SAMPLED DATA CONTROL OF BLDC MOTOR USING SMC AND FUZZY CONTROLLER AND COMPARING THEIR PERFORMANCE

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***ABSTRACT*: *BLDC(Brushless Direct Current) motors are used widely for control applications that require high performance. Conventional PID and optimal control techniques provide only satisfactory performance. In this paper, sampled data control of BLDC motor using non-linear control techniques is presented. This paper provides the comparison of two main approaches for the design of digital controller. First is discretization of controller designed for continuous time model of plant and second approach is direct discrete time design based on approximate discrete time model of system. The nonlinear control techniques used are Sliding Mode Control SMC and Fuzzy Controller. SMC is first designed in continuous and then this control technique is implemented in discrete time using discretization of (continuous time) CT controller and direct discrete time design based on approximate discrete time model of system. The fuzzy Logic based Controller has been presented in this paper which significantly minimizes the computational effort. Also the control law presented eliminates the chattering in SMC and Fuzzy SMC. Comparison of the performance of these three control techniques i.e. Discretized SMC, DT SMC and Fuzzy Controller is established in the presence of parameter uncertainty. Sliding Mode observer is employed for feedback. Discretization of sliding mode observer and direct discrete time design of observer is also done for sampled data feedback. Mathematical Calculations and Simulations results are shown to prove the performance of the proposed controller.*

Keywords: Sampled data, Chattering, Sliding Mode Control, Fuzzy Controller, Sliding Mode Observer

INTRODUCTION

For industrial purposes, there are mainly two types of DC motors that are frequently used. First is the conventional DC motor in which the current through the field coil of the stator poles structure produces the flux and second is the Brushless Direct Current Motor where the air gap flux is produced by permanent magnet instead of field poles formed by wounded wires [1]. Moreover the conventional DC motor employs the mechanical commutation that involves losses and contact uncertainties at small voltages, whereas BLDC motors are electronically commutated and do not use brushes for commutation. The fact that the life expectancy of the brush construction is restricted increases the worth of BLDC motor. The BLDC motor has been used extensively in many applications, ranging from servo to traction drives because of several distinct advantages [2]. It has the following advantages over the conventional DC motors with mechanical commutation and induction motors: higher efficiency, dynamic response, better torque vs speed characteristics, high power density, long operating life, larger torque to inertial ratio, noiseless operation and higher speed ranges [3]. Moreover, brushless DC motors are smaller by construction, easy to control, reliable and inexpensive. So it is widely applied in areas which need high performance [4]. BLDC Motor is shown in Fig. 1.

BLDC has one more advantage. The magnetic field which is generated by the stator and the rotor, rotates at the same frequency so that the BLDC motor do not experience the “slip” that is normally seen in induction motors. Hence we can say, that BLDC resembles Synchronous motor [5]. In addition, BLDC motor has better heat dissipation property and ability to operate at high speeds [6].

Despite all these advantages, BLDC has one property that makes it much difficult to handle in terms of modeling the plant and control system design for the motor, that is, coupled non-linear dynamics [7].

In the beginning compact representation of the BLDC motor model was obtained by Chee-Mun Ong [8]. This model was resembling the model of permanent magnet DC motors. As a result, PID controller can be applied to control BLDC motors easily. In recent years, the researchers have applied another algorithm to cater for nonlinear dynamics and enhance high performance system. DC motor predictive models were presented by R. Singh, this research was successful to design optimal controller [9], also speed control of separately excited DC motor was introduced by M. George. GUPTA presented a robust structure position control for DC motor [10]. Yuet al presented LQR method to optimally tune the PID gains. In this method the response of the system is near optimal but it requires mathematical calculations and solving equations. GA based PID control was applied by Lin et al for brushless DC motor [11]. However, all these researches just focused on continuous time system and thus implementation on microcontroller remained unaddressed. Tran Dinh Hu presented discrete time modeling and control but it was only limited to optimal control techniques [12]. J. A. Oyedepo and A. Folaponmile presented continuous SMC with PI tuning but in designing control law, he kept his major focus on PID techniques [13], thus the same mathematical calculations and equation solving. Young et al presented that Sliding mode control (SMC) is one of the most popular strategy to deal with uncertain control systems. Considerable work was not done on SMC with BLDC motor because there was not powerful microprocessor systems in the past and there existed considerable chattering in SMC systems.

