MODELING WEIRS DISCHARGE COEFFICIENT USING EVOLUTIONARY ALGORITHM

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ABSTRACT: *Information about flow discharge in side weirs is an important issue in hydraulic engineering. In general, the channel and side weir shapes affect the flow discharge. Nevertheless, estimating discharge coefficient (which is dependent upon flow characteristics, channel and side weir geometry) is a key issue in analyzing flow discharge over these structures. In this study, the Genetic Expression Programming (GEP) approach was used for predicting trapezoidal and rectangular sharp-crested side weirs discharge coefficient. Correlation coefficient (R), mean normalize error (MNE) and nash-sutcliffe index (NS) statistics are used as comparing criteria for the evaluation of the model's performances. The obtain results approved capability of GEP in prediction of trapezoidal and rectangular side weirs discharge coefficient. The results also showed the influence of downstream Froude number for trapezoidal side weir and upstream Froude number for rectangular side weir in prediction of the discharge coefficient for both of side weirs.*

Keywords: Discharge coefficient, Genetic expression programming, weir, evolutionary algorithm

1. INTRODUCTION

Sides weirs are measurement and flow control devices, installed on the channel's side wall to divert water over them, and are grouped mainly into sharp edge and broad crested weirs. Like normal weirs, side weirs might take different shapes (e.g. rectangular, triangular, trapezoidal, etc). Information about flow discharge in side weirs is an important issue in hydraulic engineering. In general, the channel and side weir shapes affect the flow discharge. Nevertheless, estimating the discharge coefficient (which is dependent upon flow characteristics, channel and side weir geometry) is a key issue in analyzing flow discharge over these structures [1]. There are numerous studies around side weirs hydraulics. Kumar and Pathak [2] investigated the discharge coefficient of sharp and broad-crested triangular side weirs and related the discharge coefficient of a triangular side weir to approach Froude number and apex angle of the weir. Ghodsian [3] studied hydraulic characteristics of sharp crested triangular and results showed that Di Marchi's coefficient for this weir depends upon Froude number of main channel, apex angle, and weir height to upstream depth. The studies of Kaya [4] on semi-elliptical side weirs in subcritical flow showed that the discharge coefficient of these weirs is more than classic weirs. Haddadi and Rahimpour [5] studied flow passed over broad crested trapezoidal side weirs in an experimental research and by analyzing the obtained results, suggested some functions with acceptable error for discharge coefficient of these weirs.

In the recent years, application of Machine learning (ML) [e.g. Artificial Neural Networks (ANNs), Neuro-Fuzzy models (NF), Genetic Algorithms (GA), Genetic Expression Programming (GEP), and Support vector machines (SVM).] in hydraulics studies has caught the attention of numerous researchers.

Dursun [6] applied NF model for estimating discharge coefficient of semi-elliptical side weirs and compared the results of NF with multiple linear regression (MLR) winding up that NF technique leads to better results for modeling discharge coefficient. Emiroglu [7] used NF and multiple nonlinear regression (MNR) techniques for modeling discharge coefficient, and NF resulted in more acceptable and fewer errors. Kisi [8] used GEP and ANN techniques for estimating the side weir discharge coefficient and compared the obtained results with those obtained from MLR and MNR which estimations made by GEP and ANN showed fewer errors. The present study is an attempt to evaluate the performance of GEP for prediction discharge coefficient of trapezoidal and rectangular side weirs.

2. MATERIAL AND METHODS

Genetic Expression Programming (GEP)

Genetic Expression programming (GEP) was developed by Ferreira [9] using fundamental principle of the Genetic Algorithms (GA) and Genetic Programming (GP). GEP is a procedure that mimics biological evaluation to create a computer program to model some phenomenon. In applying GEP for solving a problem, there are five major preparatory steps as follows:

Set of terminals: A set of input variables or constants. The set of primitive functions: A set of domain specific functions used in conjunction with the terminal set to construct potential solutions to a given problem. For symbolic regression this could consist of a set of basic mathematical functions, while Boolean and conditional operators could be included for classification problems. The fitness measure: Fitness is a numeric value assigned to each member of a population to provide a measure of the appropriateness of a solution to the problem in question. The parameters for controlling the run: This includes the population size and the crossover and mutation probabilities. The method for designation a result and the criterion for terminating run: This is generally a predefined number of generations or an error tolerance on the fitness [10].

