CONTROLLER DESIGN FOR PERFORMANCE ANALYSIS AND OPTIMIZATION OF TWIN ROTOR SYSTEM

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ABSTRACT—*In present work, twin rotor system (TRS) Abstract: In present work, twin rotot system (TRS) performance optimization problem is considered. Practical TRS exhibit undesired oscillatory behavior when it is commanded to attain a certain angular position along elevation or azimuth axis. Moreover, coupling between elevation and azimuth dynamics degrade its performance. Therefore in this paper, control techniques are designed to optimize TRS performance. First, the development of TRS model, its linearization process and analysis in open loop structure is presented, then control design techniques are discussed. For controller design, at first, proportional-integral-derivative (PID) control design using Ziegler-Nicholous method is presented. Effect of application of proportional gain, PI gains and PID gains is analysed. Tuning and application of three PID gains produce better results. Further optimization in performance is done using loop transfer recovery (LTR) controller that use Kalman filter in combination with control law design block to minimize error cost function. Results for LTR application on TRS are compared. Finally, for optimal performance enhancement linear quadratic gaussian (LQG) controller is designed that is tuned by adjusting the process and output noise covariances. LQG performs well to meet required time and frequency domain specifications. At the end, comparative performance analysis of all the designed controllers is done.*

NTRODUCTION

Mathematical model of a dynamical system facilitates system analyst or designer to optimize system performance to fulfill required specifications. In order to meet specs, different control techniques are devised that approach system behavior with different aspects, e.g. deterministic controller design involves tuning of control parameters by assuming the system behavior deterministic and certain, while stochastic controllers (e.g. robust, adaptive, optimal etc.) also map system uncertainties and random behavior. In both areas of controller design, many schemes for design optimization has been proposed for twin rotor system (TRS). Developing controller for this type of system is challenging due to the coupling effects between two axes and also due to its highly nonlinear characteristics, e.g. H2 and H∞ norm based optimal control has been designed in [1] for multi input multi output MIMO TRS having cross coupling between it's elevation and azimuth outputs. Also in [2] sliding surfaces based controller has been proposed to overcome the cross coupling and disturbance created during TRS movement. In [3] nonlinear inside circle is utilized to control the electrical flow of the stage, and the nonlinear outside circle permits the stage to be splendidly balanced out and situated in space. Hypothetical controller plan improvement with an arrangement of investigations is done keeping in mind the end goal to check the adequacy of the proposed nonlinear decentralized criticism controller. In [4] authors propose the control objective is to make the beam of the TRMS move quickly and accurately to the desired positions e.g. the pitch and the travel angles. They propose further exact element models of the framework for both vertical and even developments are produced. Elsewhere [5], cross coupling and disturbance in MIMO systems that interfere the performance of TRS are catered by introducing decoupling procedures like sliding and integral mode controls. In another investigations [6], another control technique which uses particle swarm optimization (PSO) for disconnected tuning of corresponding basic subordinate (PID) controller for the twin rotor multi-input multi-yield framework (TRS). The central target is to make TRS performance, better rapidly and precisely using PSO. In [7] this system include two inputs, two Outputs nonlineor system, having a solid cross coupling between its inputs and the outputs. The task is to control their position (the azimuth point and its elevator). Nonlinear conditions of azimuth and rise points are inferred in state space utilizing Euler-Lagrange conditions. The model is then linearized and controller is connected to control both positions simultanously. In [8] demonstrating of complex air vehicles is a testing undertaking because of high nonlinear conduct and critical coupling impact between rotors.Twin rotor multi-input multioutput framework (TRS) is a research facility setup intended for control tests, which takes after a helicopter with temperamental, nonlinear, and coupled progression. This paper concentrates on the configuration and examination of sliding mode control (SMC) and backstepping controller for pitch and yaw angle control of main and tail rotor of theTRS under parametric vulnerability. The proposed control methodology with SMC and backstepping accomplishes all specified impediments of TRS. In [10] presents the quantitative criticism hypothesis (QFT) based control, input unsettling influence dismissal and following of a twin rotor framework. Twin rotor framework is a class of various information various yield (MIMO) framework having complex nonlinearity.QFT is a strong recurrence space method in view of Nichols outline. This method accomplishes wanted hearty outline over a determined scope of plant vulnerability. QFT is utilized for the outline of hearty controllers for the plants with vulnerability in the parameters, information and yield unsettling influences.In [11] LQG controller for TRMS without and with sensor, actuator disappointment is composed. Execution of LQG controller for TRS is done under no disappointment of sensor, actuator. TRS yield with LQG controller and solid H infinity controller are contrasted without and sensor, actuator disappointment. The target of the proposed procedure is to demonstrate the H vastness controller is solid over LQG controller for TRS with sensor, actuator disappointment which is approved.

