THE RADIATION EFFECT ON MHD STAGNATION POINT FLOW OF MICROPOLAR FLUIDS DUE TO A POROUS SHRINKING SHEET WITH HEAT GENERATION

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ABSTRACT: *This article considers the stagnation point flow of electrically conducting micropolar fluids due to a surface with the boundary in motion (stretching/shrinking).The fluids flows under the effect of applied magnetic field in the presence of heat source and radiation. The equation of the motion have been reduced to ordinary differential equation by employing suitable similarity functions. The numerical solutions have been sought for velocity, temperature, and concentration using code of Mathematica software. The effects of the physical parameter of the interest have been observed on the heat and mass transfer kinematics of the problem. The results have been presented in the graphical form.*

Key Words: Micropolar polar fluid, Stagnation point, Radiation, Shrinking sheet, Heat generation, Porous Sheet.

1. INTRODUCTION

Eringen [1] provided a generalized theory for dynamics of micropolar fluids. Micropolar fluids consist of rigid, spherical or randomly oriented particles with their own microrotations and spins, suspended in a viscous medium. Ariman et al. [2, 3] presented a review of micropolar fluids flow. Heat transfer is important in many industrial processes. Upendar and Srinivasacharya [4] analyzed a mathematical model for the steady, mixed convection heat and mass transfer along a semi-infinite vertical plate embedded in a micropolar fluid in the presence of a first-order chemical reaction and radiation. Hayat et al. [5] investigated the effects of variable thermal conductivity on the mixed convection flow over a porous stretching surface. Hayat [6] described the effect of heat transfer on the axisymmetric flow of MHD micropolar fluid between two radially stretching sheets. The steady micropolar fluid flow between two vertical porous parallel plates in the presence of magnetic field was considered by Bhargava et al. [7]. Sajjad et al. [8] considered hydromaganetic flow of micropolar fluid between two horizontal parallel plates when the lower one to be stretching sheet and the upper being a porous stretching sheet and obtained numerical solution of the problem in the presence of a transverse magnetic field. Ashraf et al. [9] obtained numerical simulation for two-dimensional flow of a micropolar fluid between an impermeable and a permeable disk. Shafique and Rashid [10] obtained numerical solution of three dimensional micropolar fluid flows due to a stretching flat surface.

Rehman et al. [11] considered heat transfer in two-dimensional steady hyderomagnetic natural convection flow of a micropolar fluid past a non-linear stretching sheet with temperature dependent viscosity. Mohammedein and Gorla [12] analyzed the flow of micropolar fluids bounded by a stretching sheet with a prescribed wall heat flux, viscous dissipation and internal heat generation. El-Hakiem et al. [13] analyzed the effect of viscous and Joule heating on the flow of an electrically conducting micropolar fluid past a plate whose temperature varies linearly with the distance from the leading edge in the presence of a uniform transverse magnetic field.

Abdel- Rehman [14] discussed the effect of magnetic field and focused thin films for unsteady micropolar fluid flow. Veena [15] discussed the effect of change in shape and size of micro molecules of a micro polar fluid on the variation of pressure and load capacity in a squeeze film bounded by a rigid plate. The magnetohyderodynamic (MHD) viscous flow of micropolar fluid over a shrinking sheet has been solved numerically by Shafique [16].

In this work, the physical problem for the radiation effect on MHD stagnation point flow of micropolar fluids due to a porous shrinking sheet with heat generation is first modelled and then simplified by using the non- dimensional variables. The numerical solutions of the simplified equations are found and the results for axial velocity, microrotation and temperature functions are discussed through graphs for various physical parameters of the problem. After the introduction in Section 1, the outlines of this paper are as follows. Section 2 contains mathematical formulation. Discussion and results are given in sections 3.

