SOME CHARACTERIZATIONS OF AG-GROUPOID BY THEIR GENERALIZED FUZZY SOFT QUASI IDEALS

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ABSTRACT: *In this pape,r we introduce the concept of* $(\in_\gamma, \in_\gamma \vee q_\delta)$ *-fuzzy soft left ideal,* $(\in_\gamma, \in_\gamma \vee q_\delta)$ *-fuzzy soft quasi-ideal over an intra-regular AG-groupoid and investigate their fundamental properties and mutual relationship.*

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1. INTRODUCTION

Fuzzy set theory on semi-group has already been developed. The idea of belongingness of fuzzy point by Murali [25]. The idea of quasi-coincidences of fuzzy points with fuzzy sets was introduced by Bhakat and Das [3,4]. Molodtsov generalized the idea of fuzzy set theory and introduced the concept of fuzzy soft set theory [24]. It is a powerful mathematical tool for dealing with uncertainties. These uncertainties occur in many areas such as economics, engineering, environmental science, medical science, and social science. Up to the present, research on soft sets has been very active and many important results have been achieved in the theoretical aspect. Maji et al. extand the work and defined algebraic operations in fuzzy soft sets theory [23]. Yin and Zhan characterized the order semi-group in term of fuzzy soft ideals [40].

An AG-groupoid is non-associative algebraic structure lies between a groupoid and a commutative semi-group [18]. If an AG-groupoid S contain left identity then the equation hold $S^2 = S$. The left identity of AG-groupoid is unique. An AG-groupoid with right identity become commutative semi-group. If $\{a,b\}$ is any subset of AG-groupoid S with left identity $(ea)b = (ba)e.$ Now our purpose is to bring out some consistent probes for intra-regular AG-groupoids using the new generalized concept of fuzzy soft sets. The purpose of this paper is to deal with the algebraic structure of intra-regular AG-groupoid by applying fuzzy soft theory. We introduced some new types of fuzzy ideals namely $(\in_\gamma, \in_\gamma \vee q_\delta)$- fuzzy soft left ideals and $(\in_\gamma, \in_\gamma \vee q_\delta)$- fuzzy soft quasi-ideals in AG-groupoids and develop some new results. We give some characterizations for intra-regular AG-groupoids using the properties of $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy soft quasi-ideals.

2. PRELIMINARIES

A groupoid $(S,.)$, is called AG-groupoid if its elements hold the left invertive law,

$(db)c = (cb)d.$ Every AG-groupoid Satisfy the medial law which is,

$(ab)(cd) = (ac)(bd)$, for all $a,\ b,\ c,\ d \in S.$

An AG-groupoid with the left identity satisfies the following equations.

$a(bc) = b(ac)$ and $(ab)(cd) = (db)(ca).$

$(ab)(cd) = (dc)(ba)$ It is called paramedial law.

Let S be an AG-groupoid. A non-empty subset A is called AG-subgroupoid if $A^2 \subseteq A.$ A non-empty subset A of an AG-groupoid is called a left (right) ideal of S if $SA \subseteq A$ $(AS \subseteq A).$ A nonempty subset Q of an AG-groupoid is called quasi-ideal of S if $SQ \cap QS \subseteq Q.$ An AG-groupoid S with left identity is $S^2 \subseteq S.$ It is easy to see that every one sided ideal is quasi-ideal. It is given that $L[a] = a \cup Sa, Q[a] = a \cup (aS \cap Sa)$ are principal left ideal and principal quasi-ideal. Let X be a non empty set. A fuzzy subset f of X is defined as a mapping from X into $[0,1]$, where $[0,1]$ is the closed interval of real number. We denote by $\xi(X)$ the set of all fuzzy subsets of X .

A fuzzy subsetf of S of the form.

$$f(y) = \begin{cases} t(\neq 0)\ if\ \ y = x, \\ 0 \text{ otherwise,} \end{cases}$$

is said to be the fuzzy point with support x and value t and is denoted by x_t, where $t \in (0,1].$

Let f and g be any fuzzy subsets of an AG-groupoid S . Then the product $f \circ g$ is defined by

$$\left(f \circ g\right)(a) = \begin{cases} \bigvee_{a=bc} \left\{ f(b) \wedge g(c) \right\} \text{ if } a = bc, \\ 0 \qquad\qquad \text{otherwise.} \end{cases}$$

In what follows let $\gamma, \delta \in [0,1]$ be such that $\gamma \prec \delta,$ for any $Y \subseteq X$, we defined $\chi^{\delta}_{\gamma X}$ be the fuzzy subset of X by

$\chi_{\gamma X}^{\delta}(x) \geq \delta$ for all $x \in Y$ and $\chi_{\gamma Y}^{\delta}(x) \geq \gamma$ otherwise. Clearly $\chi_{\gamma Y}^{\delta}$ is the characteristic function of Y if $\gamma = 0$ and $\delta = 1,$

Let U be an initial universe set and A be the set of parameters. Let P^U denotes the power set of U. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P^U$.