In the present work, performance optimization problem for MIMO TRS has been considered. For optimization PID, LQR

And LTR techniques have been used. Zeigler Nicolus method for tuning of P, I and D parameters of PID controller is used and effect of using these gains individually and collectively on the performance of TRS is analysed. Furthermore, linear quadratic regulator (LQR) controller is designed and optimization of process and output covariance matrices is done in order to get better TRS performance. Moreover a linear transfer recovery (LTR) optimal control is designed that uses Kalman filter (KF) as a state estimator for estimation of non-measureable states. The comparative performance analysis of all the designed controllers is done and finally the discussion and conclusion is presented. Paper is organized as follows: section II contains discussion on TRS Model, it's linearization and analysis in time and frequency domain that lead to the problem formulation. Section III presents the main results regarding PID, LQR and LTR controllers design and their comparative performance analysis and discussion.

I. TWIN ROTOR SYSTEM

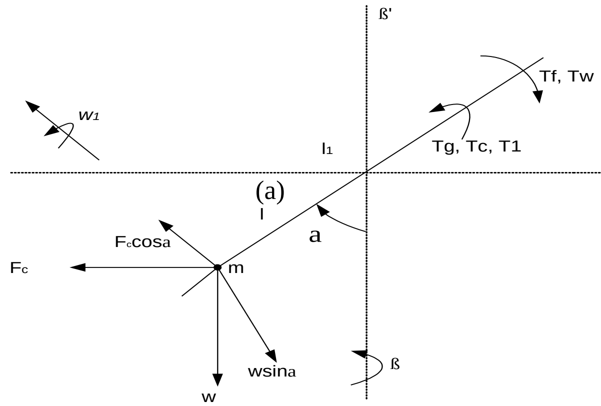
a) Modeling of TRS

The twin rotor system is a multidimensional naturally unstable system with two controlled inputs and two outputs. The shape of the CE-150 lab system is shown in Figure 2.1. The model of TRS is built by collecting the components of different torques acting on elevation, azimuth, and motor/ rotor dynamics. At the end sensor dynamics are modeled. The elevation dynamics are balanced by five torques namely Gravitational (τ_G), Gyroscopic (τ_{gy}), Centrifugal (τ_C), frictional (τ_f) and main motor (τ_1) torque. These torques are balanced for elevation dynamics by orientation shown by the free body diagram of Figure 2.2 (a) and mathematically stated in Eq (2.1).

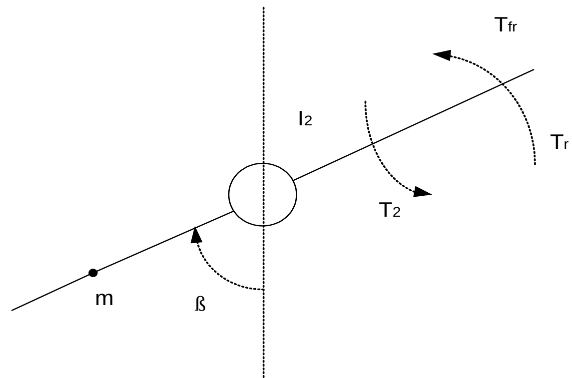
Fig 2.1: CE-150 Twin Rotor System

$$I_1 \ddot{\alpha} = \tau_1 + \tau_c + \tau_{gy} - \tau_G - \tau_f \qquad (2.1)$$

where $I_1 = \text{moment of inertia of the helicopter body around horizontal axis (kg.m}^2)$



(a)



(b)

Fig 2.2 (a) FBD for elevation dynamics modeling
(b) FBD for Azimuth dynamics modeling

The azimuth dynamics are modeled by balancing the side motor (τ_2), frictional (τ_{f2}) and main motor reaction (τ_r) torques shown in Figure 2.2 (b) and related in Eq (2.2).

$$I_2 \ddot{\beta} = \tau_2 - \tau_r - \tau_{f2} \qquad (2.2)$$

where

$$I_2 = \text{Moment of Inertia in vertical plane (Kg.m}^2)$$

The motor dynamics are approximated by first order transfer function of Eq (2.3) and motor/ rotor torques are balanced by Eq (2.4).