2. MATHEMATICAL MODEL:

Consider micropolar fluid flow towards the stagnation point on a porous stretching/shrinking surface**.** The fluid is incompressible and electrically conducting. The flow is steady and two-dimensional. The magnetic field of strength H_o is perpendicular to the surface that stretches or shrinks along x-axis. The horizontal component of velocity varies proportional to a specified distance x. The velocity of flow in the region exterior to the boundary layer is $U=cx$. The surface temperature is T. The temperature in the region exterior to the boundary layer is T_∞ . The body couple is absent. Velocity vector is $\underline{V}=V(u,v)$ and Spin vector is $\underline{r}=r(0,0,r_3)$

Under the above assumptions the equations governing the problems are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$(\mu+k)\frac{\partial^2 u}{\partial y^2}+k\frac{\partial r_3}{\partial y}-\sigma\mu_0^2 H_0^2 u+\rho U\frac{dU}{dx}+$$
$$\frac{\rho\upsilon}{K}(u_e-u)=\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}) \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}(\frac{\partial u}{\partial y})^2+$$
$$\frac{16\alpha}{3\beta\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\sigma u_0^2 H_0^2}{\rho C_p}+\frac{Q(T-T_\infty)}{\rho C_p} \qquad (4)$$

$$\gamma(\frac{\partial^2 r_3}{\partial y^2})-\kappa(\frac{\partial u}{\partial y}+2r_3)=\rho j(u\frac{\partial r_3}{\partial x}+v\frac{\partial r_3}{\partial y}) \qquad (3)$$

Where ρ is density, σ is the electrical conductivity, K is the thermal conductivity, C_p is the specific heat capacity at constant pressure, μ is dynamic viscosity, k and γ are additional viscosity coefficients for micropolar fluid and j is micro inertia ,α is Stefan Boltzmann constant, β is Roseland mean absorption coefficient and μ_0 is magnetic permeability.

The boundary conditions are:

$$r_3(x,0)=0,\ u(x,0)=cx,$$
$$v(x,0)=v_0,\quad T(x,0)=T_w \qquad (5)$$
$$r_3(x,\infty)=0,\ u(x,\infty)=ax,$$
$$T(x,\infty)=T_\infty$$

Using similarity transformations:

The velocity components are described in terms of the stream function Ψ (x, y:

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$$

$$\psi(x,y)=x\sqrt{c\upsilon}\,f(\eta),\ \xi=y\sqrt{\frac{c}{\upsilon}}$$

$$u=xcf',\quad v=-\sqrt{\upsilon a}\,f,\quad r_3=\frac{c^{\frac{3}{2}}x\omega}{\upsilon^{\frac{1}{2}}},$$

$$\theta(\xi)=\frac{T-T_\infty}{T_w-T_\infty}$$

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equations (2), (3) and (4), we get

$$(1+\Delta)f'''-\Delta L'-Hf'-\Omega(f'-C)+$$
$$C^2=f'^2-ff'' \qquad (6)$$

$$d_2L''-2\Delta d_1L+\Delta d_1 f''=$$
$$f'L-fL' \qquad (7)$$

$$(4+3R_n)\theta''+$$
$$3R_n\Pr(f\theta'+E_cf''^2+Hf'^2+B\theta)=0 \qquad (8)$$

and the boundary conditions are

$$f'(0)=\lambda,\ f(0)=A,\ L(0)=0,\ \theta(0)=1, \qquad (9)$$
$$f'(\infty)=1,\ L(\infty)=0,\ \theta(\infty)=0,$$

Where as $P_r=\frac{\mu c_p}{k}$ $E_c=\frac{u_e{}^2}{c_p(T_w-T_\infty)}$, $H=\mu_0 H_0\sqrt{\frac{\sigma}{\rho a}}$, $R_n=\frac{\beta k}{4\alpha T_\infty^3}$ and $A=-\frac{v_0}{\sqrt{\upsilon c}}$ are the physical parameters with usual meanings. The dimensional less material are constants.

$$\Delta=\frac{k}{\mu},\ d_1=\frac{\mu}{\rho ja},\ d_2=\frac{\gamma}{\rho j\upsilon}$$

3. RESULTS AND DISCUSSION:

The set of non-linear coupled ODE'S eq (6) to eq (8) with B.c's (9) are solved numerically with coding of Mathematica. Several computations are made to examine the effect of parameters of interest. Graphs have been plotted for fluid motion and thermal state of the problem.