A pair $\langle F, A\rangle$ is called a fuzzy soft set over U, where F is a mapping given by $F : A \rightarrow \Gamma(U)$ represents fuzzy sets of U.

The product and extended intersection of two fuzzy soft sets $\langle F, A\rangle$ and $\langle G, B\rangle$ over an semigroup S is a fuzzy soft set over S and is defined as

- $$\langle F \circ G\rangle(\varepsilon) = \begin{cases} F(\varepsilon) \text{ if } \varepsilon \in A - B, \\ G(\varepsilon) \text{ if } \varepsilon \in B - A, \\ F(\varepsilon) \circ G(\varepsilon) \text{ if } \varepsilon \in A \cap B, \end{cases}$$

for all $\varepsilon \in C = A \cup B$. This is denoted by $\langle F \circ G, C\rangle = \langle F, A\rangle \Theta \langle G, B\rangle$ [23].

- $$H(c) = \begin{cases} F(c) & \text{if } c \in A - B, \\ G(c) & \text{if } c \in B - A, \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

for all $c \in C = A \cup B$. This is denoted by $\langle F, A\rangle \tilde{\cap} \langle G, B\rangle = \langle H, C\rangle$.[23]

A new ordering relation is defined on $F(S)$ denoted as "$\subseteq \vee q_{(\gamma,\delta)}$", as follows.

For any $f, g \in F(S)$ $f \subseteq \vee q_{(\gamma,\delta)} g$, we mean that $x_r \in_{\gamma} f$ implies $x_r \in_{\gamma} \vee q_{\delta} g$ for all $x \in S$ and $r \in (\gamma, 1]$.

- Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two fuzzy soft sets over U. We say that $\langle F, A\rangle$ is an (γ, δ)-fuzzy soft subset of $\langle G, B\rangle$ and write $\langle F, A\rangle \subseteq_{(\gamma,\delta)} \langle G, B\rangle$ if

  (i) $A \subseteq B$.

  (ii) For any $\varepsilon \in A, F(\varepsilon) \subseteq \vee q_{(\gamma,\delta)} G(\varepsilon)$.

A fuzzy soft set $\langle F, A\rangle$ over an AG-groupoid S is called

- Fuzzy soft left (right) ideal over S if $\Sigma\langle S, E\rangle$ $\langle F, A\rangle \subseteq \langle F, A\rangle(\langle F, A\rangle$ $\Sigma\langle S, E\rangle \subseteq \langle F, A\rangle)$.
- Fuzzy soft bi-ideal over S if $\langle F, A\rangle\langle F, A\rangle \subseteq \langle F, A\rangle$ and $\langle F, A\rangle \otimes \Sigma\langle S, A\rangle \otimes \langle F, A\rangle \subseteq \langle F, A\rangle$
- An $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$-fuzzy soft left ideal over S if $\Sigma(S, A) \otimes \langle F, A\rangle \subseteq_{(\gamma,\delta)} \langle F, A\rangle$ and satisfied the condition $x_r \in_{\gamma} F(\varepsilon) \Rightarrow y_r \in_{\gamma} \vee q_{\sigma} F(\varepsilon)$ for all $x, y \in S, \varepsilon \in A$ and $r \in (\gamma, 1]$.
- An $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$-fuzzy soft quasi-ideal over S if

  (i) $\langle F, A\rangle \otimes \langle F, A\rangle \subseteq_{(\gamma,\delta)} \langle F, A\rangle$, and

$(ii) \langle F, A\rangle \otimes \Sigma(S, A) \cap \Sigma(\tilde{S}, A) \otimes \langle F, A\rangle$ $\subseteq_{(\gamma,\delta)} \langle F, A\rangle$

and satisfied the condition $x_r \in_{\gamma} F(\varepsilon) \Rightarrow y_r \in_{\gamma} \vee q_{\sigma} F(\varepsilon)$ for all $x, y \in S, \varepsilon \in A$ and $r \in (\gamma, 1]$.

Corollary1. *Let O be an ordered semigroup and $R \subseteq O$. Then R is a left (resp.,right ideal, bi-ideal, quasi-ideal) of O if and only if $\Sigma(R, A)$ is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$-fuzzy soft left ideal (resp.,right ideal,bi-ideal,quasi-ideal) over O for any $A \subseteq E$.*

Lemma1. *Let $f, g \in \xi(X)$, then $f \subseteq \vee q_{(\gamma,\delta)} g$ if and only if $\max\{g(x), \gamma\} \geq \min\{f(x), \delta\}$ for all $x \in X$.*

Proof. *It is straightforward.*

Lemma2. *Let S be an AG-groupoid and $X, Y \subseteq S$. Then*

(i) $X \subseteq Y$ if and only if $\chi_{\gamma X}^{\delta} \subseteq \vee q_{(\gamma,\delta)} \chi_{\gamma Y}^{\delta}$.