$$M_1 = \frac{1}{T_1 s + 1} \qquad (2.3)$$

where $T_1 = \text{main motor time constant}$

$$\tau_1 = a^2 u_1 + b u_1 \qquad (2.4)$$

$u_1 = \text{output of the motor}$

and finally the sensor dynamics are modeled by linear relationship of Eq (2.5)

$$y_\alpha = k_\alpha \alpha - y_{\alpha_0}$$

$$y_\beta = k_\beta \beta \qquad (2.5)$$

The dynamic equations are assigned state variables. After manipulation the mathematical model is built that is stated in Eq (2.6).

$$\left.\begin{aligned}
x_1' &= \frac{1}{T_1}(-x_1 + u_1)\\
x_2' &= x_3\\
x_3' &= \frac{1}{I_1}((a_1x_1)^2) + b_1x_1 - B_1x_3 - T_g \sin x_2 - K_{gyro}u_1x_6\cos x_2)\\
x_4' &= \frac{1}{T_2}(-x_6 + u_2)\\
x_5' &= x_6\\
x_6' &= \frac{1}{I_2}((a_2x_4)^2) + b_2x_4 - B_2x_6 - T_{pr\,x_7} - K_rT_{or}u_1)\\
x_7' &= -T_{pr}x_7 + K_rT_{or}u_1\\
y &= \begin{bmatrix} y_1\\ y_2\end{bmatrix} = \begin{bmatrix} x_2\\ x_5\end{bmatrix}
\end{aligned}\right] \qquad (2.6)$$

Where $[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7]^T = [\omega_1\ \alpha\ \dot{\alpha}\ \ \omega_2\ \beta\ \dot{\beta}\ \ x_7]$ are the main motor speed, elevation angle, angular speed in elevation, side motor speed, azimuth angle, angular speed in azimuth and angular moment caused by u_1 on azimuth respectively. The system parameters with description are tabulated in Table 2.1.

Table 2.1: TRS Parameters Description
Time constant of main roror, T1 = 0.3 s
Vertical torque, a1 = 0.105 N.m/MU
Horizontal axes Moment of friction, b1=0.00936 N.m/MU2
Vertical axes Moment of Inertia (MOI), I1 = 4.37e-3 Kg.m2
Vertical axes Rate of MOI , B1 = 1.84e-3 Kg.m2/s
Gravitational torque, Tg = 3.83e-2 N.m
Time constant of side motor, T2 = 0.25 s
Horizontal torque, a2 = 0.033 N.m/MU
Moment of friction in vertical axes, b2 = 0.0294 N.m/MU2
Coefficient of cross coupling, Tor = 2.7 s
Coupling coefficient, Tpr = 0.75 s
Nonlinear reaction torque (turning), Kr = 0.00162 N.m/MU
MOI in horizontal axes, I2 = 4.14e-3 Kg.m2
Rate of MOI in horizontal axes, B2 = 8.69e-3 Kg.m2/s
Momentum of gyroscope, Kgyro = 0.015 Kg.m/s

b) Linearization of TRS Model

For controller design we need to linearize the TRS model of (2.6) that need to first compute the Jacobean matrices given by:

$$A_{[i,j]} = \frac{\partial f_{[i]}(\hat{x}_{k-1},\ u_{k-1}, 0)}{\partial x_{[j]}}$$

$$B_{[i,j]} = \frac{\partial f(\hat{x}_{k-1}, u_{k-1}, 0)}{\partial u_{[j]}}$$

$$W_{[i,j]} = \frac{\partial f_{[i]}(\hat{x}_{k-1},\ u_{k-1}, 0)}{\partial w_{[j]}}$$

$$H_{[i,j]} = \frac{\partial h_{[i]}(\hat{x}_k, 0)}{\partial x_{[j]}}$$

$$V_{[i,j]} = \frac{\partial h_{[i]}(\hat{x}_k, 0)}{\partial v_{[j]}}$$

and then these matrices are, evaluated at Linearization point (x_k, u_k) to get the linear form of nonlinear model.