The effect of stretching /shrinking parameter on f' is presented in the fig 1, it is noticed that the flow pattern changes significantly with change in the values of ϵ. The increasing magnetic parameter H_a slows down the fluid flow for $\in>0$, as shown in fig 2, it is because of Lorentz force provides an opposing effect to the flow. But the fig 3 demonstrates the opposite behavior of fluid velocity under the effect of H_a when (the sheet is shrinking). Fig 4 shows that the velocity f' increases with increase in the value of the micropolar parameter Δ .The effect of the porosity parameter Ω on f' has been depicted in fig 5.

The microrotation component ω has been plotted under the effect of magnetic field in fig 6 and fig 7 respectively for stretching and shrinking cases. It is observed that ($\in > 0$), the microrotation increases near the surface but it decreases away from the surface with increase in H_a but opposite pattern is seen for shrinking case ($\in < 0$).The magnitude of the micronotation ω increases with increase in the values of Δ as shown in the fig 8. The microrotation ω increases near the surface but it decreases away from the surface with increase in the values of Ω as presented in fig 9.

Fig. 10 and fig .11 respectively depict the effect of the Prandtl number P_r and radiation parameter R_n on temperature distribution .Both the parameter reduced the temperature function. The increase in the Eckert number E_c and heat source parameter B, causes increase in θ as shown in fig. 12 and fig. 13 respectively.

