HE TRICYCLIC GRAPHS WITH THE EXTREMUM ZEROTH-ORDERGENERAL RANDIC INDEX

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ABSTRACT- *A* (n, n+2)-*graph G is a connected simple graph with* n *vertices and* $n + 2$ *edges. If* d_v *denotes the degree of the vertex* v, *then the zeroth-order general Randic index of the graph* G *is defined as* $R_\alpha^0(G)=\sum_{v\in V} d_v^\alpha$, *where* α *is a real number. We characterize, for any* α , *the* (n, n+2)-*graphs with the extremum* (*minimum for* $0<\alpha<1$ *and maximum for* $\alpha<0$ *or* $\alpha>1$) *zeroth-order general Randic index of* (n, $n+2$)-*graphs.*

Keywords: (n, n+2)-graph, zeroth-order general Randic index, degree sequence

1 INTRODUCTION

Let $G = (V, E)$ be a simple connected graph with the vertex set V and edge set E. For any $v \in V$, $N(v)$ denotes the neighborhood of v and $d_v = |N(v)|$ is the degree of v. The Randic index was defined [1] as, and then generalized by replacing the exponent $-\frac{1}{2}$ by an arbitrary real number α [2]. This graph invariant is called the general Randic index and denoted by R_α, i.e $R_\alpha(G)=\sum_{uv\in E}(d_u d_v)^\alpha$. Some results of the general Randic index can be found in [3] .

Kier and Hall [4] defined the zeroth-order Randic index as $R^0(G)=\sum_{v\in V} d_v^{-\frac{1}{2}}$. Eventualy LI and Zheng [5] defined the zeroth-order general Randic index of graph G as $R_\alpha^0(G)=\sum_{v\in V} d_v^\alpha$, for any real number α . Chen and Deng [6] characterized the (n, n+1)-graph with the maximum and minimum zeroth-order general Randic index for any real number α .

Let $G(k,n)$ be the set of connected simple graphs have n vertices and the minimum degree of vertices is k. Pavlovic and Gutman [7] used linear programming formulation and calculated the minimum value of Randic index for $G(1,n)$. For $k = 2$ the minimum value of Randic index was obtained in [8]. Li et al. [9] studied the famous conjecture of minimum Randic index for $G(k,n)$, however, faced with a serious problem [10]. In this paper, we investigate the zeroth-order general Randic index $R_\alpha^0(G)$ of (n, n+2)-graphs G, i.e., connected simple graphs with n vertices and n+2 edges.

