TIME VARYING EQUALIZATION OF DOUBLY SELECTIVE CHANNEL

U. Zaka[1], I.Tauqir[2], A.M. Siddiqui[3], A. Rashdi[4]
[1, 2, 4] Department of Electrical Engineering, National University of Science and Technology, Islamabad, Pakistan
[3]Department of Computer Software Engineering, National University of Science and Technology, Islamabad, Pakistan
[1]umairzaka.msee19@students.mcs.edu.pk, [2]imrantqr@mcs.edu.pk, [3]dradil@mcs.edu.pk, [4]adnanrashdi@mcs.edu.pk

ABSTRACT: *Inter symbol interference and rapid time variations makes doubly selective channels difficult to equalize. The rapid time variations entail a receiver that also adapts itself to the channel variation. This increases the implementation complexity requirement for the receiver. Therefore faster techniques are much needed to establish real time communication over such severe channels. The focus of this paper is on faster equalization techniques for doubly selective channels. The block equalization technique was used because it emphasize on low complexity equalization. MMSE equalizer is reformulated as the problem solving a system of linear equations. This allows the application of algorithms from linear algebra predominantly iterative methods for solution of system of linear equations. Jacobi, Gauss Seidel, Steepest Descent and Conjugate Gradient are used. Jacobi and Gauss Seidel are good for square matrices of lower order, but worse for higher order matrices. Steepest Descent takes more time and more number of iterations to converge than Conjugate Gradient. This paper concludes by proposing the Conjugate Gradient method is suitable for low complexity block equalization problem from the perspective of convergence time and bit error rate.*

Key Words: Doubly Selective Channel, Time Varying Equalization, Conjugate Gradient, Bit Error Rate (BER)

1. INTRODUCTION

Over the period of time wireless communication industry has evolved rapidly. The digital cellular systems which are currently in use are designed to provide services like voice, internet access and video conferencing with high data rates and greater speed. These services demands data rates ranging from some number of hundred Kbps for fast moving users to some Mbps for slow moving users. These high data rates introduce frequency-selective transmission, whereas speed of movement and carrier offsets give rise to time selectivity. This results in so-called doubly selective channel (DSC).

To battle against these DSC effects, equalizers have a vital role to play. In [1] time variant (TV) FIR equalizer was introduced, before that only time invariant (TIV) FIR equalizers were used. Basis Expansion Model (BEM) was used to approximate the DSC and serial liner equalizer (SLE) and block linear equalizer (BLE) with Minimum Mean Square Error (MMSE) and zero forcing (ZF) were studied. Generalized minimal residual (GMRES) and least squares (LSQR) were used in [2] to equalize BEM based DSC. The proposed 1-tap equalizer achieves results comparable to MMSE over Wi-MAX system. Multiple Input Multiple Output (MIMO) based Orthogonal Frequency Division Multiplexing (OFDM) channel was equalized by MMSE in [3], which results in improved BER performance with some Inter Symbol Interference (ISI) is still present. In [4] frequency domain representation of Linear TV MMSE equalizer was introduced and it made sure of a very adequate tradeoff among complexity, convergence speed, and performance. Conjugate Gradient (CG) method was used for channel estimation and equalization of DSC for OFDM in [5]. Linear MMSE and Decision Feedback Equalization (DFE) techniques were studied for equalization which concluded that LMMSE equalization provides better performance to simple DFE. This motivates us to use CG for BLE by using MMSE for DSC. In this paper we are proposing equalization of DSC with the help of MMSE block equalizer using Conjugate Gradient Method (MMSE-BLE-CG).