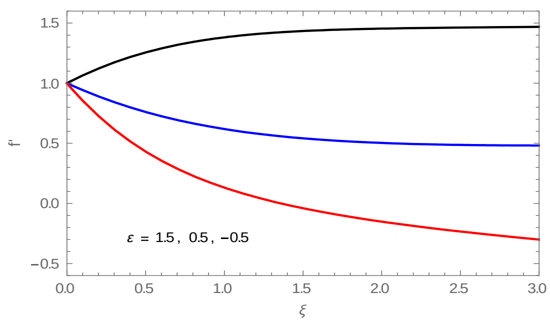

Fig.1: The plot for curves of f' under the effect ε

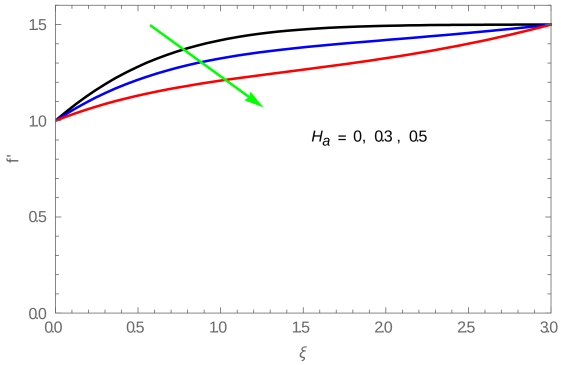

Fig.2: The plot for curves of f' under the effect of Hartmann number *H*a

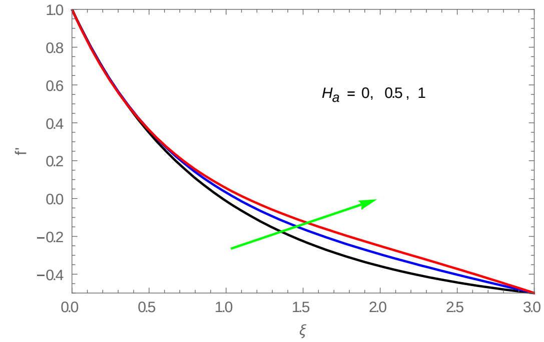

Fig.3: The plot for curves of f' under the effect of Hartmann number *H*a

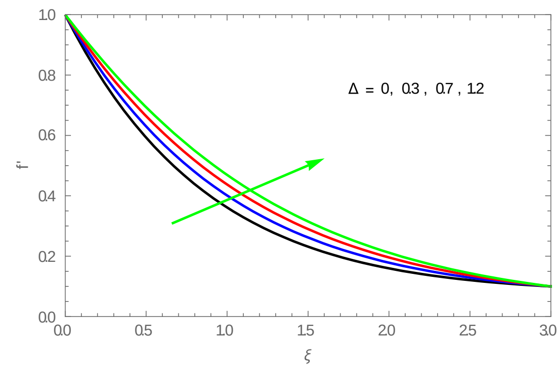

Fig.4: The plot for curves of f' under the effect of Δ

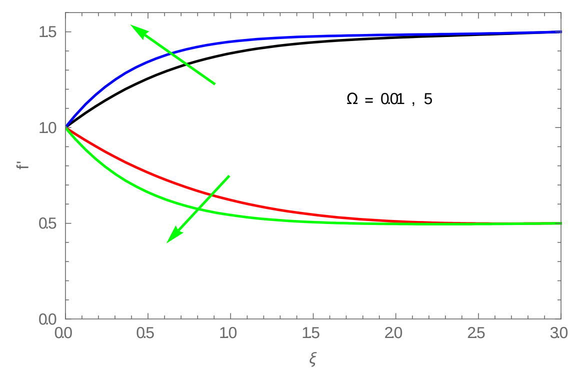

Fig.5: The plot for curves of f' under the effect of Ω

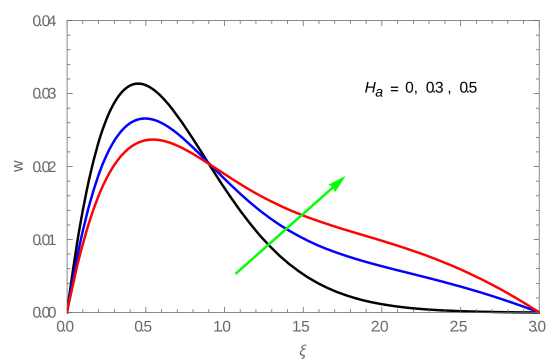

Fig.6: The plot for curves of *w* under the effect of *Ha*

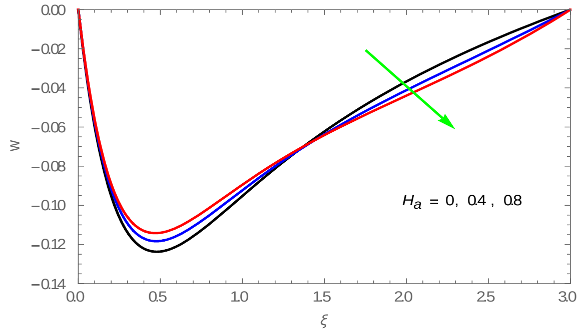

Fig.7: The plot for curves of *w* under the effect of Hartmann number *H*a