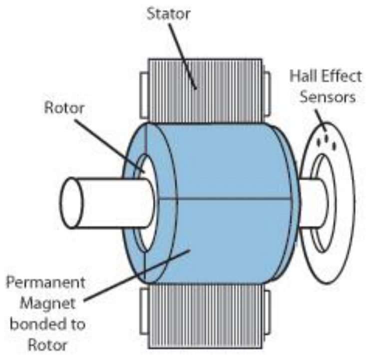

Figure 1: BLDC Motor

Control of sampled data model of plant means that we are using some digital device for implementing the control law for plant. Digital systems like microcontrollers are easy to use because to implement the control law all we need to do is programming while in continuous controllers hardware is changed every time control law is changed [14]. For economical implementation for estimation of unknown states of the system, Observers are employed, for it is costly to use sensors for measuring all states of system [15].

This paper presents sampled data control of BLDC motor using non-linear control techniques. Euler forward difference method is used to find discrete time equivalent model. The purpose of paper is to compare different sampled non-linear control techniques and to study and compare two common sampled-data control techniques namely discretization which is based on continuous-time model of plant and direct discrete-time design based on approximate discrete-time model of plant.

The work presented in this paper is better than the present work done on BLDC motor for following reasons. First, that it take care of nonlinear dynamics of BLDC Motor and second, that the techniques presented are practically implementable because they are in discrete time. Comparison of two nonlinear techniques is shown, not only that, but, two ways to get discrete time controller of continuous time controller/plant are presented in detail. Nonlinear observer is also presented in detail because only a nonlinear observer can cope up the nonlinear dynamics of plant and provides a true estimate of states [16]. Sampled versions of the observer are also presented, so that it be practically realizable.

Two main nonlinear controllers are designed for the control purpose of BLDC motor. These are Sliding Mode Control (SMC) and Fuzzy Controller. Direct implementation of Fuzzy Controller is possible but this is not the case with SMC. So continuous SMC is first discretized to obtain the Discretized SMC, also direct Discrete Time SMC is made to compare that which controller out of discretized SMC and discrete time SMC gives the better result.

SMC is robust for uncertainties and external disturbances, the desired position is perfectly tracked. Fuzzy Controller is also robust for uncertainties but not to the extent SMC is. Major problem of SMC is chattering [17], this is also cared for. The control law presented eliminates the chattering.

This paper is organized into seven sections. Section I gives introduction to the proposed work, section II deals with the mathematical modeling of BLDC motor, section III presets control law of SMC, DT SMC and DSMC, section IV describes the fuzzy controller design for the addressed problem, section V describes the observer design i.e. the Sliding Mode Observer in continuous time, discretized and discrete time, section VI section discusses the simulation results and finally conclusion is presented in section VII. Mathematical Calculations and Simulations results are shown to prove the performance of the proposed controllers and observers.

BLDC MOTOR

To rotate the BLDC motor, the stator windings should be energized in a sequence. It is important to know the rotor position in order to understand which winding will be energized following the energizing sequence. Rotor position is sensed using Hall effect sensors embedded into the stator.