The Jacobean matrices for TRS model (2.6) becomes as under

$$A = \begin{bmatrix}
-\frac{1}{T_1} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0\\
\frac{(b_1+2a_1x_1)}{I_1} & \frac{-T_g}{I_1} & \frac{-B_1}{I_1} & 0 & 0 & -K_g u_1 \cos x_2 & 0\\
0 & 0 & 0 & 0 & \frac{-1}{T_2} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & \frac{(b_2+2a_2x_4)}{I_2} & 0 & \frac{-B_2}{I_2} & \frac{T_{pr}}{I_2}\\
0 & 0 & 0 & 0 & 0 & 0 & -T_{pr}
\end{bmatrix}$$

$$B = \begin{bmatrix}
\frac{1}{T_1} & 0\\
0 & 0\\
-K_g x_6 \cos x_2 & 0\\
0 & \frac{1}{T_2}\\
0 & 0\\
-K_rT_{or} & 0\\
K_rT_{or} & 0
\end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0\end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0\\ 0 & 0\end{bmatrix}$$

and invoking the equilibrium point **x**=0, **u**=0 in above matrices gives following required linear TRS model

$$A = \begin{bmatrix}
-\frac{1}{T_1} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0\\
\frac{b_1}{I_1} & \frac{-T_g}{I_1} & \frac{B_1}{I_1} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \frac{-1}{T_2} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & \frac{b_2}{I_2} & 0 & \frac{-B_2}{I_2} & \frac{T_{pr}}{I_2}\\
0 & 0 & 0 & 0 & 0 & 0 & -T_{pr}
\end{bmatrix}$$

$$B = \begin{bmatrix}
\frac{1}{T_1} & 0\\
0 & 0\\
0 & 0\\
0 & \frac{1}{T_2}\\
0 & 0\\
-K_rT_{or} & 0\\
K_rT_{or} & 0
\end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(2.7)

a) Analysis of TRS Model

The linearized TRS model (2.7) is analysed in time and frequency domains in order to judge TRS performance in open loop. Figure 2.3 shows the impulse response of TRS system while Figure 2.4 shows its bode magnitude plot.

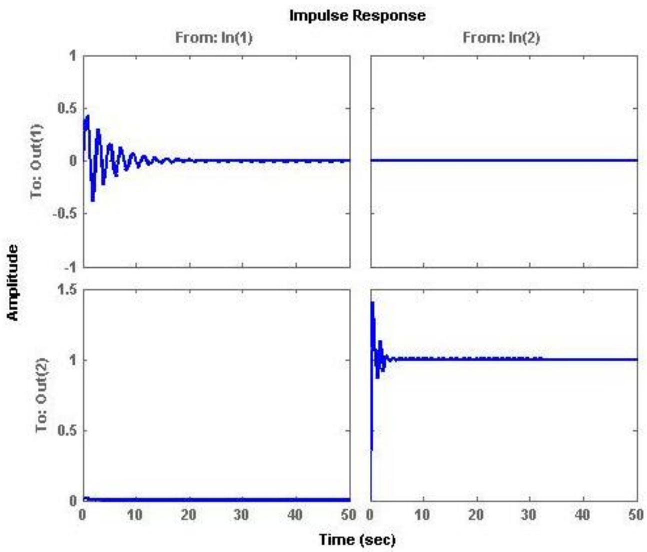


Fig 2.3: Impulse response of linearized TRS system

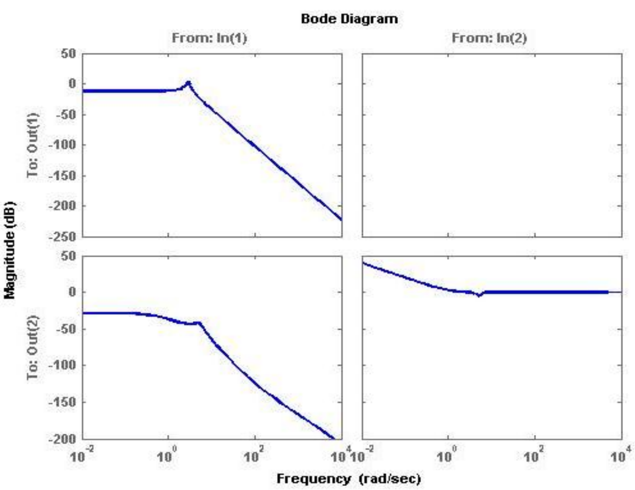


Fig 2.4:Bode plot of Linearized TRS System

In Figure 2.3, note that the elevation and azimuth positions enter steady state after taking many oscillations and large overshoots are observed, moreover settling time of the model to desired position very large that is not acceptable in a practical scenario. The response depicts that the model is expected to behave roughly in practice, therefore, its response times and overshoot need to be controlled. Figure 2.4 shows that the helicopter dynamics in elevation have poor gain margins, however, these margins are fine in azimuth dynamics. The effect of the 1st input on azimuth is significant at lower frequencies, but the elevation has very less impact of 2nd input. The bode plots give us the idea of coupling in helicopter dynamics. Based on the prior discussion, to optimize the TRS performance controller design problem for this model is considered in section III.