Experimental setup and procedure

The data used in this study those applied by Tynes [11] for trapezoidal side weir, and Emiroglu [12] for rectangular side weir. Fig. 1 represents the schematic view of the studied weirs.

a) Trapezoidal model: it is 65 ft in long with side slopes of 2.5H: 1V, longitudinal slope of 0.000385, and a Manning's n values of 0.0125. The exterior slope of the channel wall was 3H: 1V. The nearly horizontal part of the weir crest was separated longitudinally from the berm by a distance of 2.0 feet at both the upstream and downstream ends of the weir. The width of the access road was 0.48 feet.

b) Rectangular side weir: the main channel was 12 m in long, 0.5m in depth with a rectangular cross section of 0.5m depth and a longitudinal slope of 0.001. The collection channel was 0.5 m in wide and 0.7 m deep, and situated parallel to the main channel. The width of the collection channel across the side weir was 1.3 m and it was constructed as a circular shape to provide free overflow conditions over the side weir.

Model implementation

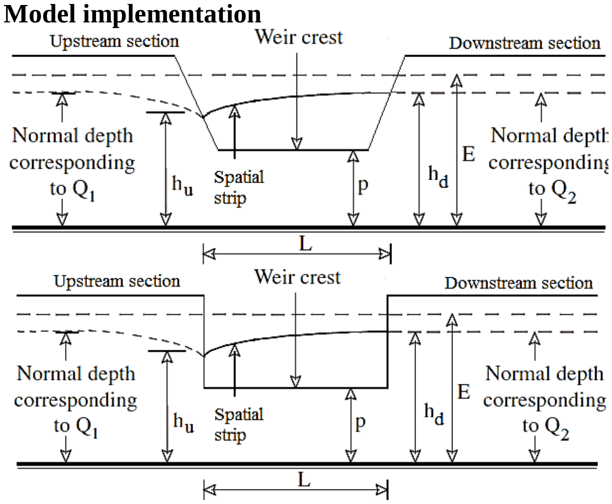


Fig. 1. Definition sketch of a sharp-crested trapezoidal and rectangular side weir.

Referring to Fig. 1, the discharge coefficient (C_d) might be considered as a function of channel width (b), flow depth at the main channel upstream (h_1), mean flow velocity at upstream (v_1), length of side weir (L), flow depth at side weir upstream (h_u), flow depth at side weir downstream of (h_d), mean flow velocity at downstream section of side weir (v_{wd}), mean flow velocity at upstream section of side weir (v_{wu}), crest height (p), the side slope of trapezoidal weir (z), deviation angle of flow (ψ), mass density of the fluid (ρ), roughness of the main channel (n), slope of main channel bed (S_0), surface tension (σ), and gravitational acceleration (g). Mathematically, the following functional relationships might be considered:

For trapezoidal side weir:

$$C_d = f\left(v_1, L, b, h_1, p, h_u, \Psi, S_0, n, v_{wd}, g, h_d, \sigma, \rho, z\right) \quad (1)$$

For rectangular side weir[13, 14]:

$$C_d = f\left(L, b, h_u, p, \Psi, S_0, n, v_{wu}, g, \sigma, \rho\right) \quad (2)$$

According to Subramanya and Awasthy [15], deviation angel ψ, might be given as the following equation:

$$\sin(\Psi) = \sqrt{1-\left(\frac{v_1}{v_s}\right)^2} \quad (3)$$

Where, v_s is velocity of flow dQ_s (discharge per unit length of side weir) over the brink. They also mentioned that the effect of S_0, n and σ on discharge coefficient for elementary flow particle is very small and negligible. El-Khashab [16] also stated that dimensionless length of side weir includes the effects of deviation angle on discharge coefficient thus this parameter is not present in discharge coefficient equation in this study. Therefore, equations reduce to:

