MATHEMATICAL KNOWLEDGE IN TEACHING FRACTION CONCEPTS

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ABSTRACT: *Teachers need various types of knowledge in order to deliver various concepts at elementary level especially the teaching of mathematics among primary school children. In this paper, Balls' framework (2008) or, Mathematical Knowledge for Teaching (MKT) is used as benchmark guideline. This paper investigates and explores component of MKT knowledge among experienced teachers of the primary school. Data collected using paper pencil test and interview. This paper, narrowed to teacher's knowledge and their practices while teaching the skill of comparing various fractions included. The data gathered from teachers were analyzed using thematic analysis techniques. The results indicated that teachers lack various components of MKT knowledge as a proposal by various researchers and assumed that teaching as procedural more than enough due to lack of deep understanding of mathematics and the various types of MKT it's not required due to the practices in the classroom, PKSR (Penilaian Kendalian Sekolah Rendah) and year six examination UPSR (Ujian Pencapaian Sekolah Rendah).*

Keywords: subject matter knowledge, pedagogical content knowledge, procedural knowledge, conceptual understanding.

1. INTRODUCTION

Teachers play an important role in the trajectory of students throughout the formal schooling experience [1]. Teachers and their knowledge play an important and valuable part in the context teaching especially in primary school. Teacher's knowledge is fundamental in shaping children learning and thinking including their actions. It is widely accepted that the knowledge a teacher holds affect the way they perform all the core tasks of teaching [8]. So, teachers' knowledge in teaching fractions is very important since it is the most important topics students need to understand in order to be successful in algebra and beyond, yet it is an area in which U.S. students struggle [7]. This may be due to major emphasis in school mathematics was on procedural knowledge, or what is now referred to as ***procedural fluency***. Rote learning was the norm, with little attention paid to understanding of mathematical concepts. Rote learning is not the answer in mathematics, especially when students do not understand the mathematics. In recent years, major efforts have been made to focus on what is necessary for students to learn mathematics, what it means for a student to be ***mathematically proficient***. We may ask the questions, what makes a teacher effective? According to [4], teacher preparation or knowledge of teaching and learning, subject matter knowledge, experience, and the combined set of qualifications measured by teacher licensure are all leading factors in teacher effectiveness. Subject matter knowledge alone is insufficient to significantly impact classroom practice. MKT encompasses, among other things, understanding the concepts that underlie procedures, performing error analysis to determine where a student's mistake lies, and choosing effective representations to model concepts.

2. MATERIAL AND METHODS

Case study approach is applied in this study. Case study approach provides answers for *how* and *why* question [12]. Case studies are one approach that supports deeper and more detailed investigation of the type that is normally necessary to answer how and why questions. Three methods were used to collect data, first paper pencil, interview, lastly fraction worksheets, which was designed by the teacher and researcher. The items adopted from LMT (Learning Mathematics for Teaching), Malaysian Primary Mathematics Text Book and finally from past year UPSR exam questions. The sub items developed based on Ball's definitions, and related literature about teachers' knowledge and teaching knowledge.

Total of 8 in-service primary school teachers teaching mathematics at primary school took part in this study. For phase 1, 10 mathematics items were answered by teachers based on their experience in teaching of mathematics at primary level. All the 8 in-services teachers' scripts were analyzed and were interviewed based on their own scripts. Their own students' responses also included from their worksheets, total students take part in this study (n =622), standard 4 students (n = 205), standard 5 students (n = 225) and finally standard 6 students (n = 192).

3. RESULTS AND DISCUSSION

For this research paper, only four items are selected, how to determine the larger or smaller fraction and related questions included. What are strategy teachers used in the classrooms to teach, how the MKT knowledge applied in teaching, how they know the students understanding, what and how the alternative methods used if students facing difficulties, how teachers find their students error and what type of errors students makes, how to overcome the misconceptions among students, how students think in order to determine the larger or smaller values of fraction or ordering fractions accordingly.

The items designed and prepared by teachers and researcher based on the syllabus in standard 4, 5 and 6. The items are validated by experienced teachers [3] and lectures from teaching institutions. Sub items on figure C adapted from Ball's Domains of Knowledge and supporting questions [2].

Figure A shows objective item with five statements. Whereas figure B shows the main items followed by sub items in

figure C. Figure A, B and C answered and responded by teachers based on their teaching experiences; their various domain of knowledge suggested by Ball's Model (2008). Figure D and E answered by their own students. For phase 1, teachers give written responses to the items in figure A, B and C, whereas for figure A, they just select the best statement from the five-given statement. Their selection for figure A, shown in table 1.