II. MAIN RESULTS

a) PID Controller Design

PID controller changes distinctive execution parameters like greatest overshoot, settling time and so on by tuning its corresponding, essential and subordinate increases. Relative pick up (kp) will have the impact of diminishing the ascent time and will decrease, however never wipe out the relentless state mistake. A fundamental control (ki) will have the impact of wiping out the enduring state blunder for a consistent or step input, yet it might make the transient reaction slower. A subordinate control (kd) will have the impact of expanding the soundness of the framework, diminishing the overshoot, and enhancing the transient reaction.

The impacts of increment in each of controller parameters and on a shut circle framework are condensed in the Table 3.1.

Gain	**T_r**	**OS**	**T_s**	**E_ss**
K_p	Dec	Inc	SC	Dec
K_i	Dec	Inc	Inc	0
K_d	SC	Dec	Dec	Inc

Table 3.1: Variation in response times as gains are increased (T_r =Rising time, OS=maximum overshoot, T_s =settling time, E_ss= steady state error)

The PID gains are computed using Ziegler-Nicholous criterion. Application of proportional gain on TRS model (2.7) yield the impulse and bode response depicted in Figure 3.1 and Figure 3.2, while response (in time and frequency domain) using PI and PID gains after tuning of parameters using the Z-N method is shown in Figure 3.3- Figure 3.6 respectively.

For further enhancement of performance, we apply LTR controller to this system.

b) LTR Controller Design

In contemporary control innovation, LTR techniques are roused particularly by the ideals of the likelihood of finish decoupling in the plan procedure between the outline of attractive circle shapes from self-assertive criteria and the plan of an implementable dynamical controller with determined execution and vigor properties in various recurrence districts. This takes into consideration blends of outline criteria that can't be taken care of by different methodologies. Besides, LTR is applicable by the goodness of the capacity of assigning particular observer and/or controller structures, for example, low request ones, to the subsequent shut circle framework.

