SOLUTION OF WAR PLANNING PROBLEM USING DERIVATIVE FREE METHODS

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ABSTRACT: *The focus of this research is to formulate optimization model of war planning problem along with fuel supply and availability of aircrafts constraints. The derivative free methods are used for the optimization of Deterministic problems. These methods are basically designed for unconstrained optimization problems. In formulated optimization war planning problems the constraints are handled by using exterior penalty functions. The results of the war planning optimization model are obtained by using MATLAB which demonstrate the effectiveness, applicability and comparison of these derivative free methods.*

Keywords: derivative free methods, penalty function, war planning problem, deterministic model, unconstrained optimization

1. INTRODUCTION

The optimization issues appear in very nearly all ranges of life like assembling, scheduling, engineering and business. Utilizing optimization procedures [2] the best results of the problem are attempted to get by using least measure of restricted assets [1].

Two principle procedures of optimization, specifically, derivative based and derivative free are, no doubt utilized frequently. Among the direct search methods we concentrated on Hooke and Jeeves (HJ) strategy [11] and Nelder and Mead (NM) system [8,9,10]. These methods are intended for unconstrained optimization issues. They can additionally connect to constrained optimization problems by changing them into unconstrained optimization problems by utilizing the penalty function [3]. The structures of the penalty function alongside the tenets for alter the penalty parameters at the end of each one unconstrained optimization stage characterize the specific method or strategy.

For calculating different sorts of optimization issues a lot of direct search methods have been produced by the analysts. A definite investigation of these systems, with recorded foundation, might be found in [5,6,7].

Direct search methods and Genetic Algorithms [12,14] are intended for the solution of unconstrained optimization issues. It is required to handle constraints in such a way, to the point that the converted issue is free of constraints. There are a few investigations of handling constraints. The least complex strategy for requirement, is to add penalty term to the objective function for level of violation of the constraints [12,13].

The thought of this system is to change the constrained optimization issue to an unconstrained one by adding/subtracting the value of or from the objective function focused around constraint violation present in the result [4,15].

A deterministic model assumes certainty in all aspects. Most models really should be stochastic or probabilistic rather than deterministic, but this is often too complicated to implement. Representing uncertainty is fraught. Some more common stochastic models are queueing models, markov chains, and most simulations.

For example when planning a school formal, there are some elements of the model that are deterministic and some that are probabilistic. The cost to hire the venue is deterministic, but the number of students who will come is probabilistic. An example of such a problem together with its mathematical model is discussed below.

The Pakistani Command receives orders to destroy the enemy nuclear power stations. The enemy has three key stations located in different cities, and certain successful interdictions could effectively halt the production of deadly nuclear power plants. The fuel supply is limited to 30,000 L for this particular mission. Any bomber aircraft sent to any particular city must have at least enough fuel for the round trip plus 100 L for safety reason [16].

Table-1 Availability of aircrafts

Bomber Type	Description	Km/L	Availability
RB-57F	Martin B-57 Canberra	2	20
HP-57H	Handley Page Halifax	3	12

Table-2 Details of Target

Plants	Cities	Distance	Probability of Destruction	
			F-57	H-57
1	City-X	880	0.15	0.10
2	City-Y	750	0.30	0.15
3	City-Z	920	0.25	0.12

How many of each type of bomber aircraft should be dispatched and how should they be allocated across the three targets to maximize the probability of success?

2. MATERIAL AND METHODS

2.1 Hooke and Jeeves Method

This method starts with an initial point. In N-dimensional problem a Set of N linearly independent search directions generate 2N points.

Exploratory Move: Exploratory move is performed on the current point systematically to find the best point around the current point.

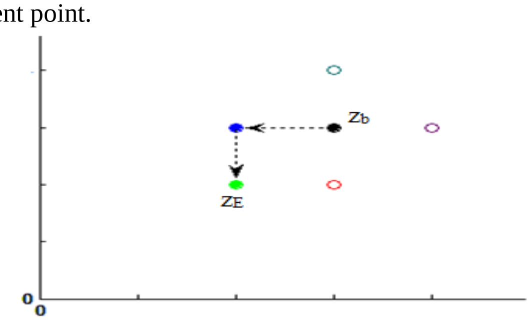

Figure-1 Successful exploratory move

Pattern Move: When exploratory move success then pattern move is perform, a new point is found by jumping from the current base point along a direction connecting to the previous.