(ii) $\chi_{\gamma X}^{\delta} \cap \chi_{\gamma Y}^{\delta} =_{(\gamma,\delta)} \chi_{\gamma(X \cap Y)}^{\delta}$.

(iii) $\chi_{\gamma X}^{\delta} \circ \chi_{\gamma Y}^{\delta} =_{(\gamma,\delta)} \chi_{\gamma(XY)}^{\delta}$ [24]

Proof. It is straightforward.

3. SOME CHARACTERIZATIONS OF INTRA-REGULAR AG-GROUPOIDS

In this section we have characterized the intra-regular AG-groupoids using the generalized fuzzy soft quasi ideals.

Theorem 1. *Let S be an AG-groupoid with left identity e. Then S is intra-regular If and only if $(\langle G_1, A\rangle \tilde{\cap} \langle F, B\rangle) \tilde{\cap} \langle G_2, C\rangle \subseteq_{(\gamma,\delta)} (\langle G_1, A\rangle \otimes \langle F, B\rangle) \otimes \langle G_2, C\rangle$, for any $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$-fuzzy soft quasi-ideals $\langle G_1, A\rangle$ and $\langle G_2, C\rangle$ and for. any $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$- fuzzy soft left-ideal $\langle F, B\rangle$ over S.*

Proof. The proof is straightforward.

Example 1. *Let* $S=\{1,2,3\}$ *and the binary operation* "⋅" *define on* S *as follows:*

$\cdot$	1	2	3
1	2	2	3
2	3	3	3
3	3	3	3

Then S *is an AG-groupoid.*

Let $E=\{0.35,0.4\}$ *and define a fuzzy soft set* $\langle F,A\rangle$ *over* S *as follows*

$$F(\varepsilon)(x)=\begin{cases}2\varepsilon \text{ if } x\in\{1,2\},\\ \frac{2}{5} \quad \text{otherwise.}\end{cases}$$

Then $\langle F,A\rangle$ *is an* $(\in_{0.3},\in_{0.3}\vee q_{0.4})$ *- fuzzy soft left ideal of* S.

Again let $E=\{0.7,0.8\}$ *and define a fuzzy soft set* $\langle G,A\rangle$ *over* S *as follows:*

$$G(\varepsilon)(x)=\begin{cases}\varepsilon \text{ if } x\in\{1,2\},\\ \frac{2}{5} \quad \text{otherwise.}\end{cases}$$

Then $\langle F,A\rangle$ *is an* $(\in_{0.2},\in_{0.2}\vee q_{0.4})$ *- fuzzy soft bi-ideal of* S.

Theorem 2. *Let* S *be an AG-groupoid with left identity* e. *Then* S *is intra-regular if and only if* $\langle G,A\rangle\tilde{\cap}\langle F,B\rangle \subseteq_{(\gamma,\delta)} (\langle G,A\rangle\otimes\langle F,B\rangle)\otimes\langle G,A\rangle$, *for any* $(\in_\gamma,\in_\gamma\vee q_\delta)$ *- fuzzy soft quasi- ideal* $\langle G,A\rangle$ *and for any* $(\in_\gamma,\in_\gamma\vee q_\delta)$ *- fuzzy soft left ideal* $\langle F,B\rangle$ *over* S.

Proof. Let S be an intra-regular and let a be an element of S, $\varepsilon\in A\cup B$ and $\langle G,A\rangle\tilde{\cap}\langle F,B\rangle=\langle H,A\cup B\rangle$.
We consider the following cases.

Case 1: $\varepsilon\in A\setminus B.$
Then $G(\varepsilon)=(G\circ F)(\varepsilon)$.

Case 2: $\varepsilon\in B\setminus A.$
Then $F(\varepsilon)=(G\circ F)(\varepsilon)$.

Case 3: $\varepsilon\in A\cap B.$
Then $(G\circ F)(\varepsilon)=(G(\varepsilon)\circ F(\varepsilon))\circ G(\varepsilon).$

Now we show that $G(\varepsilon)\cap F(\varepsilon)\subseteq_{(\gamma,\delta)}(G(\varepsilon)\circ F(\varepsilon))\circ G(\varepsilon).$ Since S is intra-regular, therefore for any a in S there exist x and y in S such that $a=(xa^2)y$.