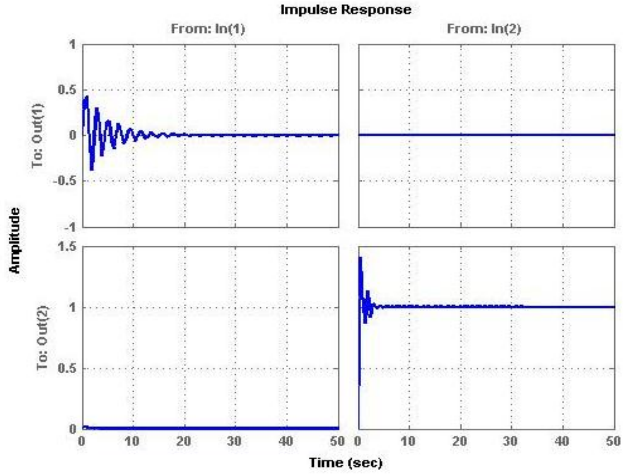


Fig 3.1:Impulse response with proportional controller

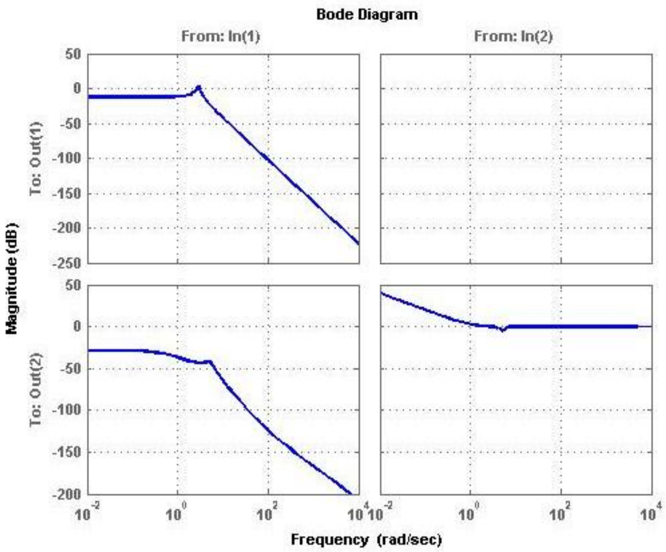


Fig 3.2:Bode Plot with proportional controller

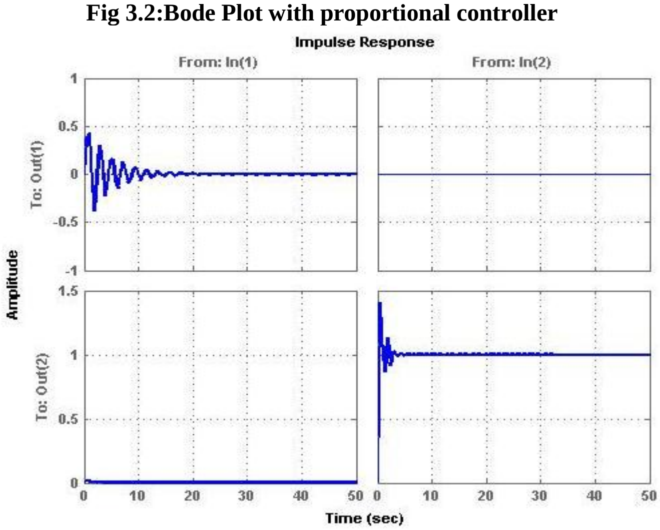


Fig 3.3:Impulse response of PI controller

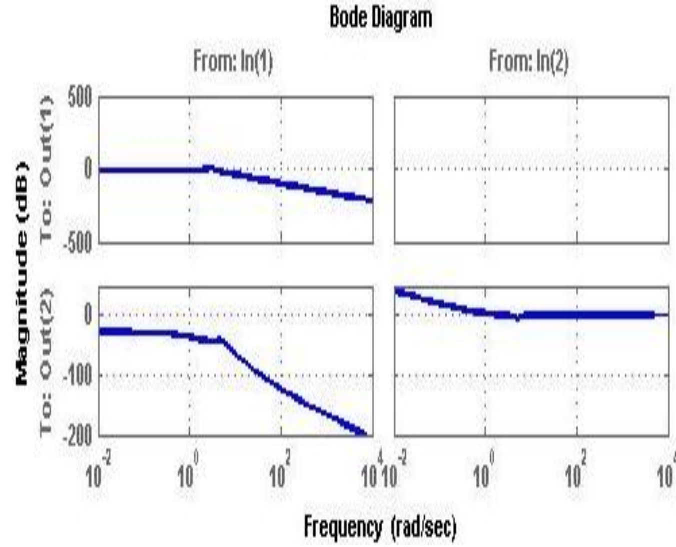


Fig 3.4:Bode Plot of PI controller

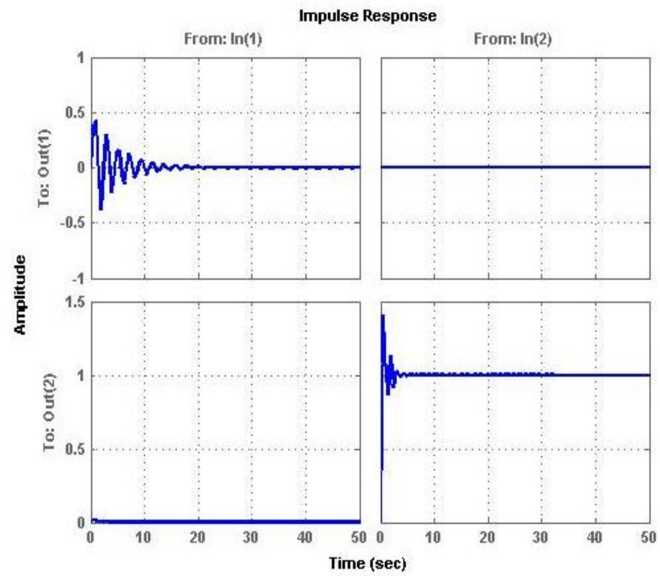


Fig 3.5:Impulse response of PID controller

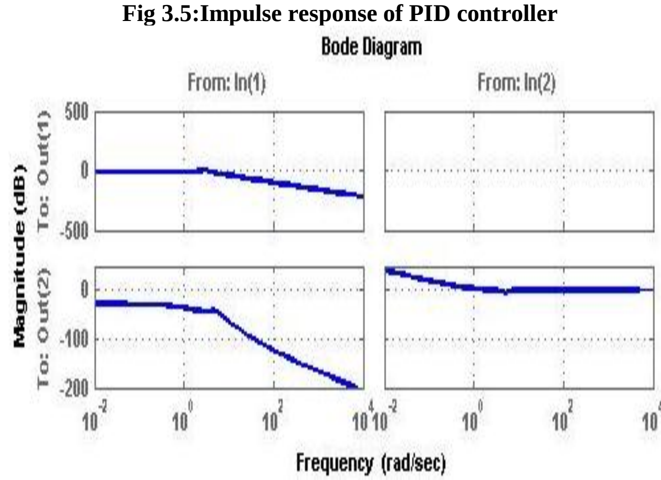


Fig 3.6:Bode Plot of PID controller

This is rather than most other optimization based techniques where particular structures cannot be allotted or can be composed just through exceptionally dull methods. In the same way, control vitality ought to likewise be incorporated into target capacity to minimize the control vitality of the framework. Figure 3.7 demonstrates the piece chart of plant alongside Linear Quadratic Regulator (LQR). Here yield of plant is controlled by fluctuating the pick up K of Linear Quadratic Regulator. The working instrument of state estimator is delineated in Figure 3.7.