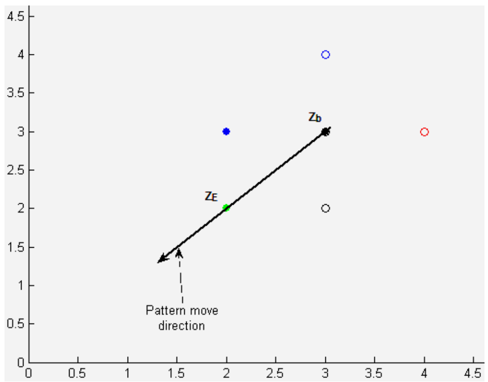

Figure-2 Pattern move direction

2.2 Nelder-Mead Simplex Method

The method uses the following operations

Reflection: Reflect the worst vertex over the centroid.

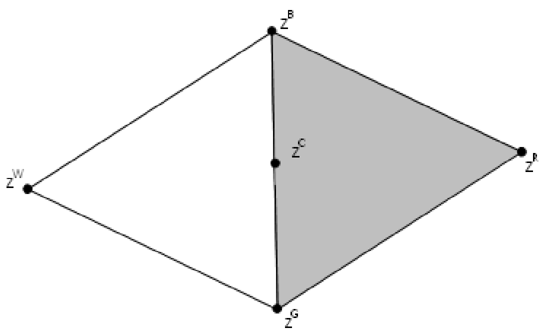

Figure-3 Reflection

Expansion: If the function value at the reflect point is less than best point the expansion is performed.

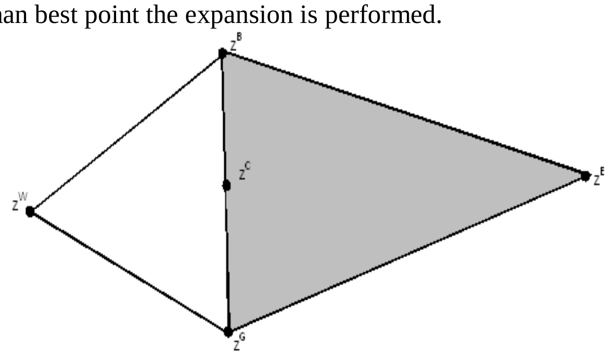

Figure-4 Expansion

Contraction: If the function value of the reflection point lies between the good and best vertex then

Inner Contraction: if the function value greater than the best point then inner contraction is performed.

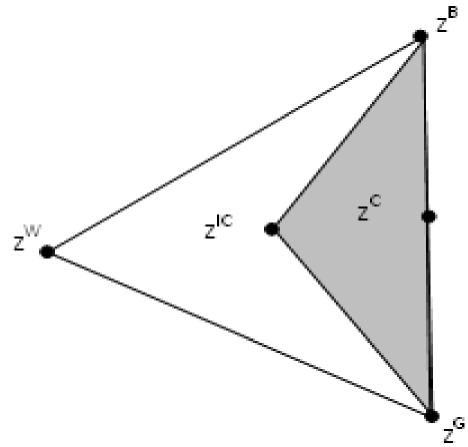

Figure-5 Inside Contraction

Outer Contraction: if the function values less than the best point then outer contraction is performed.

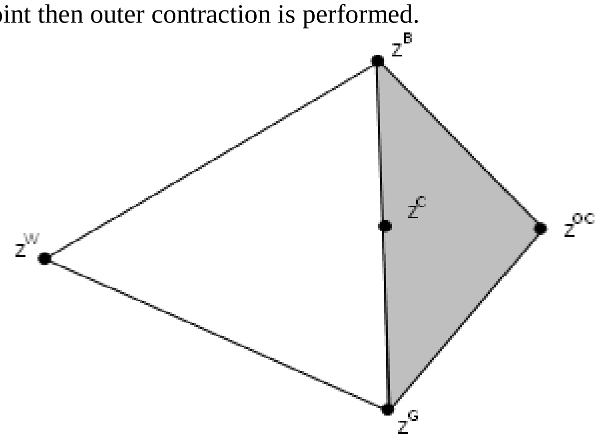

Figure-6 Outside Contraction

Shrink: If no one from the above conditions is satisfied then shrink produced.

