and $j_0 = (j_{0x}, j_{0y}) = (1,0)$

The site *(i,j)*	The value of $\frac{L(i,j)}{L}$ Infinite lattice	The site *(i,j)*	The value of $\frac{L(i,j)}{L}$ Infinite lattice
(0,0)	**0**	**(0,0)**	**0**
(1,0)	**0.24998**	**(-1,0)**	**0.32540**
(2,0)	**0.42148**	**(-2,0)**	**0.44887**
(3,0)	**0.50497**	**(-3,0)**	**0.52147**
(4,0)	**0.56122**	**(-4,0)**	**0.57318**
(5,0)	**0.60393**	**(-5,0)**	**0.61335**
(6,0)	**0.63847**	**(-6,0)**	**0.64626**
(7,0)	**0.66748**	**(-7,0)**	**0.67413**
(8,0)	**0.69258**	**(-8,0)**	**0.69838**
(9,0)	**0.71548**	**(-9,0)**	**0.71972**
(10.0)	**0.73427**	**(-10.0)**	**0.73890**

Table (4)

Calculated equivalent inductance in terms of L between the origin and the site $j = (j_x, j_y)$ for an infinite perturbed triangular lattice: the broken inductance between the site $(i_{0x}, i_{0y}) = (0,0)$ and $j_0 = (j_{0x}, j_{0y}) = (1,0)$

The site *(i,j)*	The value of $\frac{L(i,j)}{L}$ Infinite lattice	The site *(i,j)*	The value of $\frac{L(i,j)}{L}$ Infinite lattice
(0,0)	**0**	**(0,0)**	**0**
(1,0)	**0.49994**	**(-1,0)**	**0.34909**
(2,0)	**0.54123**	**(-2,0)**	**0.48646**
(3,0)	**0.59865**	**(-3,0)**	**0.56566**
(4,0)	**0.64516**	**(-4,0)**	**0.62124**
(5,0)	**0.68274**	**(-5,0)**	**0.66390**
(6,0)	**0.71415**	**(-6,0)**	**0.69858**
(7,0)	**0.74103**	**(-7,0)**	**0.72773**
(8,0)	**0.76463**	**(-8,0)**	**0.75303**
(9,0)	**0.78543**	**(-9,0)**	**0.77515**
(10,0)	**0.80425**	**(-10.0)**	**0.9889**

REFERENCES

1. Morita, T., and Horiguchi, T. Analytic properties of the lattice Green function, Journal of Physics A: General Physics. 5 (67): (1972).
2. Economou , E., N., Green's Function in Quantum Physics ,(3rd ed),Springer –Verla Berlin (2006).
3. Joyce, S. Lattice Green's Function for the simple cubic lattice, Journal of Physics A: Mathematical and general.(5): 65-68 (1972).
4. Maradudin, A. A., Montroll, E. W., Weiss, G. H., Herman, R., and Milnes,H. W. Green's functions for monatomic simple cubic lattices. Mkmoires, Academie Royal de Belgique, Classe des Sciences 14, Fascicule 7 (1960).
5. Horiguchi , T ., and Morita , T. Note on the Lattice Green's Function for the simple cubic lattice ,Journal of Physics C ,(8): 232-235 (1975).
6. Barton,G., Elements of Green's Functions and Propagation :Potentials ,Diffusion ,and Waves, Oxford University Press (1989).
7. Cserti, J., Szechenyi, G., and David, G., Uniform tiling with electrical resistors. Journal of Physics A: Mathematical and Theoretical. 44, 215201-215220(2011).
8. Joyce, G, S., Exact Results for a Body Centered Cubic Lattice Green's Function with Applications in Lattice Statistics, journal Of mathematical Physics, 12,1390 (1971).
9. Cserti, J. Application of The Lattice Green's Function for Calculating The inductance of Infinite Networks of inductors .American Journal of Physics.(68):896-906, (2000).
10. Cserti, J., David, G., and Piroth, A. Perturbation of Infinite Networks of Resistors, American Journal of Physics. (70):153-159 (2002).

11. Asad, J., Hijjawi, R., Sakaji, A and Khalifeh, J. Infinite Network Of Identical Capacitors By Green’s Function. International Journal of Modern Physics B, 19 (24): (2005)
12. Morita, T. Useful Procedure for Computing the Lattice Green's Function square ,Tetragonal ,and BBC Lattices ,Journal of Mathematical Physics .12(8): (1971).
13. Wu, F, Y. Theory of resistor networks: the two-point resistance, Journal of Physics A: Mathematical and Theoretical, 37, 6653–6673 (2004).
14. Hijjawi, R , S . Asad, J. H. Sakaji, A. and Khalifeh ,J.M. Perturbation of an Infinite Network of identical Capacitors .International Journal Of Modern Physics B . 21(2):199-209 (2007).
15. Asad ,J .H .Hijjawi ,R .S .Sakaji ,A .J .and Khalifeh ,J.M. Infinite Networks of Identical Capacitors ,Modern Physics Letter B,24 695-705 (2010).
16. Kirkpatrick, S. Percolation and Conduction, Reviews of Modern Physics, 45, 574-588 (1973).
17. Hollos, S. A Lattice Green Function Introduction, Exstrom Laboratories LLC, 662 Nelson Park Dr, Longmont, Colorado 80503, USA, 2005