THE INFLUENCE OF RADIATION ON MHD SLIP FLOW AND HEAT TRANSFER OF CASSON FLUIDS DUE T0 SHRINKING SHEET IMMERSED IN POROUS MEDIUM

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ABSTRACT: *Theoretical analysis for MHD slip flow and heat transfer of Casson fluid is considered due shrinking sheet lying in porous medium. The suitable similarity transforms have been employed to convert the governing equations to ordinary differential form. The resulting equations are solved numerically to compute the effects of emerging parameters namely Casson parameter β , slip parameter δ , Prandtl number P_r , magnetic parameter M, Eckert number. E_c , shrinking parameter, porosity parameter K, the results for flow velocity and temperature distribution are presented in graphical form.*

Keywords: Casson fluid, slip flow, shrinking sheet, similarity transforms, Eckert number

1. INTRODUCTION

Non Newtonian fluids display useful applications in many industrial processes. For these fluids, the relationship between the shear stress and the shear rate is different. Non-Newtonian fluids have applications in polymer engineering, in petroleum drilling, manufacturing of foods, certain separation processes and paper and some other industrial processes [1-2]. There are various models for formulation of such fluids.In 1959, Casson proposed his model for the flow of special type of Non Newtonian fluids named as Casson fluids. For the characterization of cement slurry, this model is used by petroleum engineers. The Casson model is more accurate at both very low and very high shear rates. Casson fluid can be perceived as a shear thinning liquid with zero viscosity at an infinite rate of shear, with infinite viscosity at zero rate of shear and a yield stress below which no flow occurs [3]. The nonlinear Casson constitutive equation describe accurately the flow curves of pigments suspensions in the lithographic varnishes [4] and for suspension silicon [5]. As given by Casson, the properties of many polymers over a wide range of shear rates are described by shear stress-shear rate relation [6]. Mustafa et al. [7] studied the unsteady boundary layer flow and of Casson fluid over a moving flat plate. Eldabe and Salwa [8] analyzed MHD flow and heat transfer of non-Newtonian Casson fluid flow between two rotating cylinders. .Hayat et al. [9] examined the cross diffusion effects on MHD Casson fluid flow. Khalid et al. [10] discussed an unsteady free convection MHD flow of Casson fluid through an oscillating vertical plate in a porous medium. Raju *et al*. [11] examined flow and heat transfer of MHD Casson fluids past an exponentially permeable stretching sheet. Sulochana *et al.* [12] considered non- linear thermal radiation on MHD 3-D Casson fluid flow. Nadeem et al. [13] discussed the magnetohydrodynamic flow of a Casson fluid over an exponentially penetrable shrinking sheet. Nadeem et.al. [14] investigated MHD 3- dimensional Casson fluid flow over a permeable linearly stretching sheet. Babu *et al*. [15] studied numerically, the effects of radiation and heat source/sink on the steady 2- dimensional flow of heat and mass transfer past a shrinking sheet..Kishore *et al.* [16] presented the hydromagnetic flow in a porous medium with radiation, variable heat and mass diffusion. Poornima et al. [17] studied convective flow of a radiating nanofluid past a non-linear boundary moving sheet. Sharma et al. [18] considered the heat transfer due to thermal radiation over an exponentially shrinking sheet. Ramana Reddy et al. [19] discussed the effect of thermal diffusion on unsteady MHD dusty fluid flow. Sandeep et al. [20] considered the unsteady MHD free convection flow past a suddenly moving vertical plate involving radiation and rotation effects.

In this work, we are intended to present MHD slip flow and heat transfer of Casson fluid past a shrinking sheet lying in porous medium. Computational analysis of the problem is carried out for wide ranges of the parameters of interest to explore the physical aspects of the study.