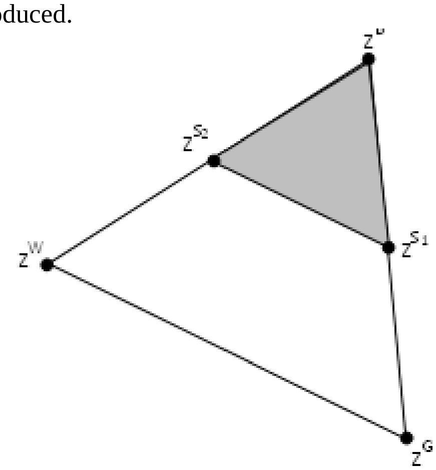

Figure-7 Shrink

3. FORMULATION OF WAR PLANNING PROBLEM

3.1 Decision Variables

z_{ij} = The number of bomber type i (i = F and H) that will be sent to location of plant j (j =1, 2 and 3).

3.2 Objective Function

The objective is to maximize the probability of success in destroying the nuclear power stations, or expressed in other words to minimize the probability of failing to destroy the nuclear power stations.

The probability of succeeding to destroy plant 1 by one F-57 bomber is 0.15. The probability, therefore, of not succeeding (failing) to destroy plant 1 by one F-57 bomber is (1 - 0.15 =) 0.85. The probability of not succeeding to destroy plant 1 by z_1 number of F-57 bombers is $(0.85)^z{}_1$. Therefore, the probability of succeeding to destroy plant 1 by z_1 F-57 bombers is $1 - (0.85)^z{}_1$.

The probability of failure and success for all other assignments can be found in a similar way. The objective then is to maximize the success of destroying all nuclear power stations or in other words to minimize the failure of the overall mission. This can be expressed as an objective function where the failure of destroying all nuclear power stations is to be minimized.

The probability of not succeeding to destroy plant 1 by z_1 number of F-57 bomber is $(0.85)^{z1}$

The probability of not succeeding to destroy plant 2 by z_2 number of F-57 bomber is $(0.70)^{z2}$

The probability of not succeeding to destroy plant 3 by z_3 number of F-57 bomber is $(0.75)^{z3}$

Similarly

The probability of not succeeding to destroy plant 1 by z_4 number of H-57 bomber is $(0.90)^{z4}$

The probability of not succeeding to destroy plant 2 by z_5 number of H-57 bomber is $(0.85)^{z5}$

The probability of not succeeding to destroy plant 3 by z_6 number of H-57 bomber is $(0.88)^{z6}$

So the nonlinear objective function is

$$\text{Minimize } Z = (0.85)^{z1} \bullet (0.70)^{z2} \bullet (0.75)^{z3} \bullet (0.90)^{z4} \bullet (0.85)^{z5} \bullet (0.88)^{z6}$$

3.3 Constraints

1. Fuel supply limitation

$$\text{Fuel required (liters) up to target for each trip of } z_1 \text{ for one side} = \frac{880}{2}$$

$$\text{Fuel required (liters) up to target for each trip of } z_1 \text{ for comeback} = \left(\frac{880}{2}\right) \cdot 2$$

$$\text{Fuel required (liters) forsafty purpose for } z_1 = \left(\frac{880}{2}\right) \cdot 2 + 100 = 980$$

Similarly

$$\text{Fuel required (liters) for safty purpose for } z_2 = \left(\frac{750}{2}\right) \cdot 2 + 100 = 850$$

$$\text{Fuel required (liters) forsafty purpose for } z_3 = \left(\frac{920}{2}\right) \cdot 2 + 100 = 1020$$

$$\text{Fuel required (liters) forsafty purpose for } z_4 = \left(\frac{880}{3}\right) \cdot 2 + 100 = 686.6 \approx 687$$

$$\text{Fuel required (liters) forsafty purpose for } z_5 = \left(\frac{750}{3}\right) \cdot 2 + 100 = 600$$

$$\text{Fuel required (liters) forsafty purpose for } z_6 = \left(\frac{920}{3}\right) \cdot 2 + 100 = 713.66 \approx 714$$

The fuel supply constraint is

$$980\, z_1 + 850 z_2 + 1020 z_3 + 687 z_4 + 600 z_5 + 714 z_6 \le 30{,}000$$

2. Constraint for the number of aircraft

Total availability of F-57 aircrafts

$$Type\ F: z_1 + z_2 + z_3 \le 20$$

Total availability of H-57 aircrafts

$$Type\ H: z_4 + z_5 + z_6 \le 12$$

So that, finally non-linear model is

$$\text{Minimize } Z = (0.85)^{z1} \bullet (0.70)^{z2} \bullet (0.75)^{z3} \bullet (0.90)^{z4} \bullet (0.85)^{z5} \bullet (0.88)^{z6}$$