So by (1), (2), (3) and (4) we get

$$a=[xa^2]y=[x\{aa\}]y$$
$$=[a\{xa\}]y=[y\{xa\}]a$$

Now $y\{xa\}=y\{x((xa^2)y)\}$

$$=y\{(xa^2)(xy)\}=(xa^2)\{y(xy)\}$$
$$=(xa^2)\{xy^2\}=\{y^2x\}(a^2x)$$
$$=a^2(\{y^2x\}x)=a^2(\{x^2y^2\})$$
$$=(\{y^2x^2\})(aa)=a((\{y^2x^2\})a)$$
$$=a(ta) \text{ Where } t=(\{y^2x^2\})$$

so $a=[a(ta)]a$

Then we have

$$\max\{((G(\varepsilon)\circ F(\varepsilon))\circ G(\varepsilon))(a),\gamma\}$$
$$=\max\left\{\bigvee_{a=mn}\min\{((G(\varepsilon)\circ F(\varepsilon))(m),G(\varepsilon)(n)\},\gamma\right\}$$
$$\geq\max\{\min\{((G(\varepsilon)\circ F(\varepsilon))(a(ta)),G(\varepsilon)(a)\},\gamma\}$$
$$=\max\left\{\min\left\{\sup_{a(ta)=rs}\min(G(\varepsilon)(r),F(\varepsilon)(s)),G(\varepsilon)(a)\right\},\gamma\right\}$$
$$\geq\max\{\min\{\min(G(\varepsilon)(a),F(\varepsilon)(ta),G(\varepsilon)(a)\},\gamma\}$$
$$\geq\max\{\min\{G(\varepsilon)(a),F(\varepsilon)(ta),G(\varepsilon)(a)\},\gamma\}$$
$$=\min\{\max(G(\varepsilon)(a),\gamma),\max(F(\varepsilon)(ta),\gamma),\max(G(\varepsilon)(a),\gamma)\}$$
$$\geq\min\{\min(G(\varepsilon)(a),\delta),\min(F(\varepsilon)(a),\delta),\min(G(\varepsilon)(a),\delta)\}$$
$$=\min\{\min\{G(a),\delta\},\min\{F(a),\delta\}\}$$
$$=\min\{\min\{G(a),F(a),\delta\}\}$$
$$=\min\{(G(\varepsilon)\cap F(\varepsilon))(a),\delta\}.$$

It follows that $G(\varepsilon)\cap F(\varepsilon)\subseteq\vee q_{(\gamma,\delta)}(G(\varepsilon)\circ F(\varepsilon))\circ G(\varepsilon).$ That is $H(\varepsilon)\subseteq\vee q_{(\gamma,\delta)}((G\circ F)\circ G)(\varepsilon)$. Thus in any case, we have $H(\varepsilon)\subseteq\vee q_{(\gamma,\delta)}((G\circ F)\circ G)(\varepsilon).$

Therefore

$\langle G,A\rangle\tilde{\cap}\langle F,B\rangle\subseteq_{(\gamma,\delta)}((\langle G,A\rangle\otimes\langle F,B\rangle)\otimes\langle G,A\rangle).$ Co nversely. Let Q is a bi-ideal and L is left-ideal of S, then by corollary $\Sigma(L,E)$ is $(\in_\gamma,\in_\gamma\vee q_\delta)$ - fuzzy soft left ideal and $\Sigma(Q,E)$ is $(\in_\gamma,\in_\gamma\vee q_\delta)$ - fuzzy soft bi-ideal of S. Now by the assumption, we have

$$\Sigma(Q,E)\tilde{\cap}\Sigma(L,E)$$
$$\subseteq_{(\gamma,\delta)} ((\Sigma(Q,E)\otimes\Sigma(Q,E))\otimes\Sigma(Q,E)).$$

Hence we have

$$\chi^{\delta}_{\gamma(Q\cap L)} =_{(\gamma,\delta)} \chi^{\delta}_{\gamma Q}\cap\chi^{\delta}_{\gamma L}$$
$$\subseteq \vee q_{(\gamma,\delta)}((\chi^{\delta}_{\gamma Q}\otimes\chi^{\delta}_{\gamma L})\otimes\chi^{\delta}_{\gamma Q})$$
$$=_{(\gamma,\delta)} \chi^{\delta}_{\gamma(QL)Q}.$$

So this implies $Q\cap L\subseteq(QL)Q$ so $a\in Q\cap L\Rightarrow a\in(QL)Q$ for a in S $L[a]=a\cup Sa, Q[a]=a\cup(Sa\cap aS)$ are left and quasi ideals of S generated by a.

$$[a\cup(Sa\cap aS)]\cap[a\cup Sa]$$
$$\subseteq([a\cup(Sa\cap aS)][a\cup Sa])[a\cup(Sa\cap aS)]$$
$$\subseteq\{(Sa)(Sa)\}[a\cup(Sa\cap aS)]$$
$$=(Sa^2)[a\cup(Sa\cap aS)]$$
$$=(Sa^2)\cup(Sa^2)[(Sa\cap aS)]\subseteq Sa^2$$

Hence S is intra-regular.

Theorem 3. *Let S be an AG-groupoid with left identity e. Then S is intra-regular if and only if $\langle F,A\rangle\tilde{\cap}\langle G,B\rangle\subseteq_{(\gamma,\delta)}\langle F,A\rangle\otimes\langle G,B\rangle$, for any $(\in_{\gamma},\in_{\gamma}\vee q_{\delta})$- fuzzy soft quasi-ideals $\langle F,A\rangle$ and for. any $(\in_{\gamma},\in_{\gamma}\vee q_{\delta})$- fuzzy soft left-ideal $\langle G,A\rangle$ over S.*

Proof. Let S be an intra-regular and let a be an element of S, $\varepsilon\in A\cup B$ and $\langle F,A\rangle\tilde{\cap}\langle G,B\rangle=\langle H,A\cup B\rangle$.