2. MATHEMATICAL MODEL:

The slip flow of electrically conducting, incompressible Casson fluid is considered owing to horizontal shrinking sheet that is immersed in a porous medium. The flow is steady and two-dimensional. An external magnetic field of strength H_o is applied normed to the plane $(b>0)$ of flow. The induced magnetic flied is neglected. The fluid temperature is T , T_w is temperature shrinking sheet and T_∞ is ambient fluid temperature. The flow velocity ha components u , v along *x*-axis and *y*-axis respectively. The velocity of external flow is *U* and $U_W(x)$ is velocity of the shrinking surface.

The rheological stress tensor of state for an isotropic and incompressible flow of Casson fluids as given by

$$\tau_{ij}=\begin{cases}2\left(\dfrac{\mu_B+p_y}{\sqrt{2\pi}}\right)e_{ij},\pi>\pi_c\\ 2\left(\dfrac{\mu_B+p_y}{\sqrt{2\pi_c}}\right)e_{ij},\pi<\pi_c\end{cases}$$

Here $\pi=e_{ij}e_{ij}$ and e_{ij} are the $(i,j)^{th}$ component of the deformation rate, π_c is a critical value, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, and p_y is the yield stress of the fluid.

Under the above assumptions the equations governing the problems are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad , \qquad (1)$$

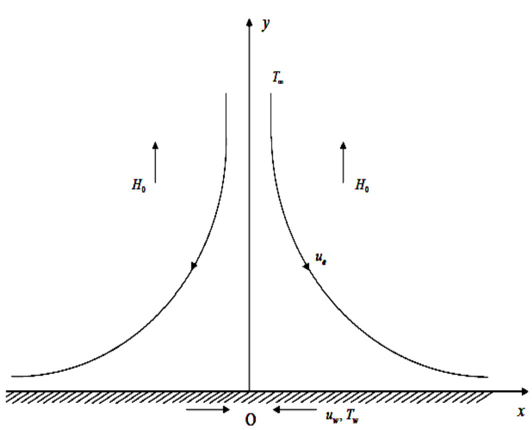
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+(1+\frac{1}{\beta})\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_e\mu_e{}^2H_o{}^2u}{\rho}-\frac{\upsilon}{K}\left(1+\frac{1}{\beta}\right)u \qquad (2)$$

$$\rho C_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=k\frac{\partial^2 T}{\partial y^2}+\mu(\frac{\partial u}{\partial y})^2+\sigma_e\mu_e{}^2H_o{}^2u^2+\frac{16\alpha}{3\beta}\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

where ρ is the density, C_p the specific heat at constant pressure, υ is the coefficient of kinematic viscosity, σ_e the electrical conductivity, μ_e the magnetic permeability, k the thermal conductivity and μ the coefficient of viscosity. The other symbols have their usual meanings, $\beta=\frac{\mu_B\sqrt{2\pi_c}}{p_y}$ is the Casson parameter.

Figure 1. Physical model of two-dimensional stagnation point flow past a shrinking sheet.

The boundary conditions are



$$y=0: u=u_w=cx+L(\frac{\partial u}{\partial y}), v=0; T=T_w \qquad (4)$$

$$y=\infty: u=u_e(x)=ax; T=T_\infty$$

Where c is a proportionality constant of the velocity of shrinking sheet, L is a slip length and a is a constant proportional to the free stream velocity away from the sheet.

3. ANALYSIS OF THE VELOCITY AND THE THERMAL BOUNDARY LAYERS

The continuity Equation (1) is identically satisfied by stream function $\Psi(x,y)$, defined as

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \qquad (5)$$

For the solution of the momentum and the energy equations (2) and (3), the following dimensionless variables are defined:

$$\Psi(x,y)=\sqrt{a\upsilon}xf(\eta),\eta=\sqrt{\frac{a}{\upsilon}}y,\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \qquad (6)$$

quation (6), transform equations (2) and (3) into

$$f'''+ff''-f'^2-Mf'+1=0 \qquad (7)$$

$$\theta''+P_rf\theta'+P_rE_cf''^2+P_rE_cHf'^2=0 \qquad (8)$$

where a prime (') denotes differentiation with respect to η, $M=\mu_eH_o\sqrt{\frac{\sigma_e}{\rho a}}$ is the Magnetic parameter, $P_r=\frac{\mu C_p}{k}$ is the Prandtl number and $E_c=\frac{U^2}{C_p(T_w-T_\infty)}$ is the Eckert number.