The model of BLDC motor can be represented as:

$$J\,\ddot{\theta} + b\,\dot{\theta} = T_m = k\,i \qquad (1)$$

$$L\,\frac{di}{dt} + Ri = V - k\,\dot{\theta} \qquad (2)$$

Combining these two, we get

$$L\,J\,\dddot{\theta} + (L\,b + R\,J)\ddot{\theta} + (R\,b + K^2)\,\dot{\theta} = K\,V \qquad (3)$$

Putting

$$\dot{\theta} = \frac{d\theta}{dt} = \omega \qquad (4)$$

Hence (3) becomes:

$$L\,J\,\ddot{\omega} + (L\,b + R\,J)\dot{\omega} + (R\,b + K^2)\,\omega = K\,V \qquad (5)$$

Where

R : Armature resistance [Ω].
L : Armature inductance [H].
K : Electromotive force constant [Nm/A].
Kt : Torque constant [Nm/A].
Ke : Voltage constant [Vs/rad].
V : Source voltage [V].
$\dot{\theta}$: Angular velocity of rotor [rad/s].
J : Moment of inertia of the rotor [kgm2].
b : Damping ratio of the mechanical system [Nms].

"b" imposes non-linear dynamics in the plant, so the requirement is to design such a control law, that the angular velocity be independent of it and hence we can track the given input accurately.

State Space Model is given as

$$\dot{x_1} = x_2$$

$$\dot{x_2} = -f(b,R,L,J)\,x_1 - \frac{K^2}{L\,J}\,x_1 - g(b,J)\,x_2 - \frac{R}{L}\,x_2 + \frac{K\,V}{L\,J}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -f(b,R,L,J) - \frac{K^2}{L\,J} & -g(b,J) - \frac{R}{L} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K}{L\,J} \end{bmatrix} V$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Or

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K}{L\,J} \end{bmatrix} V \qquad (6)$$

Where

$k_1 = -f(b,R,L,J) - \frac{K^2}{L\,J}$ (7)

$k_2 = -g(b,J) - \frac{R}{L}$ (8)

Hence

$$\dot{x} = A\,x + B\,V \qquad (9)$$
$$y = C\,x$$

Now since it is the tracking problem, we have to made velocity vector y to track reference signal y_r.

We assume that 1st and 2nd derivatives of reference signal exist. Let

$$x = \begin{bmatrix} x_r \\ \dot{x_r} \end{bmatrix}$$

we define error as:

$$e = x - x_r = \begin{bmatrix} y - y_r \\ \dot{y} - \dot{y_r} \end{bmatrix}$$

Putting in (9)

$\dot{e_1} = e_2$ (10)

$\dot{e_2} = k_1\,x_1 + k_2 x_2 + \frac{K\,V}{L\,J} - \ddot{y}$ (11)

The discrete time system equations of the BLDC motor can be obtained as following:

Euler discretization technique is used for finding Approximate Discrete time equivalent model of the system.

$$x(k+1) = (I + T_s A\,)x(k) + T_s B\,V(k)$$

Let $A_{cd} = (I + T_s A\,)$ and $B_{cd} = T_s\,B$

$$x(k+1) = A_{cd}\,x(k) + B_{cd}\,V(k) \qquad (12)$$

Similarly

$$e_1(k+1) = e_1(k) + T_s\,e_2(k)$$

and

$$e_2(k+1) = e_2(k) + T_s\,k_1\,x_1(k) + T_s\,k_2 x_2(k) + \frac{T_s\,K\,V(k)}{L\,J} - \nabla^2 y_r(k)$$

Where

$$\nabla^2 y_r(k) = y_r(k+2) - 2y_r(k+1) + y_r(k)$$

CONTROL LAW DESIGN

Control law and its derivation and implementation of the mentioned controllers is given below:

SLIDING MODE CONTROL (SMC)

Sliding Mode Control is designed to drive the system states onto a surface in the state space, and this surface is called sliding surface. Once the state reach the sliding surface, sliding mode control keeps the states on the sliding surface. Hence we can split the sliding mode control in two parts. The first part is the design of a sliding surface so that the design specifications are met with sliding motion. The second part involves the selection of control law such that the states reach the sliding manifold in no time and stay there.