We consider the following cases.

Case 1: $\varepsilon\in A\setminus B$.

Then $F(\varepsilon)=(F\circ G)(\varepsilon)$.

Case 2: $\varepsilon\in B\setminus A$.

Then $G(\varepsilon)=(F\circ G)(\varepsilon)$.

Case 3: $\varepsilon\in A\cap B$.

Then $F(\varepsilon)\cap G(\varepsilon)$ and $(F\circ G)(\varepsilon)=F(\varepsilon)\circ G(\varepsilon)$. Now we show that $F(\varepsilon)\cap G(\varepsilon)\subseteq_{(\gamma,\delta)}F(\varepsilon)\circ G(\varepsilon)$. Since S is intra-regular, therefore for any a in S there exist x and y in S such that $a=(xa^2)y$. So by (1), (2), (3), and (4) we have

$$a=[xa^2]y=[x\{aa\}]y$$
$$=[a\{xa\}]y=[y\{xa\}]a$$
$$=[y\{xa\}][\{xa^2\}y]$$
$$=\{xa^2\}[[y\{xa\}]y]$$
$$=[y[y\{xa\}]]\{a^2x\}$$
$$=a^2\{[y[y\{xa\}]]x\}$$
$$=\{x[y[y\{xa\}]]\}(aa)$$
$$=a(\{x[y[y\{xa\}]]\}a)=a(ta)$$

where $t=\{x[y[y\{xa\}]]\}$

so $a=a(ta)$.

Then we have

$$\max\{(F(\varepsilon)\circ G(\varepsilon))(a),\gamma\}$$
$$=\max\left\{\sup_{a=uv}\min\{F(\varepsilon)(u),G(\varepsilon)(v)\},\gamma\right\}$$
$$\geq\max\{\min\{F(\varepsilon)(a),G(\varepsilon)(ta)\},\gamma\}$$
$$=\min\{\max\{F(\varepsilon)(a),\gamma\},\max\{G(\varepsilon)(ta),\gamma\}\}$$
$$\geq\min\{\min\{F(\varepsilon)(a),\delta\},\min\{G(\varepsilon)(a),\delta\}\}$$
$$=\min\{\min\{F(\varepsilon)(a),\delta\},\min\{G(\varepsilon)(a),\delta\}\}$$
$$=\min\{\min\{F(\varepsilon)(a),G(\varepsilon)(a),\delta\}\}$$
$$=\min\{(F(\varepsilon)\cap G(\varepsilon))(a),\delta\}.$$

It follows that $F(\varepsilon)\cap G(\varepsilon)\subseteq\vee q_{(\gamma,\delta)}F(\varepsilon)\circ G(\varepsilon)$.

That is $H(\varepsilon)\subseteq\vee q_{(\gamma,\delta)}(F\circ G)(\varepsilon)$.

Thus in any case, we have

$$H(\varepsilon)\subseteq\vee q_{(\gamma,\delta)}(F\circ G)(\varepsilon).$$

Therefore,

$$\langle F,A\rangle\tilde{\cap}\langle G,B\rangle\subseteq_{(\gamma,\delta)}\langle F,A\rangle\otimes\langle G,B\rangle$$

Conversely. Let Q and L are any two left ideals of S, then $\Sigma(Q,E)$ and $\Sigma(L,E)$ are $(\in_{\gamma},\in_{\gamma}\vee q_{\delta})$- fuzzy soft quasi ideals and $(\in_{\gamma},\in_{\gamma}\vee q_{\delta})$- fuzzy soft left ideals of S. Now by the assumption, we have

$$\Sigma(Q,E)\tilde{\cap}\Sigma(L,E)\subseteq_{(\gamma,\delta)}\Sigma(Q,E)\otimes\Sigma(L,E).$$

Hence we have

$$\chi^{\delta}_{\gamma(Q\cap L)}=_{(\gamma,\delta)}\chi^{\delta}_{\gamma Q}\cap\chi^{\delta}_{\gamma L}$$
$$\subseteq\vee q_{(\gamma,\delta)}\chi^{\delta}_{\gamma Q}\otimes\chi^{\delta}_{\gamma L}=_{(\gamma,\delta)}\chi^{\delta}_{\gamma QL}$$

for a in $S, Q[a] = a \cup (Sa \cap aS)$ and $L[a] = a \cup Sa$ are quasi ideal and left ideal generated by a.