The corresponding boundary conditions are:

$$\eta=0: f=f_0, f'=\frac{c}{a}+\delta f'', \theta=1 \qquad (9)$$

$$\eta=\infty: f'=1, \theta=0$$

where $\frac{c}{a}$ is the velocity ratio parameter and $\delta=L(\frac{a}{\upsilon})^{\frac{1}{2}}$ is the slip parameter.

4. RESULTS AND DISCUSSION:

In order to have a physical insight of the study, the finally governing non linear equations (7) and (8) along with boundary conditions in eq. (9) are solved numerically. The derivatives of higher order involved in these equations are reduced to first order. The system of above mentioned equation take the form.

$$p=f',\ q=f'',\ q'=Mp+p^2-fq-1$$

$$g=\theta',\ g'=-P_r(fg+E_cq'^2+E_cHp^2)$$

And the boundary conditions become:

$$P(0)=\frac{c}{a}+\delta q, g(0)=1$$

$$P(\infty)=1, \theta(\infty)=0$$

This first order system of differential equations is then solved by coding in computational software Mathematica software 11.

Fig 2. Illustrates the effect of magnetic parameter M on horizontal velocity component f'. The velocity increases in magnitude with increase in magnetic flied strength. Several computational have been made for sufficient range of the emerging parameter. The results for representative values of these parameters are displayed in form of plots for velocity and temperature function. It is because the viscous effects suppress the flow and the Lorentz force pushes the speed up.

The sheet shrinking parameter $\frac{c}{a}$ causes increase in the boundary layer thickness as shown in fig. 3.

The result correspond with no slip conditions. Similarly the porosity of medium also shows increasing effect on fluid flow as depicted in fig. 4.

But increase in the values of casson parameter and slip parameter causes decrease in flow speed as shown respectively in fig. 5 and fig. 6.

Fig. 7 demonstrates the effect of magnetic field on temperature distribution $\theta(\eta)$. It is noticed that temperature curve rises with increase in magnetic field strength.

The thermal boundary layer thickness decreases with increase in Prandtl number as illustrated in fig. 8. It is because increase in Prandtl number decreases thermal conductivity. But increase in shrinking parameter causes an increase in temperature function $\theta(\eta)$ as show in fig. 9. Similarly Fig 10 shows that the radiation on parameter causes a significant decrease in temperature field. But Eckert number causes increase in temperature function as demonstrated in fig 11.

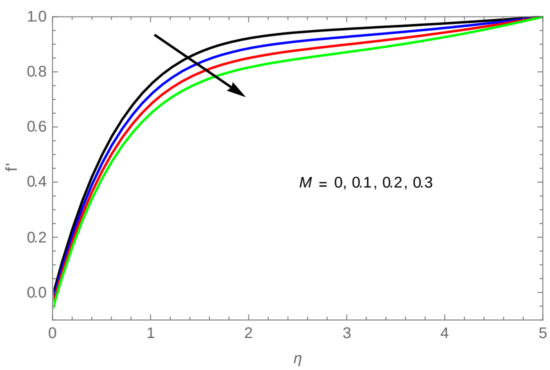


Fig.2: The plot for curves of f' under the effect of magnetic parameter M when

$$f_0 = 1,\ K = 0.1,\ \frac{c}{a} = -0.7, \delta = 0.5,\ \beta = 1,$$

P_r=0.7, E_c=0.1 and R_n=0.1.