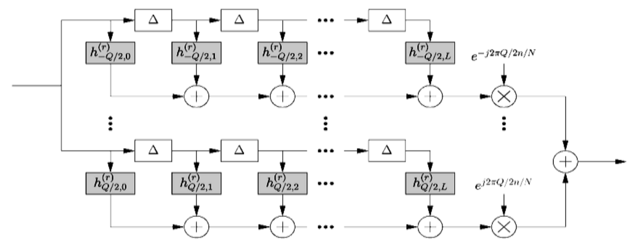

Figure 1 Block Diagram of a Doubly Selective Channel

In figure 1 the block diagram of DSC has been shown, which was an approximation using BEM coefficient. The parameters 'Q' and 'L' are the Delay Spread and Doppler Spread respectively. Following is the equation which illustrates the doubly selective channel

$$\boldsymbol{H^r} = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} h_{q,l} \boldsymbol{D_q} \boldsymbol{Z_l}$$

Where Dq is a diagonal matrix of the form $\mathbf{D_q} \coloneqq \mathbf{diag}\{\left[\mathbf{1}, \ldots\ldots, \mathbf{e}^{\frac{\mathbf{j2\pi q(N-1)}}{\mathbf{N}}}\right]^{\mathbf{T}}\}$ and $\mathbf{Z_l}$ is the circular shifted matrix of order L+LXL+1, and $h_{q,l}$ is a scalar variable having random values. Furthermore MMSE Turbo, bi-directional, TV FIR DFE, frequency domain extended models were also studied for equalization of DSC in [6], [7], [8] and [9] respectively.

This paper is organized as follows. In section 2 brief description and derivation of MMSE Linear Equalizer is discussed. Then we moved on to the implementation of CG method in section 3. We compare through MATLAB simulations the BER of our proposed MMSE-BLE-CG with MMSE-BLE-BEM equalization in section 4. Finally we draw our conclusion in section 5.

2. MMSE LINEAR EQUALIZER

MMSE-LE is a balanced equalizer, which minimizes the mean square error. The problem of ZF equalizer has been removed in this technique. It does not boost up the noise as it is done in the case of ZF. MMSE-LE tries to remove the ISI and only permits equalized symbols to pass. In reality, it does not totally remove the ISI. It passes some of the remaining ISI. If this remaining ISI is removed by force then the noise will increase by default. Due to this remaining ISI, the performance of the equalizer suffers but still the result of MMSE-LE is better than ZF equalizer.

2.1 Derivation of MMSE-LE:

To find out the predefined equation weight a cost function is used. The cost function minimizes the Mean Square Error (MSE). The entities used in derivation of MMSE-LE are discussed below.

J = Cost Function
h = Channel
η = AWGN noise
Y = Distorted Received symbols
w = MMSE Linear Equalizer
sn = Transmitted Symbols
ŝn = Output Estimated Symbols
E = Estimation
Leq = Length of Equalizer

$$\mathrm{J} = \mathrm{E}\{|\mathrm{s_n} - \hat{\mathrm{s}}_\mathrm{n}|^2\} \qquad (2.1)$$

$$\mathbf{Y} = \mathbf{Hs} + \boldsymbol{\eta} \qquad (2.2)$$

$$\begin{bmatrix} y(n) \\ y(n-1) \\ y(n-2) \\ \vdots \\ y(n-Leq) \end{bmatrix} = \begin{bmatrix} h(n,0) & \dots & h(n,l) & 0 \dots & 0 \\ 0 & \ddots & \cdot & \ddots & \vdots \\ \vdots & 0 & . & \cdot & \cdot \\ \vdots & \vdots & \ddots & \cdot & . \\ 0 & 0 & h(n,0) & \dots & h(n,l) \end{bmatrix} \begin{bmatrix} s(n) \\ s(n-1) \\ s(n-2) \\ \vdots \\ s(n-Leq) \end{bmatrix} + \begin{bmatrix} \eta(n) \\ \eta(n-1) \\ \eta(n-2) \\ \vdots \\ \eta(n-Leq) \end{bmatrix}$$

The equalization symbols ŝn are produced by passing received symbols Y from the equalizer.

$$\hat{s}_n = \boldsymbol{w^H y} = \boldsymbol{w^H}(\boldsymbol{Hs} + \boldsymbol{\eta}) \qquad (2.3)$$

By substituting 2.3 in 2.1, the resultant is

$$\mathrm{J} = \mathrm{E}\{|\mathbf{s_n} - \mathbf{w^H}(\mathbf{Hs} + \boldsymbol{\eta})|^2\} \qquad (2.4)$$