Refer to the state space model in terms of error function, (10) and (11). Now if we define the sliding surface as:

$$s = a\,e_1 + e_2$$

Then for $s = 0$ we have

$$e_2 = -\,a\,e_1$$

Substituting in (11) we get

$$\dot{e_1} = -a\,e_1$$

Choosing $a > 0$ guarantees that $e(t)$ tends to zero as t tends to infinity and rate of convergence can be controlled by choice of a . So the choice of sliding surface is correct.

Now,

$\dot{s} = a\,\dot{e_1} + \dot{e_2}$ (13)

$\dot{s} = a\,e_2 + k_1\,x_1 + dk_2 x_2 + \frac{K\,V}{L\,J} - \ddot{y}$

For making $\dot{s} < 0$

We get

$$V = \frac{L\,J}{K}\Big[\widehat{-k_1}\,x_1\,\widehat{-k_2}\,x_2 + \ddot{y_r} - a\,e_2 - k\,sat(\tfrac{s}{\varepsilon})\Big] \qquad (14)$$

DISCRETIZED SLIDING MODE CONTROL (DSMC)

From (13), Discretized Sliding mode control law is:

$$V(k) = \frac{L\,J}{K}\Big[\widehat{-k_1}\,x_1(k)\,\widehat{-k_2}\,x_2(k) + \frac{\nabla^2 y_r(k)}{T_s} - a\,e_2(k) - k\,sat(\tfrac{s(k)}{\varepsilon})\Big] \qquad (15)$$

Where T_s is the sampling time

and

$$\nabla^2 y_r(k) = y_r(k+2) - 2y_r(k+1) + y_r(k)$$

DISCRETE TIME SLIDING MODE CONTROL (DT SMC)

Direct discrete time sliding mode control can be obtained as follows. Discrete time error obtained by Euler forward difference method are given below:

$$e_1(k+1) = e_1(k) + T_s\,e_2(k)$$

$$e_2(k+1) = e_2(k) + T_s\,k_1\,x_1(k) + T_s\,k_2 x_2(k) + \frac{T_s\,K\,V(k)}{L\,J} - \nabla^2 y_r(k)$$

where

$\nabla^2 y_r(k) = y_r(k+2) - 2y_r(k+1) + y_r(k)$

Discrete sliding surface chosen was

$s(k) = a\,e_1(k) + e_2(k)$

Lyapunov function is

$v(k) = s^2(k)$

$\nabla v = s^2(k+1) - s^2(k)$

Making

$$\nabla v < 0$$

gives DT SMC Control law in final form as:

$$V(k) = \frac{L\,J}{K}\Big[\widehat{-k_1}x_1(k)\widehat{-k_2}x_2(k) + \frac{\nabla^2 y_r(k)}{T_s} - ae_2(k) - k\,sat\left(\frac{s(k)}{\varepsilon}\right) - {e_1}^2(k) - {e_2}^2(k)\Big] \qquad (16)$$

FUZZY CONTROLLER

The Fuzzy logic based speed controller is designed. It has two antecedents and a single consequent. The motor speed calculated through the mathematical model is fed back and is compared with the reference speed. This difference of actual and the reference speeds is the speed error `error` and is the first input of the controller. The other input `derivative_error` is the derivative of this error. The output of the controller is the change in the control action and not the control action. The control action can be the input voltage or current of the motor to match the actual and reference speed.

Membership Function:

The fuzzy membership functions for both the inputs are Gaussian functions and there are five membership functions for each input. The membership function Gaussian curves of the input `error` and `derivative_error `

The membership functions of the output are not fuzzy terms but they are definite values negative big, negative medium, zero, positive medium and positive big (NB, NM, Z, PM and PB). The output will be one of the above mentioned five values depending upon the value of the inputs manipulated by some definite rules.

Fuzzy Rules

There are two antecedents and all the antecedents consist of five linguistic variables. Every antecedent has different universe of discourse and it is covered by linguistic variables. These linguistic variables are defined by Gaussian membership functions. In the consequent part there are twenty five linguistic variables and all are defined by constant membership functions. Thus 25 fuzzy rules are to be defined and implemented for the output. Corresponding to different modes of operation of the motor the outputs of the fuzzy logic controller will change accordingly. The fuzzy rules are defined in the form of Table 1.