Therefore using (1), (2), (3), and (4) we get

$$[a \cup (Sa \cap aS)] \cap [a \cup Sa]$$
$$\subseteq [a \cup (Sa \cap aS)][a \cup Sa]$$
$$= a[a \cup Sa] \cup a(Sa) \cup (Sa \cap aS)a \cup (Sa \cap aS)Sa$$
$$\subseteq Sa^2.$$

Hence S is intra-regular.

Theorem 4. *Let S be an AG-groupoid with left identity e. Then S is intra-regular if and only if $(\langle G_1, A\rangle \tilde{\cap} \langle G_2, B\rangle) \tilde{\cap} \langle F, C\rangle \subseteq_{(\gamma,\delta)} (\langle G_1, A\rangle \otimes \langle G_2, B\rangle) \otimes \langle F, C\rangle$, for any $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy soft quasi-ideals $\langle G_1, A\rangle$ and $\langle G_2, B\rangle$ for any $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy soft left ideal $\langle F, C\rangle$ over S.*

Proof . Let S be an intra-regular and let a be an element of S, $\varepsilon \in (A \cup B) \cup C$ and $(\langle G_1, A\rangle \tilde{\cap} \langle G_2, B\rangle) \tilde{\cap} \langle F, C\rangle = \langle H, A \cup B \cup C\rangle$. We consider the following cases.

Case 1: $\varepsilon \in A \setminus B \cap C$.

Then $G_1(\varepsilon) = ((G_1 \circ G_2) \circ F)(\varepsilon)$.

Case 2: $\varepsilon \in B \setminus A \cap C$.

Then $G_2(\varepsilon) = ((G_1 \circ G_2) \circ F)(\varepsilon)$.

Case 3: $\varepsilon \in C \setminus A \cap B$.

Then $F(\varepsilon) = ((G_1 \circ G_2) \circ F)(\varepsilon)$.

Case 4: $\varepsilon \in (A \cap B) \cap C$.

Then $(G_1(\varepsilon) \cap G_2(\varepsilon)) \cap F(\varepsilon)$ and $((G_1 \circ G_2) \circ F)(\varepsilon) = (G_1(\varepsilon) \circ G_2(\varepsilon)) \circ F(\varepsilon)$. Now we show that $(G_1(\varepsilon) \cap G_2(\varepsilon)) \cap F(\varepsilon) \subseteq_{(\gamma,\delta)} (G_1(\varepsilon) \circ G_2(\varepsilon)) \circ F(\varepsilon)$.

Since S is intra-regular, therefore for any a in S there exist x and y in S such that $a = (xa^2)y$.

So by (1), (2), (3), and (4) we have

$$a = (xa^2)y = [x(aa)]y$$
$$= [a(xa)]y = [y(xa)]a$$
$$= [y(xa)][(xa^2)y] = (xa^2)[\{y(xa)\}y]$$
$$= \{y[y(xa)]\}(a^2x)$$
$$= (aa)(\{y[y(xa)]\}x)$$
$$= (aa)(\{y[y(x((xa^2)y))]\}x)$$
$$= (aa)(\{y[y((xa^2)(xy))]\}x)$$
$$= (aa)(y[(xa^2)(xy^2)]\}x)$$
$$= (aa)(\{(xa^2)[xy^3]\}x)$$
$$= (aa)\{x[xy^3]\}(xa^2)$$
$$= (aa)\{t(xa^2)\} \text{ where } t = (x(xy^3)).$$

Then we have

$$\max\{((G_1(\varepsilon) \circ G_2(\varepsilon)) \circ F(\varepsilon))(a), \gamma\}$$
$$= \max\left\{\bigvee_{a=uv} \min\{((G_1(\varepsilon) \circ G_2(\varepsilon))(u), F(\varepsilon)(v)\}, \gamma\right\}$$
$$\geq \max\{\min\{((G_1(\varepsilon) \circ G_2(\varepsilon))(aa)), F(\varepsilon)(t(xa^2))\}, \gamma\}$$
$$= \max\left\{\min\left\{\sup_{(aa)=rs} \min(G_1(\varepsilon)(r), G_2(\varepsilon)(s)), F(\varepsilon)(xa^2)\right\}, \gamma\right\}$$
$$= \max\left\{\min\left\{\sup_{(aa)=rs} \min(G_1(\varepsilon)(r), G_2(\varepsilon)(s)), F(\varepsilon)(a^2)\right\}, \gamma\right\}$$
$$= \max\left\{\min\left\{\sup_{(aa)=rs} \min(G_1(\varepsilon)(r), G_2(\varepsilon)(s)), F(\varepsilon)(a), F(\varepsilon)(a)\right\}, \gamma\right\}$$
$$\geq \max\{\min\{\min(G_1(\varepsilon)(a), G_2(\varepsilon)(a), F(\varepsilon)(a)\}, \gamma\}$$
$$\geq \min\{\min(G_1(\varepsilon)(a), \delta), \min(G_2(\varepsilon)(a), \delta), \min(F(\varepsilon)(a), \delta)\}$$
$$= \min\{\min\{G_1(a), \delta\}, \min G_2(a), \delta\}, \min\{F(a), \delta\}\}$$
$$= \min\{\min\{G_1(a), G_2(a), F(a), \delta\}\}$$
$$= \min\{(G_1(\varepsilon) \cap G_2(\varepsilon))(a) \cap F(\varepsilon)(a), \delta\}.$$