Where

$$|\mathbf{s_n} - \mathbf{w^H}(\mathbf{Hs} + \boldsymbol{\eta})|^2 = (\mathbf{s_n} - \mathbf{w^H Hs} - \mathbf{w^H}\boldsymbol{\eta})\,(\mathbf{s_n^*} - \mathbf{s^H H^H w} - \boldsymbol{\eta}^\mathbf{H}\mathbf{w})$$

So, the equation 2.4 becomes

$$\mathrm{J} = \mathrm{E}\{(\mathbf{s_n} - \mathbf{w^H Hs} - \mathbf{w^H}\boldsymbol{\eta})\,(\mathbf{s_n^*} - \mathbf{s^H H^H w} - \boldsymbol{\eta}^\mathbf{H}\mathbf{w})\} \qquad (2.5)$$

$$J = E\{s_n s_n^* - s_n \boldsymbol{s^H H^H w} - s_n \boldsymbol{\eta^H w} - \boldsymbol{w^H Hs} s_n^* + \boldsymbol{w^H H s s^H H^H w} + \boldsymbol{w^H H s \eta^H w} - \boldsymbol{w^H \eta} s_n^* + \boldsymbol{w^H \eta s^H H^H w} + \boldsymbol{w^H \eta \eta^H w}\} \qquad (2.6)$$

Noise (η) and Transmitted Symbols (s) are random variables. The properties which are being used for the estimation of correlation are $E\{ss^H\} = I$, $E\{ss^*\} = 1_\delta$, $E\{\eta\eta^H\} = \sigma_n{}^2$, $E\{s\eta\} = 0$ Where σn2 is noise power. Now the equation 2.6 becomes

$$J = \boldsymbol{w^H H}\, E\{\boldsymbol{ss^H}\}\boldsymbol{H^H w} - \boldsymbol{w^H H} E\{\boldsymbol{s s_n^*}\} + \boldsymbol{w^H} E\{\boldsymbol{\eta\eta^H}\}\boldsymbol{w} - E\{s_n \boldsymbol{s^H}\}\boldsymbol{H^H w} + E\{s_n s_n^*\}$$

$$J = \boldsymbol{w^H H H^H w} - \boldsymbol{w^H H 1_\delta} + \sigma_n{}^2 \boldsymbol{w^H w} - \boldsymbol{1_\delta H^H w} + 1 \qquad (2.7)$$

The purpose of this derivation is to design an MMSE-LE 'w' and to reduce the cost function J of the equalizer with respect to equalizer 'w'. To minimize the cost of equalizer apply the derivative on equation 2.7 w.r.t. 'w*'. So

$$\frac{\partial J}{\partial w*} = \boldsymbol{HH^H w} - \boldsymbol{H1_\delta} + \sigma_n{}^2 \boldsymbol{w} + 0 + 0 \qquad (2.8)$$

Place equation 2.8 equals to zero to find the equation of the equalizer

$$\boldsymbol{HH^H w} - \boldsymbol{H1_\delta} + \sigma_n{}^2 \boldsymbol{w} = \boldsymbol{0}$$

$$\boldsymbol{HH^H w} + \sigma_n{}^2 \boldsymbol{w} = \boldsymbol{H1_\delta}$$

$$\boldsymbol{w} = (\boldsymbol{HH^H} + \sigma_n^2 \boldsymbol{I})^{-1} \boldsymbol{H1_\delta} \qquad (2.9)$$

The equation 2.9 is desired equation for MMSE-LE 'w', which sets the weights of equalizer. In the absence of noise it will work as a zero forcing equalizer. In the worst noise condition the results will not be so good but the equalizer will try to minimize the MSE.