Trapezoidal side weir:

$$C_d = f\left(v_1, L, b, h_1, p, h_u, v_{wd}, g, h_d, \rho, z\right) \quad (4)$$

Rectangular Side Wear:

$$C_d = f\left(L, b, h_u, p, v_{wu}, g, \rho\right) \quad (5)$$

Bukingham's theory is used in order to change variables into dimensionless and to reach this purpose variables are chosen as repeating parameters and thus dimensional analysis based on Bukingham's theorem, the non-dimensional variables can be written as:

Trapezoidal side weir:

$$C_d = f\left(Fr_{wd}, Fr_{wu}, \frac{h_1}{p}, \frac{L}{b}, \frac{p}{h_u}, \frac{h_d}{b}, \frac{L}{h_u}, z\right) \quad (6)$$

Rectangular side weir:

$$C_d = f\left(Fr_{wu}, \frac{L}{b}, \frac{p}{h_u}, \frac{L}{h_u}\right) \quad (7)$$

Where Fr_{wu} is the upstream weir Froude number and Fr_{wd} is the downstream weir Froude number. Meanwhile Froude number which represents the effect of the gravity on the flow is a dynamical parameter and other dimensionless parameters show geometrical effects of the channel and side weirs. Input models are applied, in this research, by using of these non-dimensional parameters that are given in Table 1.

Table 1: Applied input configuration for GEP

Model	Trapezoidal side weir	Model	Rectangular side weir
M1	$\left(Fr_{wd}, \frac{h_1}{p}\right)$	M1	$\left(Fr_{wu}\right)$
M2	$\left(Fr_{wd}, \frac{L}{b}, \frac{h_1}{p}\right)$	M2	$\left(Fr_{wu}, \frac{p}{h_u}\right)$
M3	$\left(Fr_{wd}, \frac{h_u}{L}\right)$	M3	$\left(Fr_{wu}, \frac{L}{b}\right)$
M4	$\left(Fr_{wd}, z\right)$	M4	$\left(Fr_{wu}, \frac{p}{h_u}, \frac{L}{b}\right)$
M5	$\left(Fr_{wd}\right)$	M5	$\left(Fr_{wu}, \frac{p}{h_u}, \frac{L}{b}, \frac{L}{h_u}\right)$
M6	$\left(Fr_{wd}, \frac{L}{b}, z\right)$	M6	$\left(Fr_{wu}, \frac{p}{h_u}, \frac{L}{h_u}\right)$

M7	$\left(Fr_{wd},\frac{L}{b},\frac{p}{h_u}\right)$	M7	$\left(\frac{L}{b},\frac{p}{h_u},\frac{L}{h_u}\right)$
M8	$\left(Fr_{wu},\frac{L}{b},\frac{h_1}{p},z\right)$	M8	$\left(\frac{p}{h_u},\frac{L}{h_u}\right)$
M9	$\left(Fr_{wu},\frac{L}{b},\frac{p}{h_u}\right)$		
M10	$\left(\frac{L}{b},\frac{h_1}{p},\frac{h_d}{L},z\right)$		

GEP models implementation

The major step while investigating on GEP operators is to select the appropriate fitness function. Therefore, we used the default basic function set of GeneXpro Program(i.e.) $+,-,\div,\times,\sqrt[3]{\ },\sqrt{\ },ln,e^x,x^2,x^3,\sin x,\cos x,Arctgx$ for the selecting fitness function. In the proposed model 70% of experimental Data were used for training and 30% of them used for testing of the model. Run of each model (consist of ten terminal set and five function set or totally 50 models) is done more than one hundred times for both of training and testing of Data.