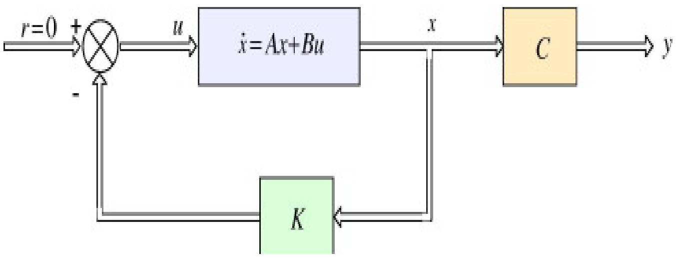


Fig 3.7: Working of Kalman state estimator

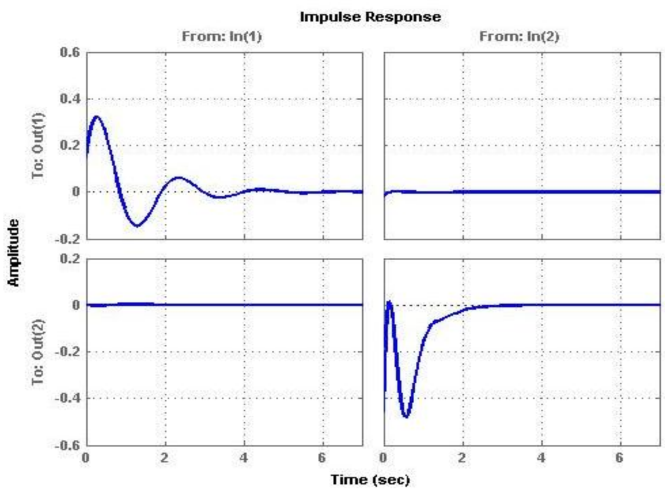


Fig 3.8:Impulse response of LTR

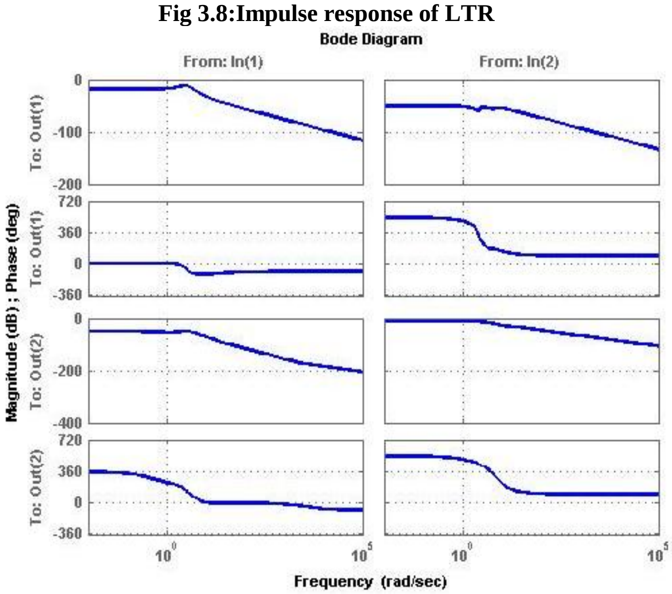


Fig 3.9:Bode Plot of LTR

As compared to Figure 3.5 and Figure 3.6, Figure 3.8 and Figure 3.9 shows that the LTR controller performs better. For further performance optimization, LQR controller is designed.

c) LQR Controller Design

The settings of a (managing) controller administering either a machine or process (like a plane or substance reactor) are found by utilizing a numerical calculation that minimizes a cost work with weighting components provided by a human (designer). The cost capacity is regularly characterized as a whole of the deviations of key estimations, sought height or process temperature, from their craved qualities. The calculation consequently finds those controller settings that minimize undesired deviations. The extent of the control activity itself may likewise be incorporated into the cost work.