OBSERVER DESIGN

For estimating states of BLDC Motor, Sliding Mode Observer (SMO) is designed. Same as SMC, for implementation purpose, SMO is first discretized and then direct discrete time SMO is also designed. So that, it can be compared that which observer out of discretized SMO and discrete time SMO gives the better result.

Table 1: Rule Base of Fuzzy Logic Controller

	Input 'error'				
Input 'derivative_error'	NB	NM	Z	PM	PB
NB	NB	NB	NM	NM	NM
NM	NB	NM	Z	Z	Z
Z	NB	NM	Z	PM	PB
PM	Z	Z	Z	PM	PB
PB	PM	PM	PM	PB	PB

SLIDING MODE OBSERVER (SMO)

The observer design is given as follows:

According to (9)

$$\dot{x} = A\,x + BV$$
$$y = C\,x$$

And

$$\dot{\hat{x}} = A\,\hat{x} + B\,V + L\,u\,(\widehat{x_1} - x_1) \qquad (17)$$

Table 2: Linguistic Variables and there parameters

Input	Linguistic Variable	[Variance, Mean]
Error	NM	[0.106 -0.4978]
	NB	[0.106 -0.25]
	Z	[0.106 0]
	PM	[0.1062 0.25]
	PB	[0.1062 0.5]
Derivative_error	NM	[0.106 -0.4978]
	NB	[0.106 -0.25]
	Z	[0.106 0]
	PM	[0.1062 0.25]
	PB	[0.1062 0.5]

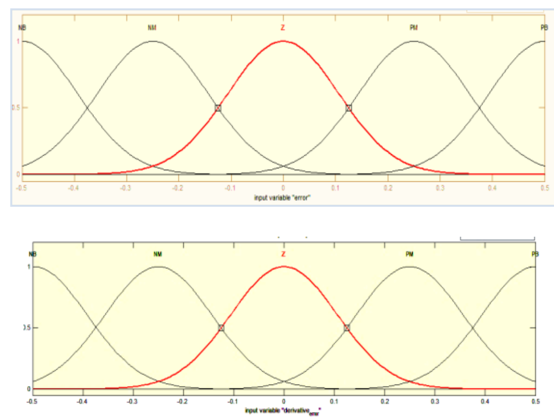

Figure 2-3: Linguistic variables of antecedents are shown with their universe of discourse. All of them have Gaussian membership functions.

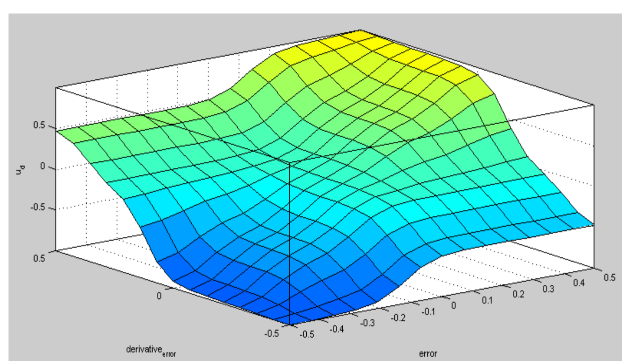

Figure 4: Surface generated against error, derivative_error and u_d by defining FIS Rules

Where $u : R \rightarrow R$ is a non-linear function of the error between estimated state $\widehat{x_1}$ and the output $y = x_1$, and $L \in R^n$ is an observer gain vector

$$L = \begin{bmatrix} -1 \\ L_2 \end{bmatrix}$$

Where $L_2 \in R^{n-1}$ is a column vector.

Let

$$e = \hat{x} - x$$
$$\dot{e} = \dot{\hat{x}} - \dot{x}$$
$$= A\,e + L\,u\,(e_1)$$

Where $e_1 = \widehat{x_1} - x_1$.