The next step is to pick the set of terminals and the set of functions for creating the chromosomes. The terminal sets used in this study are $\left(Fr_{wd},Fr_{wu}\ \frac{h_1}{p},\frac{L}{b},\frac{p}{h_u},\frac{h_d}{b},\frac{L}{h_u},z\right)$ and $\left(Fr_{wu},\frac{L}{b},\frac{p}{h_u},\frac{L}{h_u}\right)$. Beside five functions set (F1, F2, F3, F4, and F5) contains different combinations of mathematical operators are defined as follows:

$$F1=\{+,-,\times,\div\},\ F2=\left\{+,-,\div,\times,\sqrt[3]{\ },\sqrt{\ },x^2\right\}$$

$$F1=\left\{+,-,\div,\times,\sqrt[3]{\ },\sqrt{\ },\sin x,\cos x,tanx,Arctgx\right\}$$

$$F4=\left\{+,-,\times,\div,\sqrt{\ },\sqrt[3]{\ },power,\ln x,\log x,e^x\right\}$$

$$F5=\left\{+,-,\times,\div,\sqrt{\ },\sqrt[3]{\ },x^2,x^3,\ln x,\sin x,\cos x,Arctgx,power\right\}$$

The third step is to decide on the chromosomal architecture. The prevailing applied values for this architecture are: length of head, h=8, and three genes per chromosomes. The fourth step is to choose the linking function. The linking function should be chosen as "addition" or "multiplication" for algebraic sub trees [9]. The final step is to choose the genetic operators.

Performance criteria

The statistical measurements that were used to evaluate the performance of the different models, namely Pearson's Correlation Coefficient (R), Mean Normalize Error (MNE), and Nash-Sutcliffe index (NS), expressions for which are as below:

$$R=\frac{\sum_{i=1}^{N}\left(O_i-\overline{O}_i\right)\left(P_i-\overline{P}_i\right)}{\sqrt{\sum_{i=1}^{N}\left(O_i-\overline{O}_i\right)^2\sum_{i=1}^{N}\left(P_i-\overline{P}_i\right)^2}} \quad (8)$$

$$MNE=\frac{1}{N}\sum_{i=1}^{N}\left|\frac{(O_i-P_i)}{O_i}\right| \quad (9)$$

$$NS=1-\frac{\sum_{i-1}^{N}\left(O_i-P_i\right)^2}{\sum_{i-1}^{N}\left(O_i-\overline{O}_i\right)^2} \quad (10)$$

3. RESULTS AND DISCUSSION

In this study, GEP model is applied to modeling discharge coefficient. A trial and error procedure is used to obtain the best percent of data blocks for training and testing phases. The aim of this procedure is to select the best train-test blocks sizes for estimating the discharge coefficient with high performance criteria. So, two models were considered, including 30-70 (i.e. 30% of whole data are considered for testing and 70% for training), and 35-65 modes. Between two modes, the 35-65 scenario showed more accurate results for trapezoidal weir and 30-70 scenario provided better results for rectangular weir, using the GEP models. Therefore, the first scenario was selected for trapezoidal weir, while the second one was employed for rectangular weir. Consequently, 154 and 93 data series were introduced as training data for trapezoidal and rectangular weirs, respectively. Nonetheless, 84 and 42 data series were reserved as testing patterns for trapezoidal and rectangular weirs, respectively.

Trapezoidal side weir results

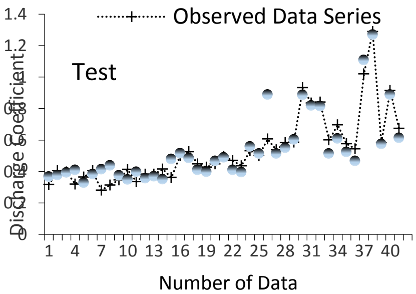
Ten different models were established to estimate discharge coefficient for trapezoidal side weir. In order to detect the effect of the prevailing parameters on C_d, the sensitivity analysis was performed using GEP model. The results of applying GEP model indicate that the discharge coefficient of trapezoidal side weir, are closely affected by downstream weir Froude number. Moreover, the sensitivity analysis indicates that the other important parameters affecting Cd for trapezoidal side weir, are $\frac{h_1}{p}$ and $\frac{L}{b}$ ratios.

Among ten models, two best models were selected according to ranking of performance criteria. Table 2 shows performance evaluation for training and testing stage for trapezoidal side weir.