The LQR calculation diminishes the measure of work done by the control frameworks architect to upgrade the controller. In any case, the architect still needs to indicate the cost work parameters, and contrast the outcomes and the predetermined plan objectives. Regularly this implies controller development will be an iterative procedure in which the specialist judges the "ideal" controllers created through reproduction and afterward conforms the parameters to deliver a controller more predictable with outline objectives. The LQR calculation is basically a mechanized method for finding a fitting state-input controller. Accordingly, it is normal for control specialists to favor elective techniques, similar to the full state input, otherwise called shaft situation, in which there is a clearer relationship between controller parameters and controller conduct. Trouble in finding the right weighting components restrains the utilization of the LQR based controller combination.

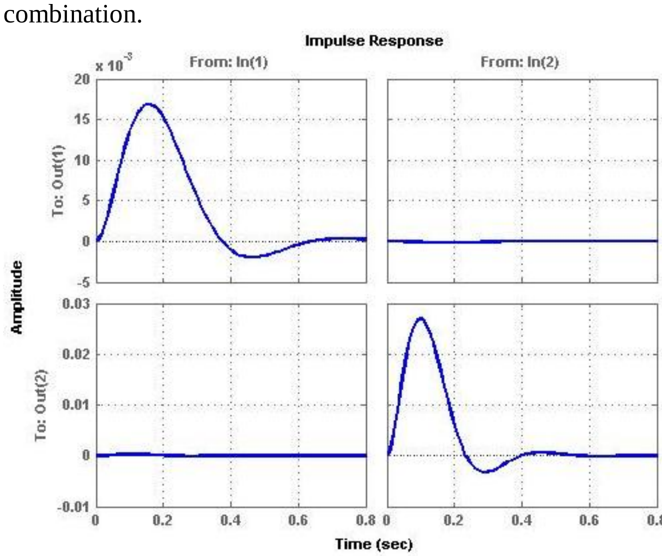


Fig 3.10:Impulse response using LQR controller

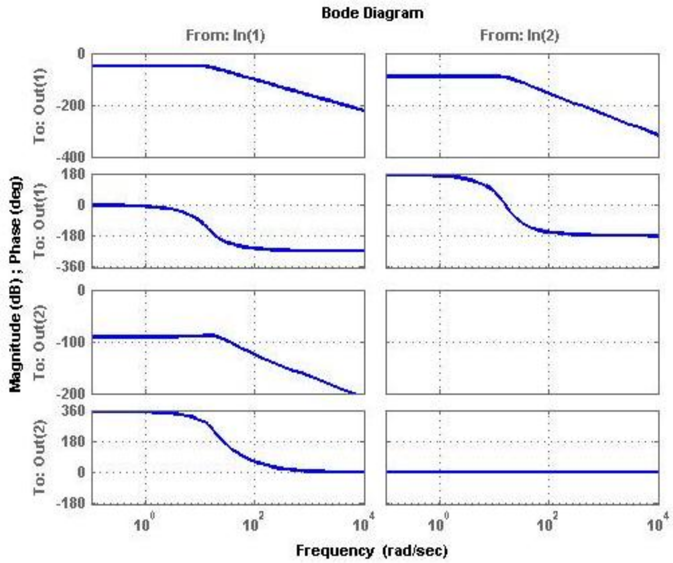


Fig 3.11: Bode Plot using LQR controller

As compared with PID and LTR controller results, Figure 3.10 and Figure 3.11 show that the impulse and bode responses using LQR controller are best out of these three control techniques.

III. CONCLUSION

Twin rotor system (TRS) performance optimization is considered in the present work as experimental TRS exhibit undesired oscillatory behavior when it is commanded to attain a certain angular position along elevation or azimuth axis. Therefore, in this paper, control techniques are designed to optimize TRS performance. The development of TRS model, its linearization process and analysis in open loop structure is presented, then control design techniques are discussed. For controller design, at first, proportional-integral-derivative (PID) control design using Ziegler-Nicholous method is presented. Effect of application of proportional gain, PI gains and PID gains is analysed. Tuning and application of three PID gains produce better results. Further optimization in performance is done using loop transfer recovery (LTR) controller that use a Kalman filter in combination with control law design block to minimize error cost function. Results for LTR application on TRS are compared. Finally, for optimal performance enhancement linear quadratic gaussian (LQR) controller is designed that is tuned by adjusting the process and output noise covariances. LQR performs well to meet required time and frequency domain specifications. At the end, comparative performance analysis of all the designed controllers is done.

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