The non-linear control law u can be designed to enforce the sliding manifold

$$0 = \widehat{x_1} - x_1$$

so that estimate $\widehat{x_1}$ tracks the real state x_1 after some finite time (i.e. $\widehat{x_1} = x_1$,).

Hence, the sliding mode control switching function

$$\sigma\,(\widehat{x_1}, x_1) \triangleq e_1 = \widehat{x_1} - x_1$$

To attain the sliding manifold, $\dot{\sigma}$ and σ must always have opposite signs (i.e. $\sigma\,\dot{\sigma} < 0$ for essentially all x). However,

$$\dot{\sigma} = \dot{e_1} = A_{11}e_1 + A_{12}e_2 - u(e_1)$$
$$= A_{11}e_1 + A_{12}e_2 - u(\sigma)$$

To ensure $\sigma\,\dot{\sigma} < 0$

Put $u(\sigma) = M\,sat(\sigma)$

Where $M > \max\{|A_{11}e_1 + A_{12}e_2|\}$

That is, positive constant M must be greater than a scaled version of the maximum possible estimator errors for the system (i.e., the initial errors, which are assumed to be bounded so that M can be picked large enough; al). If M is sufficiently large, it can be assumed that the system achieves $e_1 = 0$ (i.e. $\widehat{x_1} = x_1$). Because e_1 is constant (i.e., 0) along this manifold, $\dot{e_1} = 0$ as well.

Hence, we replace the discontinuous control $u(\sigma)$with the equivalent continuous control u_{eq} where

$$0 = \dot{\sigma} = \dot{e_1} = A_{11}e_1 + A_{12}e_2 - u_{eq}$$
$$0 = A_{12}e_2 - u_{eq}$$

Similarly

$$e_2 = \widehat{x_2} - x_2$$
$$\dot{e_2} = A_{22}e_2 + L_2\,u(e_1)$$
$$= (A_{22} + L_2\,A_{12})\,e_2$$

So, to ensure the estimator error e_2 for the unmeasured states converges to zero, L_2 is chosen so that $A_{22} + L_2\,A_{12}$ is Hurwitz .

The final version of the observer is thus

$$\dot{\hat{x}} = A\,\hat{x} + B\,V + L\,M\,sat\,(\widehat{x_1} - x_1) \quad (18)$$
$$= A\,\hat{x} + B\,V + \begin{bmatrix} -1 \\ L_2 \end{bmatrix} M\,sat\,(\widehat{x_1} - x_1)$$
$$= A\,\hat{x} + B\,V + \begin{bmatrix} -M \\ L_2\,M \end{bmatrix} sat\,(\widehat{x_1} - x_1)$$
$$= A\,\hat{x} + \begin{bmatrix} B & \begin{bmatrix} -M \\ L_2\,M \end{bmatrix} \end{bmatrix} \begin{bmatrix} V \\ sat\,(\widehat{x_1} - x_1) \end{bmatrix} \quad (19)$$

Hence

$$\dot{\widehat{x_1}} = \widehat{x_2} - M\,sat\,(\widehat{x_1} - x_1) \quad (20)$$
$$\dot{\widehat{x_2}} = k_1\,\widehat{x_1} + k_2\,\widehat{x_2} + \frac{K}{L\,J}\,V + L_2\,M\,sat\,(\widehat{x_1} - x_1) \quad (21)$$

Where

$$V = \frac{L\,J}{K}\left[\widehat{-k_1}\,x_1\,\widehat{-k_2}\,x_2 + \ddot{y_r} - a\,e_2 - k\,sat(\frac{S}{\varepsilon})\right] \quad (22)$$
$$k_1 = -f(b, R, L, J) - \frac{K^2}{L\,J}$$
$$k_2 = -g(b, J) - \frac{R}{L}$$

DISCRETIZED SLIDING MODE OBSERVER (DSMO)