Table 2: Statistics of the two best models of total data for training and testing periods for trapezoidal side weir

Machine Learning		Training			Testing		
GEP	M2	0.98	0.91	11	0.98	0.87	15
	M7	0.93	0.91	13	0.88	0.86	17

According to results in Table 2, GEP model can give good prediction performance and could be successfully applied to establish the estimating models that could provide accurate and reliable prediction. It also showed that the model M2 for trapezoidal side weir had the smallest value of the MNE as well as higher value of R and NS in the training as well as testing period, so, they were selected as the best fit models



for predicting the discharge coefficient in this study. To evaluate the accuracy and capability of the applied models in prediction of discharge coefficient for this side weir, a comparison between observed and predicted, for the best model is shown in Fig. 2.

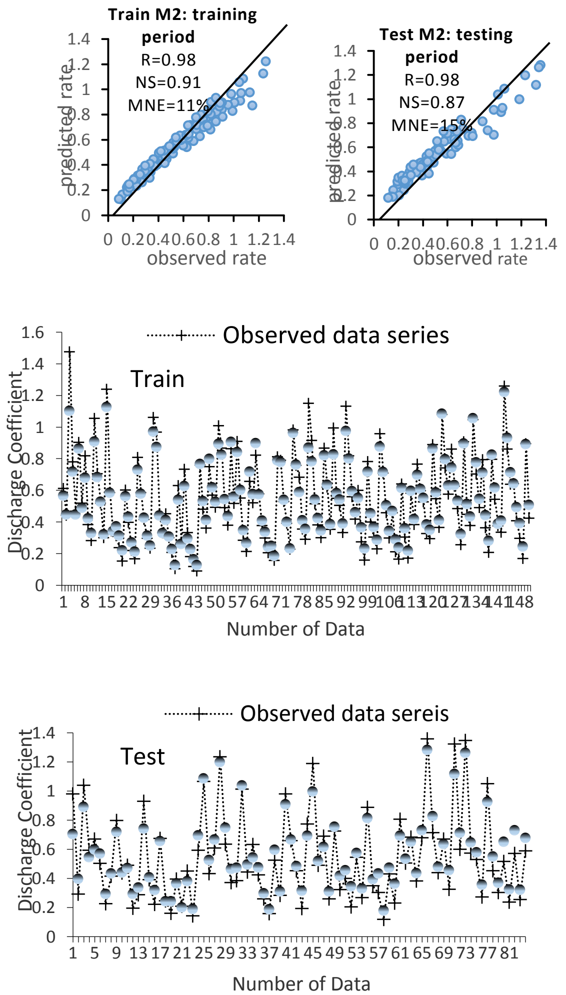


Fig. 2. Comparison of observed and predicted discharge coefficients for observed and predicted data using M2 model, (trapezoidal side weir).

For trapezoidal side weir, the best result, for GEP, is obtained from M2-F5 model performance. The model equations obtained for trapeziodal side weir is given below:

$$C_d = \sin\left[\left(\left(\frac{h_1}{p}-3.08\right)\times\left(-Fr_{wd}\right)\right)^3\right]+\sin\left(\frac{Fr_{wd}\times\sqrt{5.32}}{\frac{h_1}{p}}\right)+\sin\left(\cos\left(\sin\left(\frac{1.24}{\frac{L}{b}}\right)\right)\right)\times\sin\left(Fr_{wd}^{\ 2}\right)$$

The main reason of this complexity is due to nonlinear relation between flow characteristics, the geometry of the weir and discharge coefficient that none of these relationships are simplified in literature formulas.

Rectangular side weir results

In this part of results, eight different models have been established to predict the discharge coefficient of a rectangular side weir. In order to evaluate GEP models for the prediction of discharge coefficient, sensitivity analysis have been performed. The results indicated that the discharge coefficient of the rectangular side weir predicted by the above model is more related to upstream weir Froude number (Fr_{wu}), $\frac{p}{h_u}$ ratio and $\frac{L}{h_u}$ ratio compared to others. Among eight models, the best two models have been selected according to the ranking of performance criteria. Table 3 shows the performance of GEP models in the training and testing stages.