It follows that

$$(G_1(\varepsilon) \cap G_2(\varepsilon)) \cap F(\varepsilon) \subseteq \vee q_{(\gamma,\delta)} (G_1(\varepsilon) \circ G_2(\varepsilon)) \circ F(\varepsilon)$$

That is $H(\varepsilon) \subseteq \vee q_{(\gamma,\delta)} ((G_1 \circ G_2) \circ F)(\varepsilon)$. Thus in any case, we have

$$H(\varepsilon) \subseteq \vee q_{(\gamma,\delta)} ((G_1 \circ G_2) \circ F)(\varepsilon).$$

Therefore

$(\langle G_1, A\rangle \tilde{\cap} \langle G_2, B\rangle) \tilde{\cap} \langle F, C\rangle$
$\subseteq_{(\gamma,\delta)} (((\langle G_1, A\rangle \otimes \langle G_2, B\rangle) \otimes \langle F, C\rangle).$

Conversely let Q_1 and Q_2 are quasi-ideal and L is left-ideal of S, then corollary 123 $\Sigma(Q_1, E)$ and $\Sigma(Q_2, E)$ are $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy soft quasi ideal and $\Sigma(L, E)$ is $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy soft left ideal of S. Now by the assumption. Hence we have

$$\chi^\delta_{\gamma(Q_1 \cap Q_2)\cap L} =_{(\gamma,\delta)} (\chi^\delta_{\gamma Q_1} \cap \chi^\delta_{\gamma Q_2}) \cap \chi^\delta_{\gamma L}$$
$$\subseteq \vee q_{(\gamma,\delta)} ((\chi^\delta_{\gamma Q_1} \otimes \chi^\delta_{\gamma Q_2}) \otimes \chi^\delta_{\gamma L})$$
$$=_{(\gamma,\delta)} \chi^\delta_{\gamma(Q_1 Q_2)L}$$

So this implies ($Q_1 \cap Q_2) \cap L \subseteq (Q_1 Q_2)L$ so $a \in (Q_1 \cap Q_2) \cap L$ this implies that $a \in (Q_1 Q_2)L$ for a in S, $L[a] = a \cup Sa, Q[a] = a \cup (Sa \cap aS)$ are left and quasi ideals of S generated by a

$$([a \cup (Sa \cap aS)] \cap [a \cup (Sa \cap aS)]) \cap [a \cup Sa]$$
$$\subseteq ([a \cup (Sa \cap aS)][a \cup (Sa \cap aS)])[a \cup Sa]$$ He
$$\subseteq Sa^2$$
nce S is intra-regular.

Theorem 5.*Let S be an AG-groupoid with left identity e. Then S is intra-regular if and only if*
$(\langle F_1, A\rangle \tilde{\cap} \langle F_2, B\rangle) \tilde{\cap} \langle G, C\rangle$
$\subseteq_{(\gamma,\delta)} (\langle F_1, A\rangle \otimes \langle F_2, B\rangle) \otimes \langle G, C\rangle$
for any $(\in_\gamma, \in_\gamma \vee q_\delta)$ *-fuzzy soft Left ideals* $\langle F_1, A\rangle$ *and* $\langle F_2, B\rangle$ *and for any* $(\in_\gamma, \in_\gamma \vee q_\delta)$ *-fuzzy soft quasi ideal* $\langle G, C\rangle$ *over* S *.*

Proof. Let S be an intra-regular and let a be an element of S, $\varepsilon \in (A \cup B) \cup C$ and

$(\langle F_1, A\rangle \tilde{\cap} \langle F_2, B\rangle) \tilde{\cap} \langle G, C\rangle = \langle H, A \cup B \cup C\rangle$. We consider the following cases.

Case 1: $\varepsilon \in A \setminus B \cap C$.
Then $F_1(\varepsilon) = ((F_1 \circ F_2) \circ G)(\varepsilon)$.
Case 2: $\varepsilon \in B \setminus A \cap C$.
Then $F_2(\varepsilon) = ((F_1 \circ F_2) \circ G)(\varepsilon)$.
Case 3: $\varepsilon \in C \setminus A \cap B$.
Then $G(\varepsilon) = ((F_1 \circ F_2) \circ G)(\varepsilon)$
Case4: $\varepsilon \in A \cap B \cap C$.
Then $(F_1(\varepsilon) \cap F_2(\varepsilon)) \cap G(\varepsilon)$ and $(F_1 \circ F_2) \circ G)(\varepsilon) = (F_1(\varepsilon) \circ F_2(\varepsilon)) \circ G(\varepsilon)$. Now we show that $(F_1(\varepsilon) \cap F_2(\varepsilon)) \cap G(\varepsilon) \subseteq_{(\gamma,\delta)} (F_1(\varepsilon) \circ F_2(\varepsilon)) \circ G(\varepsilon).$