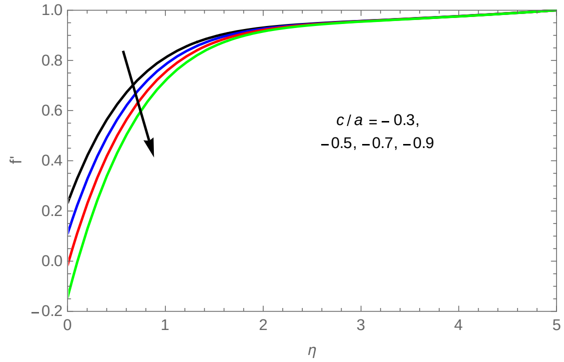


Fig.3: The plot for curves of f' under the effect of velocity ratio parameter $\frac{c}{a}$ when f_0=1, M=0.1, $\delta = 0.5$, $\beta = 1$, P_r=0.7, E_c=0.1 and R_n=0.1.

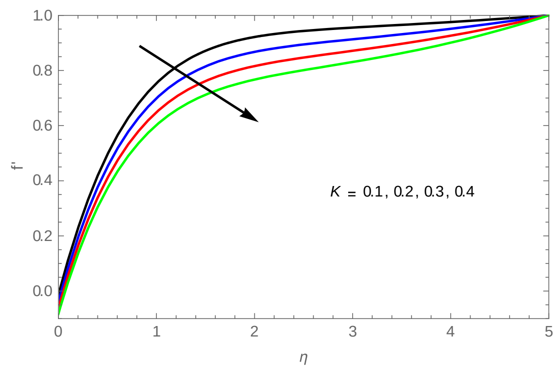


Fig.4: The plot for curves of f' under the effect of porosity parameter K when M=0.1, $\frac{c}{a} = -0.7, \delta = 0.5$, $\beta = 1$, P_r=0.7, E_c=0.1 and R_n=0.1.

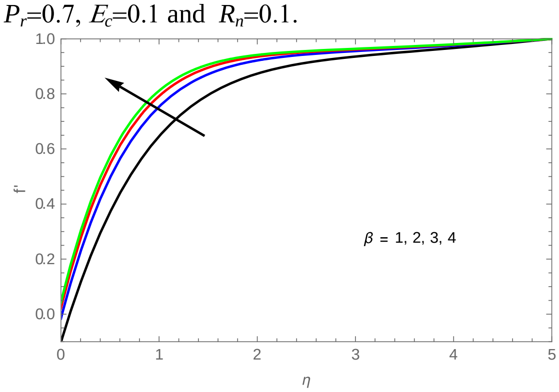


Fig.5: The plot for curves of f' under the effect of parameter β when

M=0.1, $\frac{c}{a} = -0.7, \delta = 0.5$ P_r=0.7, E_c=0.1 and R_n=0.1.

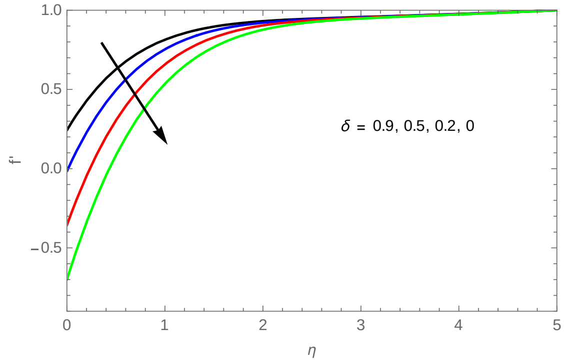


Fig.6: The plot for curves of f' under the effect of slip parameter δ when

M=0.1, $\frac{c}{a}=-0.7, \beta=1$ P_r=0.7, E_c=0.1 and R_n=0.1

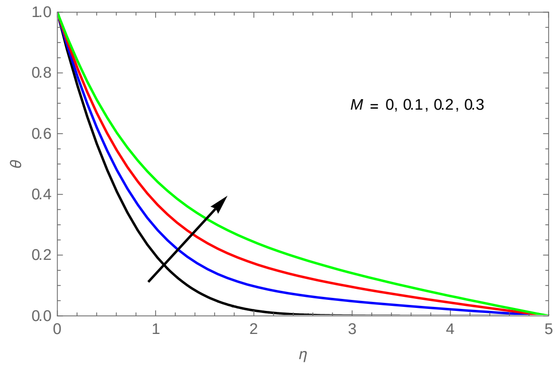


Fig.7: The plot for curves of θ under the effect of magnetic parameter M

when $\frac{c}{a}=-0.7, \delta=0.5, \beta=1$, E_c=0.1,P_r=0.7 and R_n=0.1.