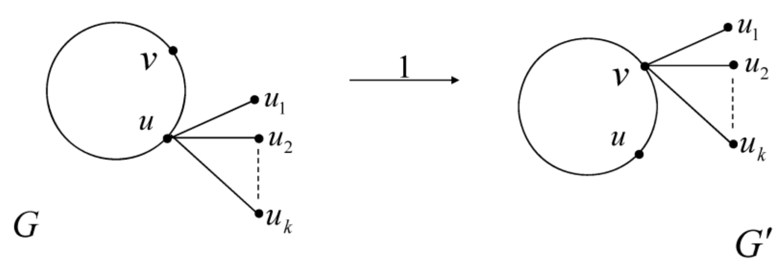

Figure 1: Transformation 1

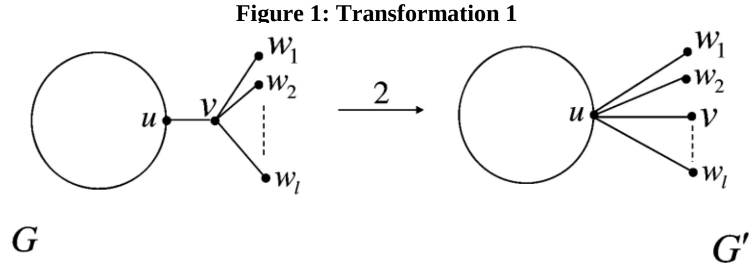

Figure 2: Transformation 2

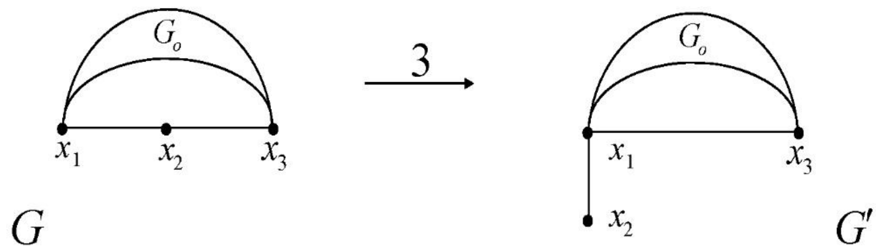

Figure 3: Transformation 3

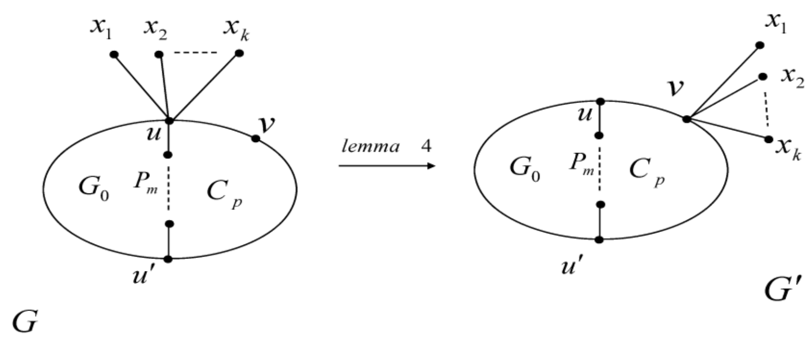

Figure 4: Lemma 2.4

We use the classification of (n, n+2)-graphs given in [11] and find the extremum of $R_\alpha^0(G)$ in all disjoint classes. Then, the extremum of $R_\alpha^0(G)$ of (n, n + 2)-graphs can be found by comparing the extremum of $R_\alpha^0(G)$ of all classes. Note that if $\alpha = 0$ then $R_\alpha^0(G) = n$, and if $\alpha = 1$ then $R_\alpha^0(G) = 2m$. Therefore in the following we always assume that $\alpha \neq 0,1$.

2 Increasing and decreasing transformations

For convenience, we use some transformations given in [6] and some new results in order to increase or decrease the zeroth-order general Randic index.

Transformation 1: If there are two vertices u and v in G such that $d_u=p>1$, $d_v = q > 1$ and $p \le q$, in which, the vertices $u_1, u_2, \dots, u_k$ are adjacent to u,then we put $G' = G - \{uu_1, uu_2, \dots, uu_k\} + \{vu_1, vu_2, \dots, vu_k\}$,$1 \le k \le p$, as shown in Figure 1.

Lemma 2.1. For the two graphs G and G' were defined in Transformation 1, we have

(I) $R_\alpha^0(G') > R_\alpha^0(G)$ for $\alpha < 0$ or $\alpha > 1$,

(II) $R_\alpha^0(G') < R_\alpha^0(G)$ for $0 < \alpha < 1$.

Proof: See Ref [6].

Transformation 2: Let uv be an edge G, the degree $d_G(u)$ of u is $p > 1$, $N_G(v)$ is the neighborhood of v and $N_G(v)$-u = w_1, w_2, ...,w_l. We put $G' = G - \{vw_1, vw_2, \dots, vw_k\} + \{uw_1, uw_2, \dots, uw_k\}$, as shown in Figure 2.

Lemma 2.2. For the two graphs G and G' were defined in Transformation 2, we have

(I) $R_\alpha^0(G') > R_\alpha^0(G)$ for $\alpha < 0$ or $\alpha > 1$,

(II) $R_\alpha^0(G') < R_\alpha^0(G)$ for $0 < \alpha < 1$.

Proof: See Ref [6].

Transformation 3: Let G be a graph include subgraph G_0 and the interior path W: $x_1x_2x_3$, with $d_G(x_2)$ = 2, and vertices x_1 and x_3 do not joined in G_0. as shown in Figure 3.

Lemma 2.3. For the two graphs G and G' were defined in Transformation 3, we have

(I) $R_\alpha^0(G') > R_\alpha^0(G)$ for $\alpha < 0$ or $\alpha > 1$,

(II) $R_\alpha^0(G') < R_\alpha^0(G)$ for $0 < \alpha < 1$.

Proof: By the definition of $R_\alpha^0(G)$, we have

$$\Delta = R_\alpha^0(G') - R_\alpha^0(G) = [(p+1)^\alpha + 1^\alpha] - [p^\alpha + 2^\alpha] = [(p+1)^\alpha - p^\alpha] - [2^\alpha - 1^\alpha] = \alpha(\xi^{\alpha-1} - \eta^{\alpha-1})$$

where $\eta \in (1,2)$, $\xi \in (p, p+1)$ and $d_G(x_1) = p$.