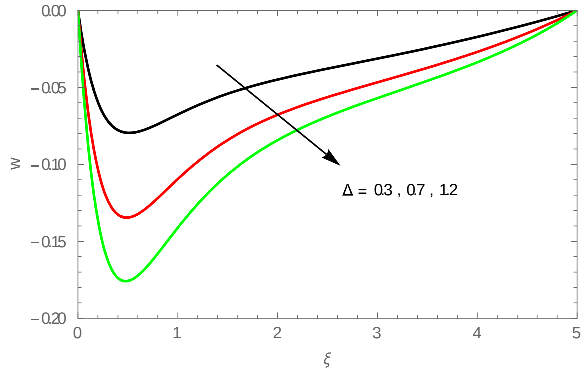

Fig.8: The plot for curves of *w* under the effect of Δ

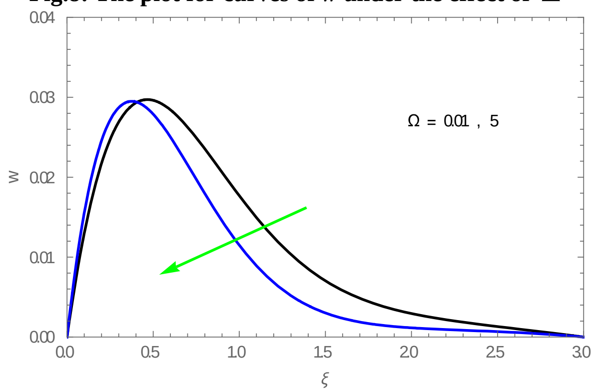

Fig.9: The plot for curves of *w* under the effect of Ω

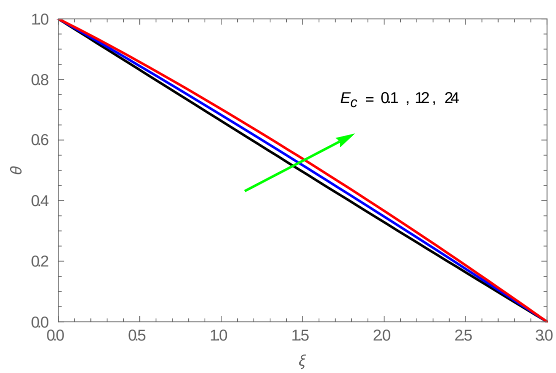

Fig.10: The plot for curves of *θ* under the effect of Eckert number E_c

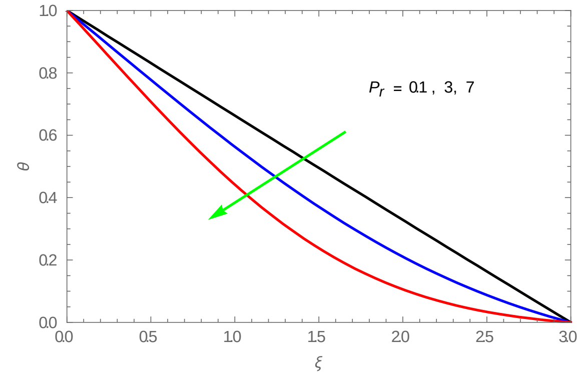

Fig.11: The plot for curves of *θ* under the effect of Prandtl number P_r

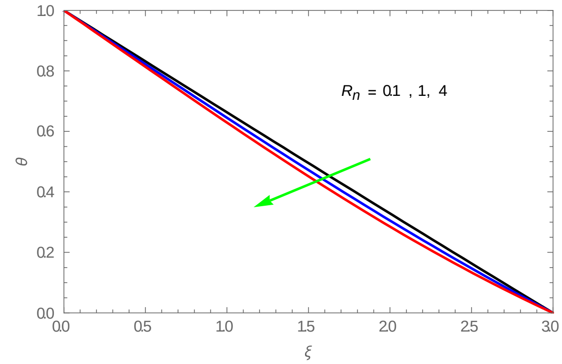

Fig.12: The plot for curves of *θ* under the effect of R_n

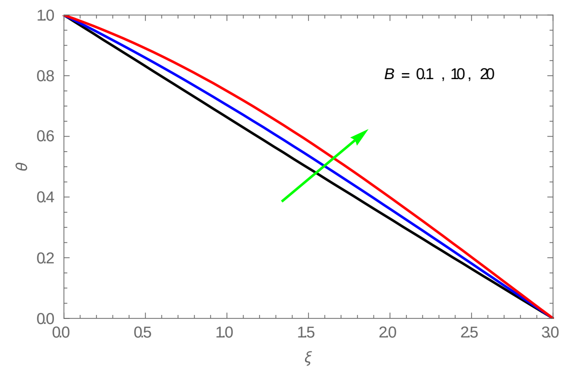

Fig.13: The plot for curves of *θ* under the effect of *B*.

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