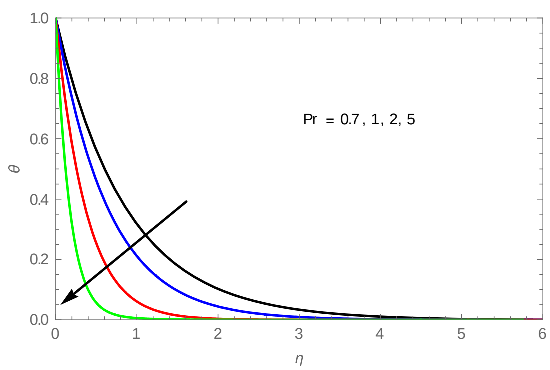


Fig.8: The plot for curves of θ under the effect of Prandtl number P_r when

M=0.1, $\frac{c}{a}=-0.7, \delta=0.5\ \beta=1$, K=0.1, E_c=0.1 and R_n=0.1.

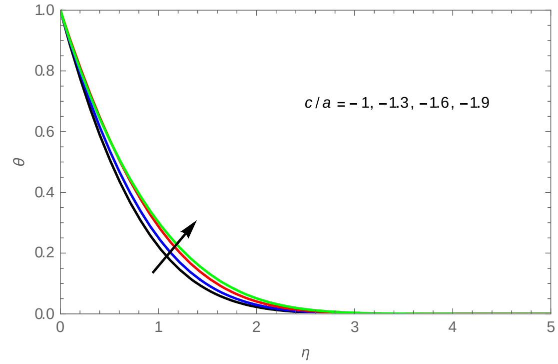


Fig.9: The plot for curves of θ under the effect velocity ratio parameter $\frac{c}{a}$ when M=0.1, $\frac{c}{a}=-0.7, \delta=0.5$, P_r=0.7 and R_n=0.1.

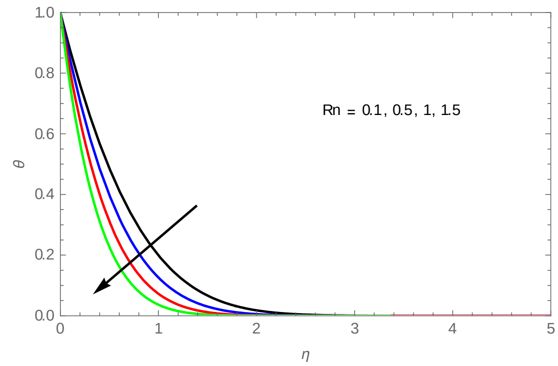


Fig.10: The plot for curves of θ under the effect of radiation parameter R_n

when f_0=1,M=0.1, $\frac{c}{a}=-0.7, \delta=0.5, \beta=1$, E_c=0.1 and P_r=0.7

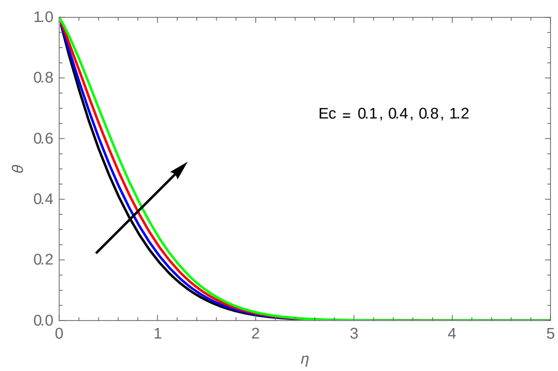


Fig.11: The plot for curves of θ under the effect of Eckert number E_c

when f_0=1,M=0.1, $\frac{c}{a}=-0.7, \delta=0.5\ \beta=1$, P_r=0.7 and R_n=0.1.

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