According to the results of the Table 3, GEP model can achieve a good performance prediction and they can be successfully utilized to predict different models.

Table 3: Statistics of the two best models of total data for training and testing periods for rectangular side weir

Machine Learning		Training			Testing		
GEP	M6	0.97	0.93	10	0.93	0.86	13
	M7	0.98	0.90	12	0.90	0.74	15

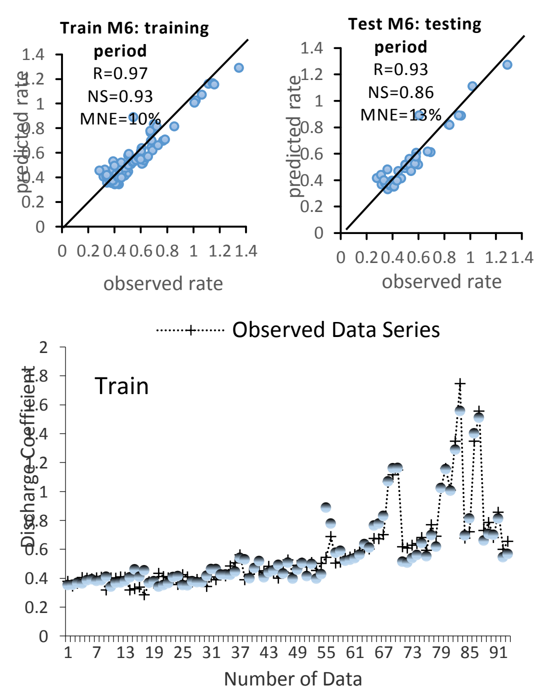


Fig. 3. Comparison of observed and predicted discharge coefficients for observed and predicted data using M6 model, (rectangular side weir).

This Table also indicates that M6 model has the least MNE as well as the most R and NS in training and testing periods. Thus, it has been chosen as the best model to predict the discharge coefficient of the rectangular side weir. In order to assess the accuracy and capability of the chosen models in the prediction of discharge coefficient, for this side weir, the comparison of observed and predicted data for the best model has been shown in Fig. 3.

The result of the best model for GEP is acquired from M6-F5 model. The model equations obtained for rectangular side weir is given below:

$$C_d = 0.449 \times \left(0.008\sqrt{\frac{L}{h_u}}\right)^{\frac{1}{6}} + sin^2\left\{\left(\frac{p}{h_u} \times Fr_{wu}\right)^3 \times \left(ln\left(\frac{L}{h_u}\right) + \frac{p}{h_u}\right)\right\} + \left\{\sin\left(Fr_{wu}\right) \times \left(\frac{p}{h_u}\right)^3\right\} \times \left\{\frac{\frac{p}{h_u}}{8.461} \times \left(\frac{L}{h_u} - \frac{p}{h_u}\right)\right\} \quad (11)$$

4. CONCLUSIONS

In this study, the accuracy of genetic expression programming (GEP) has been investigated for the estimation of discharge coefficient of trapezoidal and rectangular side weirs. According to results for trapezoidal side weir, M2 model comprising as input variables, is selected as best model. For rectangular side weir, M6 model comprising is ranked as the first-best model. These models show influence of downstream Froude number for trapezoidal side weir and upstream Froude number for rectangular side weir in prediction of the discharge coefficient for both of side weirs. The testing results obtained from GEP for M2 model of trapezoidal side weir are 0.98, 0.87 and 15% (for M6 model of the rectangular side weir are 0.93, 0.86 and 13%) that is related to R, NS and MNE respectively. The results confirm the capability and workability of GEP as an efficient machine learning approach in modeling of discharge coefficient. This research showed that GEP can be successfully applied to formulate the discharge coefficient of side weirs where

(i) The interrelationships among the relevant variables are poorly understood.

(ii)Finding the size and shape of the ultimate solution is difficult and a major part of the problem.

(iii)Conventional mathematical analysis methods do not (or cannot) provide analytical solutions.

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