Since S is intra-regular,therefore for any a in S there exist x and y in S such that $a = (xa^2)y$.
So by (1), (2), (3), and (4) we have

$$a = (xa^2)y = (x(aa))y$$
$$= (a(xa))y = (y(xa))a$$
$$\text{Now } y(xa) = y(x((xa^2)y)) = (y(xa^2)(xy))$$
$$= (xa^2)(y(xy))$$
$$= (xa^2)(xy^2) = (y^2x)(a^2x)$$
$$= a^2((y^2x)x) = (aa)(x^2y^2)$$
$$= (y^2x^2)(aa) = (y^2a)(x^2a)$$
$$\text{So } a = ((y^2a)(x^2a))a$$

Then we have

$$\max\{((F_1(\varepsilon) \circ F_2(\varepsilon)) \circ G(\varepsilon))(a), \gamma\}$$
$$= \max\left\{\bigvee_{a=uv} \min\{((F_1(\varepsilon) \circ F_2(\varepsilon))(u), G(\varepsilon)(v)\}, \gamma\right\}$$
$$\geq \max\min\{((F_1(\varepsilon) \circ F_2(\varepsilon))((y^2a)(x^2a))), G(\varepsilon)(a)\}, \gamma$$
$$= \max\left\{\min\left\{\sup_{((x^2a)(y^2a))=rs} \min(F_1(\varepsilon)(r), F_2(\varepsilon)(s)), G(\varepsilon)(a)\right\}, \gamma\right\}$$
$$\geq \max\{\min\{\min(F_1(\varepsilon)(x^2a), F_2(\varepsilon)(y^2a)), G(\varepsilon)(a)\}, \gamma\}$$
$$= \max\{\min\{\min(F_1(\varepsilon)(a), F_2(\varepsilon)(a), G(\varepsilon)(a)\}, \gamma\}$$
$$\geq \min\{\min(F_1(\varepsilon)(a), \delta), \min(F_2(\varepsilon)(a), \delta), \min(G(\varepsilon)(a), \delta)\}$$
$$= \min\{\min\{F_1(a), \delta\}, \min F_2(a), \delta\}, \min\{G(a), \delta\}\}$$
$$= \min\{\min\{F_1(a), F_2(a), G(a), \delta\}\}$$
$$= \min\{(F_1(\varepsilon) \cap F_2(\varepsilon))(a) \cap G(\varepsilon)(a), \delta\}.$$

Thus $H(\varepsilon) \subseteq \vee q_{(\gamma,\delta)} ((F_1 \circ F_2) \circ G)(\varepsilon)$.
Therefore

$$H(\varepsilon) \subseteq \vee q_{(\gamma,\delta)} ((F_1 \circ F_2) \circ G)(\varepsilon)$$

$(\langle F_1, A\rangle \tilde{\cap} \langle F_2, B\rangle) \tilde{\cap} \langle G, C\rangle$
$\subseteq_{(\gamma,\delta)} ((\langle F_1, A\rangle \otimes \langle F_2, B\rangle) \otimes \langle G, C\rangle).$

Conversely Let L_1 and L_2 are left ideal and Q is quasi-ideal of S, then $\Sigma(L_1, E)$ and $\Sigma(L_2, E)$ are $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy soft left ideal and $\Sigma(Q, E)$ is

$(\in_{\gamma},\in_{\gamma}\vee q_{\delta})$ -fuzzy soft left ideal of S. Now by the assumption, we have

Hence we have

$$\chi^{\delta}_{\gamma(L_1\cap L_2)\cap Q} =_{(\gamma,\delta)} (\chi^{\delta}_{\gamma L_1}\cap\chi^{\delta}_{\gamma L_2})\cap\chi^{\delta}_{\gamma Q}$$
$$\subseteq \vee q_{(\gamma,\delta)}((\chi^{\delta}_{\gamma L_1} ? \chi^{\delta}_{\gamma L_2}) ? \chi^{\delta}_{\gamma Q})$$
$$=_{(\gamma,\delta)} \chi^{\delta}_{\gamma(L_1L_2)Q}.$$

So this implies ($L_1\cap L_2)\cap Q\subseteq (L_1L_2)Q$ so $a\in(L_1\cap L_2)\cap Q\Rightarrow a\in(L_1L_2)Q$ for a in S $L[a]=a\cup Sa, Q[a]=a\cup(Sa\cap aS)$ are left and quasi ideals of S generated by a. So

$$((a\cup Sa)\cap(a\cup Sa))\cap a\cup(Sa\cap aS)$$
$$\subseteq ((a\cup Sa)(a\cup Sa))a\cup(Sa\cap aS)$$
$$\subseteq Sa^2$$

Hence S is intra-regular.

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