2.2 MMSE Block Linear Equalizer (BLE)

MMSE-BLE takes the whole convolution matrix, inverts it and then by inverting it performs equalization. BLEs are quite better than SLEs. They have better bit error rate (BER) results. But problem occurs when there is a huge data block involved. The equalization of that huge block itself is a tough job to do. The whole convolution matrix has to pass through the equalizer and then the equalizer retrieves the original signal from that matrix. The equation for MMSE-BLE is $\mathbf{w} = (\mathbf{HH^H} + \boldsymbol{\sigma}_\mathbf{n}^\mathbf{2}\mathbf{I})^{-1}\mathbf{H^H}$

3. IMPLEMENTATION OF ITERATIVE METHODS

3.1 Comparison of Iterative Methods:

Jacobi and Gauss Seidel are good for square matrices of lower order. Gauss Seidal takes less iterations and less time to converge than Jacobi. According to [10] Jacobi takes 40 iterations in 0.82 seconds to solve a linear equation of order 3x3 where Gauss Seidal takes 21 iterations in 0.44 seconds. For a linear equation of order of 4x4 Jacobi takes 48 iterations in 2.09 seconds and Gauss Seidal takes 27 iterations in 1.37 seconds.

As the order of the linear equation increases it becomes difficult for both these methods to converge. Hence to solve higher order equations SD and CG are used. CG takes only 50% of the iterations than SD [11]. SD searches in the pattern of "Zig Zag" in each iteration whereas CG searches for the lowest possible solution. In view of above it is concluded that CG is best among three other methods so it is used as an algorithm to implement MMSE-BLE.

To recover the transmit block, various equalization techniques have been suggested in literature such as ZF, MMSE-LE, ML, DFE and iterative techniques. ZF solution can be obtained as

$$\hat{\mathbf{x}}_\mathbf{ZF} = \mathbf{H}^\dagger \mathbf{y}$$

$H^\dagger$ is the Pseudo inverse of the convolution matrix H. Although ZF solution completely eliminates the ISI, it tends to enhance noise. The MMSE solution which provides a compromise between noise and ISI is obtained as

$$\hat{\mathbf{x}}_{\mathbf{MMSE}} = (\mathbf{H}^{\mathbf{H}}\mathbf{H} + \mathbf{I}\boldsymbol{\sigma}_{\boldsymbol{\omega}}^{2})^{-1}\mathbf{H}^{\mathbf{H}}\mathbf{y}$$

One approach to obtaining the block equalization solution directly (without evaluating the block equalizer itself because the interest lies in the transmit data) is to use the CG method.

3.1 Conjugate Gradient (CG) Method

Consider the equation for the MMSE solution which is rewritten as

$$\underbrace{(\mathbf{H}^{\mathbf{H}}\mathbf{H} + \boldsymbol{\sigma}^{2}\mathbf{I})}_{\mathbf{A}}\ \underbrace{\mathbf{x}}_{\mathbf{x}} = \underbrace{\mathbf{H}^{\mathbf{H}}\mathbf{y}}_{\mathbf{b}} \qquad (3.1)$$

To solve such systems, CG method iteratively searches for a Krylov sequence, i.e. a set of points x_q in a sequence of Krylov subspace κ_q

$$x_q = \arg\min_{x\,\in\kappa_q} f(x) \qquad (3.2)$$

Where $f(x) = x^H A x - b^H x - x^H b$ and the qth Krylov subspace is the space spanned by the columns of the matrix

$$[b \;\; Ab \;\; \ldots \;\; A^{q-1}b]$$

It is known by the Cayley Hamilton theorem that the solution to the system of equations must lie in the Krylov subspace of order N, even if it does not span R^N. Define the residual at iteration q as $r_q = Ax_q - b$ and the normalized conjugate directions as

$$\mathbf{p}_q = \frac{\|\mathbf{r}_{q-1}\|^2}{(\mathbf{x}_q - \mathbf{x}_{q-1})^H \mathbf{r}_{q-1}}(\mathbf{x}_q - \mathbf{x}_{q-1}) \qquad (3.3)$$

4. SIMULATION RESULTS AND DISCUSSION

4.1 Equalization using Conjugate Gradient Method

4.1.1 Discussion

In figure 2, a DSC has been equalized by CG method. As it is an iterative method the performance of iterations has been shown as a separate BER curve. 15 iterations were considered here. It is clear from the graph that iteration after iteration the solution is converging. to the original signal that was transmitted. In comparison to the results of [1] in figure 3 MMSE-BLE-BEM for one receive antenna results in BER curve starts getting smooth at 32dB whereas using MMSE-BLE-CG results in BER curve between 15dB to 20dB. Note that the maximum Doppler spread of 100 Hz corresponds to a vehicle speed of 120 km/h and a carrier frequency of 900 MHz .Following parameters were used;