Since $p \ge 2$ therefore $\xi \succ \eta$ and therefore $\Delta < 0$ for $0 < \alpha < 1$ and $\Delta > 0$, for $\alpha < 0$ or $\alpha > 1$.

Lemma 2.4. For the two graphs G and G' including a cycle C_p and k pendant edges in Figure 4, we have

(I) $R_\alpha^0(G) > R_\alpha^0(G')$ for $\alpha < 0$ or $\alpha > 1$,

(II) $R_\alpha^0(G) < R_\alpha^0(G')$ for $0 < \alpha < 1$.

Proof: Note that

$$\Delta = R_\alpha^0(G) - R_\alpha^0(G') = [(\mathrm{q}+\mathrm{k}+1)^\alpha + 2^\alpha] - [(q+1)^\alpha + (k+2)^\alpha] = [(\mathrm{q}+\mathrm{k}+1)^\alpha - (q+1)^\alpha] - [(k+2)^\alpha - 2^\alpha] = \alpha k\,(\xi^{\alpha-1} - \eta^{\alpha-1})$$

Where $d_G(u) = q$, $\eta \in (2, k+2)$, $\xi \in (q+1, \mathrm{q}+\mathrm{k}+1)$, If $q + 1 > k + 2$. Therefore $\Delta < 0$ for $0 < \alpha < 1$ and $\Delta > 0$, for $\alpha < 0$ or $\alpha > 1$. On the other hand

$$\Delta = R_\alpha^0(G) - R_\alpha^0(G') = [(\mathrm{q}+\mathrm{k}+1)^\alpha + 2^\alpha] - [(q+1)^\alpha + (k+2)^\alpha] = [(\mathrm{q}+\mathrm{k}+1)^\alpha - (k+2)^\alpha] - [(q+1)^\alpha - 2^\alpha] = \alpha(q-1)(\xi^{\alpha-1} - \eta^{\alpha-1})$$

Where $d_G(u)=q$, $\eta \in (2, q+1)$, $\xi \in (k+2, \mathrm{q}+\mathrm{k}+1)$, If $k+2>q+1$. Therefore $\Delta < 0$ for $0 < \alpha < 1$ and $\Delta > 0$, for $\alpha < 0$ or $\alpha > 1$

3 The extremum zeroth-order general Randic index of (n, n+ 2)-graphs

Let $G(n, n + 2)$ be the set of simple connected graphs with n vertices and n+2 edges. In this chapter, we use the classification of $G(n, n + 2)$ given in [11] and find an extremum zeroth-order general Randic index $R_\alpha^0(G)$ of each class of $G(n, n + 2)$ (see Figure 5). The extremum (minimum for $0 < \alpha < 1$ and maximum for $\alpha < 0$ or $\alpha > 1$) of $R_\alpha^0(G)$ will obtain by comparing the extremum of $R_\alpha^0(G)$ of all classes.

To this end, we use three increasing/decreasing transformations 1, 2, 3 and the lemma 2.4, have been described in previous chapter. By using and repeating these transformations we will increase/decrease $R_\alpha^0(G)$ of each class of $G(n, n + 2)$ as much as possible.

Initially, repeating transformation 2, any graph G in $G(n, n + 2)$ can be changed into a graph, in which, the edges not on the cycles are pendant edges. In the second step, by using Transformation 1, we reach a graph, in which, the pendant edges have been attached to a single vertex. Then we apply the Transformation 3 to minimize the length of cycles. Transformation 3 is repeated till it is possible. This step reduces the cycles to C_3 or C_4. Now, we have a graph that all its pendant edges were attached to a set of vertices. Once again, applying the Transformation 1, we reach a graph, in which, all pendant edges have been attached to a single vertex. By using the Lemma 2.4, we give up some cases (those their pendant edges have been attached to the vertex of degree 2 of cycles). Now we calculate the $R^0(G)$ of extremal graphs in each class.

All nineteen classes of $G(n, n + 2)$ have been Shawn in the second column of Figure 5, the third column represents the final graph(s) obtained by using the above process in each classes as described.