In phase 2 teachers was interviewed by the researcher based on their opinions, ideas and thinking. Finally, their own students give responses to the items shown in Figure D and Figure E. Table 2, shows their students responses based on the item in (Figure D and E) which required their students need to choose the correct statement (a, b, c, and d). The statement actually more on comparisons of the value of two or three proper fractions or maybe it is ordering fractions. The students need to understand the value of each fraction and at the same time they able to do comparisons. Based on findings from the table 2, it is used to get teachers views and opinion using methods of interview. Table 2, shows students responses based on figure D and figure E.

Mr. Lee asked his students to compare $\frac{5}{9}$ to $\frac{3}{7}$. Which of the following should he accept as a correct explanation?

a $\frac{5}{9}$ is greater than $\frac{3}{7}$ because 5 is greater than 3.

b They are equal because each is missing four pieces from the whole.

c They are equal because adding two to the numerator in $\frac{3}{7}$ and two to the denominator in $\frac{3}{7}$ produces $\frac{5}{9}$.

d $\frac{3}{7}$ is greater beacause the pieces will be bigger

e $\frac{5}{9}$ is greater because it is more than one-half, while $\frac{3}{7}$ is less than one-half

Fig (A)

Which is the larger fraction? $\frac{14}{24}$ or $\frac{17}{36}$

Fig (B)

a) How will the above questions can be explained verbally? Explain in writing.
b) Based on your experience as a teacher, how many students can solve these questions? Write your explanation.
c) Based on your experience as a teacher, what are the errors made by the students when solving the questions above. Give an explanation.
d) What caused the error described in (c)?
e) How are the errors described in (c) could be resolved? Explain if there are specific guidelines that have been used.
f) How the idea of benchmark fraction can be used when teaching a fraction as above? Describe your experience.
g) Based on your experience, the idea of benchmark fraction is discussed in mathematics textbooks of KSSR? If yes, explain.
h) Some say these questions have to be converted to decimal numbers to be resolved easily. Give your opinion

Fig (C)

Figure D shows three number cards.

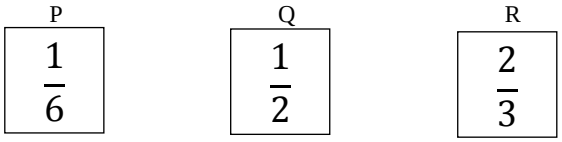


Fig (D)

Choose the correct statement based on figure D.

a Q greater than R
b P greater than R
c P less than Q
d P equal to R

Figure E shows three number cards

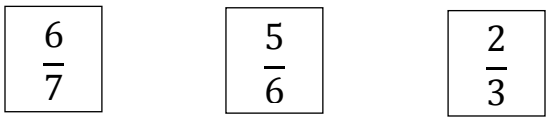


Fig (E)

Choose the correct statement based on figure E.

a Q greater than R but less than P
b P greater than R but less than Q
c P less than Q but greater than R
d P equal to R but greater than Q

Findings are categorized based on domain knowledge which was adapted from Ball's model (2008). Some of the ideas categorized accordingly and suggested questions also adapted from Ball's model (2008). The definition of teacher knowledge used as a guideline to analyze teachers' knowledge which most of the literature call as MKT (Mathematical Knowledge for Teaching). Teachers easily find the solutions regarding greater or larger fraction as shown in fig A. All most of them provided the correct answer. The number of ways suggested by various teachers' basically uniform or the same method. They listed few steps to solve. First write down multiplication table then equalize denominator, then do the comparison (Fig 1). Some even when further and suggested converting the fraction to decimals, but in Malaysian context decimal topic learned after fractions, even from standard 1 to standard six. Some of them suggested drawing diagram, in order to do a comparison as shown in Fig 2. Teachers give various ideas and opinions how to solve and make the questions solvable or understandable to the kids as requested in Fig C. Basically teachers experience speaks in most of the time. Effective teaching is one of them, but what is effective teaching? Defining effective teaching is of course problematic. Ideally, we might define effective teaching as that which leads to high achievement by students in valued outcomes, other things being equal. We also acknowledge that 'other things being equal' maybe open to different interpretations about what factors should

or can be taken into account. But what are the things especially in Malaysia was higher achievement among students as interpret as good teaching and effective teaching and in context in Malaysia the teachers will be rewarded as *'guru cemerlang'* or 'excellent teacher'. Based on Fig A, one of the participants select 'D' as her own opinion compared to others spot E (see table 1).