Subject to

$$980\, z_1 + 850 z_2 + 1020 z_3 + 687 z_4 + 600 z_5 + 714 z_6 \le 30{,}000$$

$$z_1 + z_2 + z_3 \le 20$$

$$z_4 + z_5 + z_6 \le 12$$

$$z_{ij} \ge 0 \qquad \forall\, i \,\&\, j$$

The constrained problem is changed into unconstrained optimization problem using following penalty function

$$P(x) = r\,(\max[0, g_1(x), g_2(x), g_3(x)])$$

Final unconstrained optimization model becomes

$$\text{Minimize } Z = (0.85)^{z1} \bullet (0.70)^{z2} \bullet (0.75)^{z3} \bullet (0.90)^{z4} \bullet (0.85)^{z5} \bullet (0.88)^{z6} + 100[\max(0, [980Z_1+850Z_2+1020Z_3+687Z_4+600Z_5+714Z_6-30{,}000], [Z_1+Z_2+Z_3-20], [Z_4+Z_5+Z_6-12])]$$

4. DISCUSSION

The initial guess (1,2,0,1,2,3) for HJ method converges to its local minimum at (7,8,5,3,4,5) with function value f (z)=0.00088 with step length = $\Delta = (0.5, 0.5)^t$, parameter = α = 2 and NM method converges to its local minimum at (6.9058, 7.4245, 2.8665, 3.1737, 3.3790, 4.2369) with function value f (z)=0.0003927, whatever the initial guess is provided with reflection coefficient = δ_r =1, expansion coefficient = δ_e =2, outer-contraction coefficient = δ_{ic} = - 0.5, inner-contraction coefficient = δ_{oc} = 0.5. It is observed that the step length Δz_i for each variable should be chosen in such a way that it is possible to find the optimal minima. And we also find out the optimal point every worst point is replaced with the best one in NM method. The comprehensive summary of the entire discussion is presented in the form of table as shown in table.

Table -3 Performances of HJ and NM methods in Air Attack Planning Problem.

	Methods	
	Nelder & Mead	**Hooke's & Jeeves**
Initial Guess	z_1=(0,0,0,0,0,0)	(1,2,0,1,2,3)
	z_2= (2,0,0,0,0,0)	
	z_3= (0,2,0,0,0,0)	
	z_4= (0,0,2,0,0,0)	
	z_5= (0,0,0,2,0,0)	
	z_6= (0,0,0,0,2,0)	
	z_7= (0,0,0,0,0,2)	
Iterations	57	13
Function Evaluations	105	165
Function Value	0.00039273	0.00088
Optimal Point	(6.9058, 7.8245, 5.0665, 3.1737, 3.3790, 4.2369)	(7,8,5,3,4,5)

The performance of both the method is compared in the form of graph. Red spots in the graph represent the function values of HJ method and blue spots in the graph represent the function values of NM method. This function is solved with the HJ method and the NM method and observed that the performances of the two techniques are equally good.

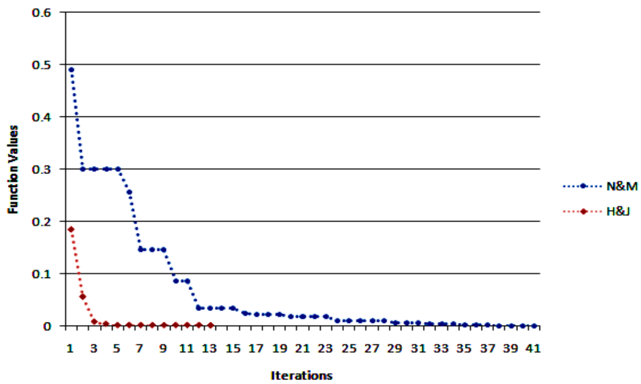

Figure-8 Graph between function values and iterations of air attack planning problem

5. RESULTS

Martin B-57 Canberra bomber aircraft are dispatched =7+8+5=20

Handley Page Halifax bomber aircraft are dispatched =3+4+5=12

Probability of failure destroying all nuclear power stations by bomber aircraft = 0.00039273

6. CONCLUSION

We applied Hooke and Jeeves method and Nelder-Mead method on air attack planning problem, we implemented these two methods in MATLAB on the formulated problems for many times at various initial guesses and for a number of step sizes. We can conclude that result of Nelder and Mead method is acceptable and batter then Hooke and Jeeves method due to its function value and its number of function evaluations.

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