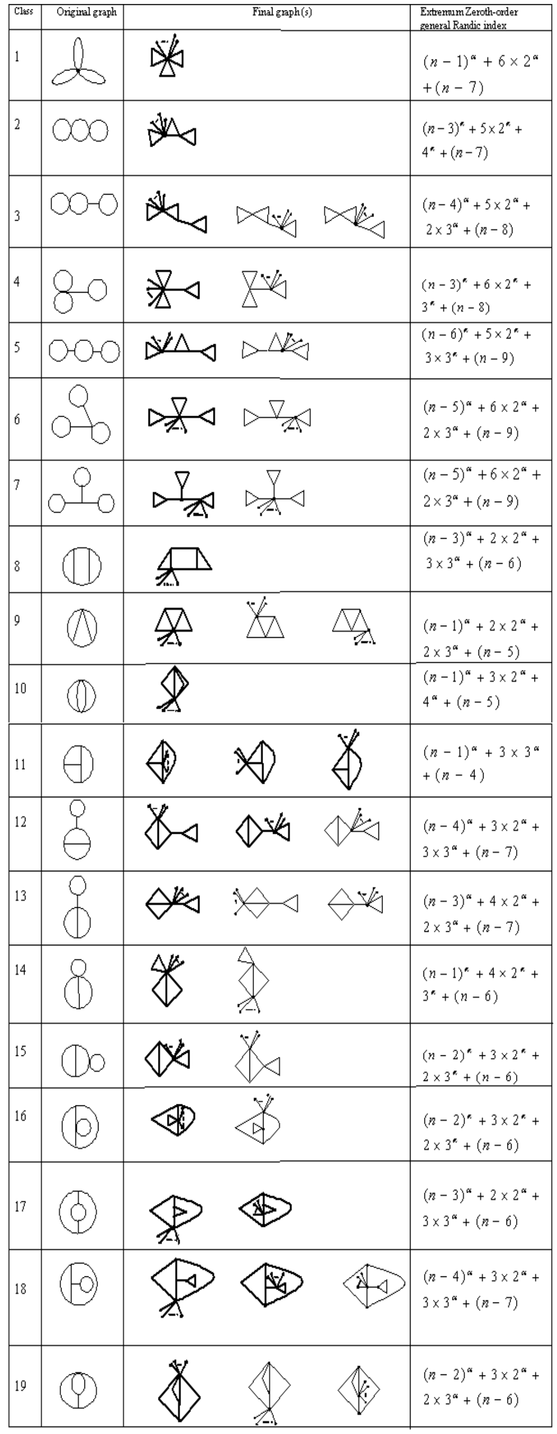

Figure 5: Original graph, Final graph(s) and The extremum zeroth-order general Randic index in 19 classes.

The forth column represents the extremum zeroth-order general Randic index of classes (maximum for $0 < \alpha < 1$ and minimum for $\alpha < 0$ or $\alpha > 1$). For any real number α, the extremum zeroth-order general Randic index can be found by comparing the values of forth column. As an example, the maximum zeroth-order general Randic index for α =0.5 and n=15 is equals 21.509861 which is related to class 7.

REFERENCES

[1] M. Randic, On the characterization of molecular branching, *J. Am. Chem. Soc.* 97 (1975) 6609-6615.

[2] L.B. Kier, L.H. Hall, Molecular Connectivity in Structure-analysis, *Re-search Studies Press, Wiley, Chichester*, UK, 1986.

[3] X.Li and H. Zaho, Trees with the rst three smallest and largest gener-alized topological indices, *MATCH Commun.Math. Comput.Chem.* 52 (2004) 157-210.

[4] L.B. Kier, L.H. Hall, The nature of struture-activity relationships and their relation to molecular Connectivity, *Eur. J. Med. Chem*. 12 (1977) 307-312.

[5] X. Li and J. Zheng, A uniedaooroach to the extremal trees for di rent indices, *MATCH Commun. Math. Comput. Chem*. 54 (2005) 195-208.

[6] S. Chen and H. Deng, Extremal (n, n + 1)-graphs with respected to zeroth- order general Randic index, *J. Math. chem.* (2006) 555-564.

[7] L. Pavlovic, I. Gutman, Graphs with extermal connectivity index, *Novi. Sad. J. Math*, Vol. 31, No. 2, (2001) 53-58.

[8] L. Pavlovic , Graph with extermal Randic index when the minimum degree of vertices is two, *Kragujevac J. Math*. 25 (2003) 55-63.

[9] X. Li, B. Liu, J. Liu, Complete solution to a conjecture on Randic index, *European Journal of Operational Research*, 200 (2010) 9-13.

[10] L. Pavlovic, Comment on Complete solution to a conjecture on Randic index, *European Journal of Operational Research*, 207 (2010) 539-542.

[11] A. Dolati, M. Haghighat, S. Golalizadeh, M. Safari: The Smallest Hosoya Index of Connected Tricyclic Graphs, *MACTH Commun. Math. Comput. Chem.* 65(2011) 57-70.