Doppler spread fmax= 100Hz

Delay spread $\mathcal{T}$max =75μs

Block size N=1024

Symbol/sample period T= 25μs;

Discrete Doppler spread Q/2= [fmaxNT] =2

Discrete delay spread L = [$\mathcal{T}$max/T] = 3

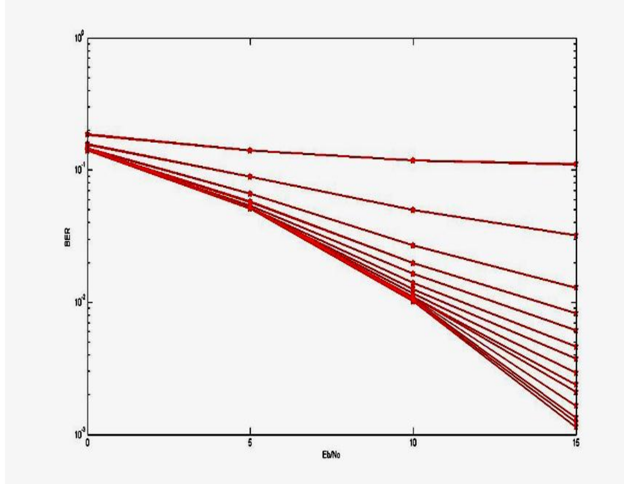

Figure 2 MMSE-BLE-CG

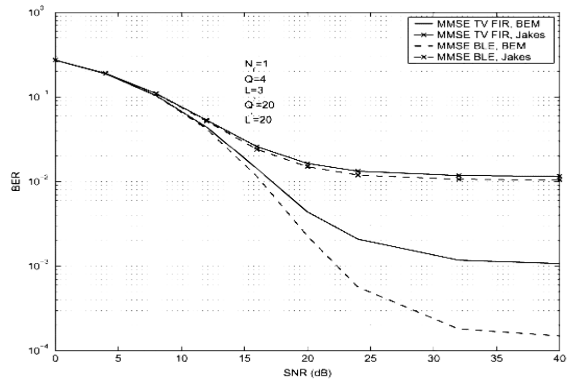

Figure 3 MMSE-BLE-BEM

4.2 Complexity Analysis

To implement the MMSE-BLE-CG, we require $\mathcal{O}(N^2)$ flops for DSC. The implementation complexity associated with the BLE requires N^2 Multiplication and addition operations. Comparing the complexity of proposed solution MMSE-BLE-CG with the complexity of MMSE-BLE-BEM of [1] it is noted that complexity of both the techniques is $\mathcal{O}(N^2)$ but there is difference in convergence time and BER vs SNR results. Runtime of MMSE-BLE-BEM in [1] is 352800 whereas runtime of MMSE-BLE-CG is 16506. This is a significant reduction is convergence time. These complexities are shown in Table I for MMSE-BLE-BEM- and MMSE-BLE-CG criterion.

Table 1 Equalizers Complexity Table

MMSE-BLE-BEM		MMSE-BLE-CG	
Complexity $\mathcal{O}(N^2)$	**CT**	**Complexity $\mathcal{O}(N^2)$**	**CT**
640,000	352800	640,000	16506

5. CONCLUSION

In communication systems, a channel through which the information flows is represented by a matrix, and the received information is the result of the transmitted information manipulated by that channel. Mathematics has a best tool for studying such a scenario, namely system of linear equations. This research is based on a comparative study of traditional iterative methods of solving a system of linear equations. The research was started off by studying how these methods differ from each other, and which method would be most suitable in a specific scenario. Once the different factors involved were studied, it was realized that there was room for improvement in the MMSE equalization technique. Efforts were put in to reduce its convergence time and were successful. MMSE is generally considered the best solution to linear equalization problems. Its attractiveness can be more improved by decreasing its convergence time by bringing in a new technique i.e. method of conjugate gradient.

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