Number of teachers	Selected Statement
1	D
7	E

Table 1: Based on Fig A

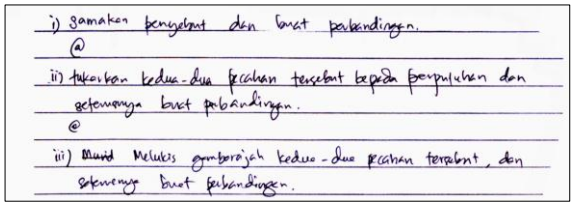


Fig1

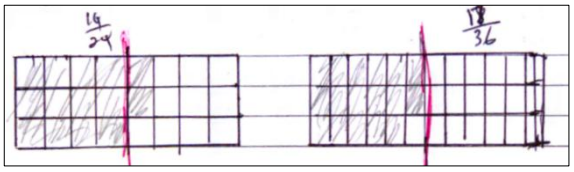
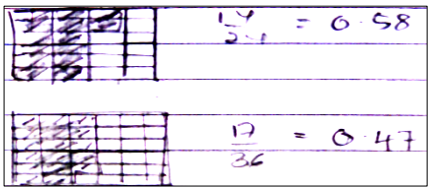


Fig 2



There is a participant draw a diagram (Fig 2) to show the relationship between the two proper fractions. The diagram clearly shows parts of the whole. The first diagram is more than half of the parts shaded while in the second diagram, less than half of the 36 small squared. According to one of the participants, the Fig 2 maybe will provide better-visualized understanding for comparing value of $\frac{1}{2}$. Participant (P8) explains that shaded part in first diagram more than half, whereas in the second diagram, less than half. But there is something not correct in the diagram, the drawing is drawn without considering the concept of the denominator. There are possibilities students will get confused with the diagram. Some of the participants explain and justify that the value of 14 more than half compared to 24, whereas 17 is less than half compared to 36. Participant (P8) simplify the fraction before doing the comparisons as shown in Fig 3. They are using method of 'benchmark' to solve the greater fraction.

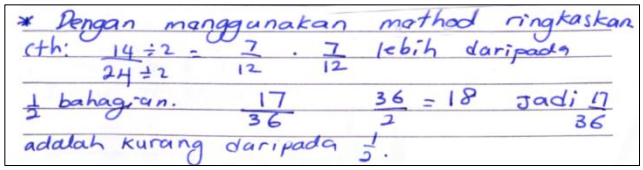


Fig 3

For the misconceptions component teachers said that students are not giving sufficient attention while teaching and learning process in the classroom, this is proven that the fraction which consists of a larger number is a greater fraction. This may be influenced by whole numbers concepts. There are differing opinions about why students tend to overgeneralise whole number properties to fractions. Evolutionary and conceptual change theories suggested that whole numbers are easier to learn than fractions because instruction primarily focuses on the one-to-one counting properties of whole numbers [1, 8]. Whole numbers are successive, discrete, and have predictable calculation properties. According to the participant they concluded that poor grasp of fraction concepts contributes to this scenario. Teachers also suggest some basic ideas how to overcome this types of misconceptions, change fraction component to the decimal form which is common in their daily lives because decimal related to the topic of currency or money is in daily lives. The topic of currency introduced after fraction in Malaysian context but they argued that money or currency is involved in everyday life so they assumed students able to do converting process.Teachers said that comparing fraction using benchmark strategy, even though they are not aware the term 'benchmark' fraction such as 0, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$ and 1. Participant elaborates that benchmark strategy is not covered in Malaysian Mathematics Primary Syllabus. All the while they use a standard method to equalize denominator as shown in

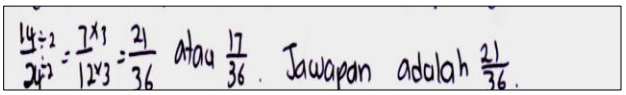


Fig4

Participant (P5) explained that benchmark strategy actually exists in the textbook by referring to the fig 5. While participant (P5) explained that easy to compare fraction $\frac{1}{3}$ and $\frac{1}{2}$ for example see figure 5. What actually in figure 5 is fraction strips. Each fraction strips were divided into equal parts, it helps students to understand the smaller and greater value but is hard for fraction $\frac{14}{24}$ or $\frac{17}{36}$.Other strategies needed in order to explain or teach. One of the participants said that benchmark strategy will help reduce time-consuming. Participant is reminded that benchmark strategy is not a complete strategy compared to the standard algorithm. Another participant said that their students more comfortable with the method in Fig 1 but without drawing any diagrams. According to the participant it will take time and space in exercise book as long it produces correct solutions is more than enough.

The repetition methods or known as drill, is very useful to improve maths skills of student. But, in fact maths is not only about solving every equation and problem given to students, so we cannot hold on this method so long. According to the observation of Development Mathematical Thinking Institute, drill is also improving the basic skills of maths for children in calculation, but not for the logic.

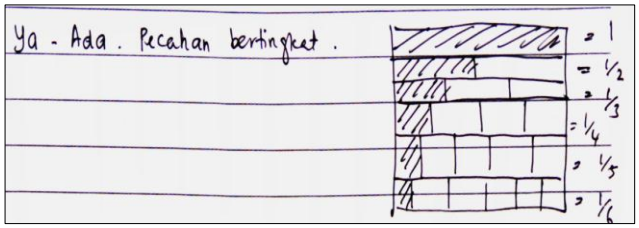


Fig 5

Based on table 1, Only 233 (34%) students out of 622 students answer correctly. This shows that, in every three students only one student answered correctly. Standard 4 students almost 66.34% unable to find correct statement about greater and smaller fraction, similarly for standard 5 (37.33%) and standard 6 (41.67%) able to find the correct statement about ordering of fractions. This may happen most probably because they unable to recall the method to find the greater and smaller fraction. 71 kids in standard 4 select B as their answers, P greater than R or $\frac{1}{6}$ greater than $\frac{2}{3}$. It shows that they are not completely understood the concept of fractions. What participant said may true as shown in figure 6. Greater either in numerator denominator will be great without considering the value of the fraction as a whole. So, teachers need to emphasis on the meaning of rational numbers rather than on calculation procedures. The influence of this method such as, student just only can count but not relating the problem and solution, this method makes an assumption that maths is only about calculation, and also, they will not have good basic theory in solving the problem, so I think teachers need another teaching method such as fact fluency to improve the mathematics skill of students. In such case, students unable to relate problem and solution that make them unfit to apply such knowledge in another problem. None of using number line to teach in order to find the greater and smaller fraction. At the same time application of fraction in daily lives less compared to whole numbers and decimals. This may contribute to poor understanding among children. Too much time is devoted to teaching the procedures of manipulating rational numbers and too little time is spent teaching their conceptual meaning. Teachers do encourage spontaneous or invented strategies, thereby discouraging children from attempting to understand these numbers on their own. On how to overcome their misconceptions participant reminds about conceptual explanation. They conceptual teaching is important but they do not show interest on teaching for better understanding. Participant is not prepared to approach kids with conceptual approach. On the other side, they neglected the poor students because they are unable to master with drill and practice which most of participant not interested.

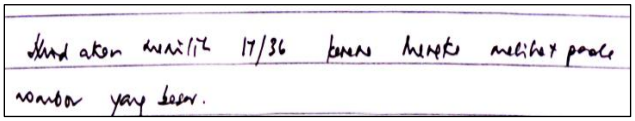


Fig 6

Stand. / Statement	a (*n*, %)	b (*n*, %)	c (*n*, %)	d (*n*, %)	*x* (*n*, %)	Total
Stand. 4 (Fig. D)	(40,19 .52)	(71,34 .63)	(69,33 .66)	(21,10 .24)	(4,1.9 5)	205
Stand. 5 (Fig. E)	(84,37 .33)	(69,30 .66)	(41,18 .22)	(15,6. 67)	(16,7. 12)	225
Stand. 6 (Fig. E)	(80, 41.67)	(39,20 .31)	(42,21 .88)	(11,5. 73)	(20,10 .41)	192
Total						622

Table 2: Students' responses based on Figure D and E

X: leave blank [No answer selected]

4.0 CONCLUSION

In general, teachers scored more highly on questions requiring a response based on their content knowledge than on questions where responses required them to describe the key ideas involved or the actions they would take with a student. Teachers able to identify there is a mistake but they are less patience to identify why misconceptions exist among students. A teacher will find it extremely difficult to answer various questions from students about a subject matter if the teacher has little knowledge about it. Teachers are required to know more and to be well qualified. According to Hammond, "this emphasis may be a reason for stronger student achievement and less public concern with teacher effectiveness" [3]. Many studies have shown that a high proportion of primary teachers lack sufficient content knowledge and Mathematical Knowledge for Teaching (MKT) to teach the fraction concepts of primary mathematics effectively [4]. A significant number of teachers have weak knowledge of fractional concepts [9, 10]. Hill and Charalambous's research (2012) supports that Mathematical Knowledge for Teaching (MKT) contributed to instructional quality. Therefore, it is being fundamental for teachers to be effective in teaching and learning especially in mathematics classrooms.

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