Discretized SMO is obtained from SMO as follows:

Applying Euler Forward Difference Method to discretize (19) we get:

$$\widehat{x_1}(k+1) = \widehat{x_1}(k) + T_s\,\widehat{x_2}(k) - T_s\,M\,sat\,(\widehat{x_1}(k) - x_1(k)) \quad (23)$$

Similarly, applying Euler Forward Difference on (20) we get:

$$\widehat{x_2}(k+1) = T_s\,k_1\,\widehat{x_1}(k) + (T_s\,k_2 + 1)\,\widehat{x_2}(k) + \frac{K\,T_s}{L\,J}\,V(k) + T_s\,L_2\,M\,sat\,(\widehat{x_1}(k) - x_1(k)) \quad (24)$$

where

$$V(k) = \frac{L\,J}{K}\left[\widehat{-k_1}\,x_1(k)\,\widehat{-k_2}\,x_2(k) + \frac{\nabla^2 y_r(k)}{T_s} - a\,e_2(k) - k\,sat(\frac{S(k)}{\varepsilon})\right] \quad (25)$$

Where T_s is the sampling time, And

$$\nabla^2 y_r(k) = y_r(k+2) - 2y_r(k+1) + y_r(k)$$

Also after discretizing plant (12), we have

$$x(k+1) = \begin{bmatrix} 1 & T_s \\ T_s\,k_1 & 1 + T_s\,k_2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K\,T_s}{L\,J} \end{bmatrix} V(k)$$

DISCRETE TIME SLIDING MODE OBSERVER (DT SMO)

Discrete Time Sliding Mode Observer is designed by using the discrete time model of plant.

$$x(k+1) = A\,x(k) + BV(k)$$
$$y(k) = C\,x(k)$$

DT SMO is given as

$$\hat{x}(k+1) = A\,\hat{x}(k) + B\,V(k) + L\,u(k)\,(\widehat{x_1}(k) - x_1(k)) \quad (26)$$

Where $u : R \rightarrow R$ is a non-linear function of the error between estimated state $\widehat{x_1}$ and the output $y = x_1$, and $L \in R^n$ is an observer gain vector

$$L = \begin{bmatrix} -1 \\ L_2 \end{bmatrix}$$

The non-linear control law u is designed to enforce the sliding manifold

$$0 = \widehat{x_1} - x_1$$

so that estimate $\widehat{x_1}$ tracks the real state x_1 after some finite time (i.e. $\widehat{x_1} = x_1$,).

Similar to continuous SMO, we have

$$e_1(k+1) = A_{11}e_1(k) + A_{12}e_2(k) - u(k)e_1(k)$$

And

$$e_2(k+1) = (A_{22} + L_2\,A_{12})\,e_2(k)$$

In the continuous case, we had $A_{22} + L_2\,A_{12}$ must be Hurwitz. Now in DT design, eigen values of $A_{22} + L_2\,A_{12}$ must lie on origin , in origin to make next step of error to be zero, despite of the fact whatever the value of initial error may be.

Hence $L_2 = -\frac{A_{22}}{A_{12}}$

Finally,

$$\widehat{x_1}(k+1) = \widehat{x_1}(k) + T_s\,\widehat{x_2}(k) - T_s\,M\,sat\,(\widehat{x_1}(k) - x_1(k)) \quad (27)$$

$$\widehat{x_2}(k+1) = T_s\,k_1\,\widehat{x_1}(k) + (T_s\,k_2 + 1)\,\widehat{x_2}(k) + \frac{K\,T_s}{L\,J}\,V(k) + T_s\,L_2\,M\,sat\,(\widehat{x_1}(k) - x_1(k)) \quad (28)$$

Where

$$L_2 = -k_2$$

$$V(k) = \frac{L\,J}{K}\left[\widehat{-k_1}\,x_1(k)\,\widehat{-k_2}\,x_2(k) + \frac{\nabla^2 y_r(k)}{T_s} - a\,e_2(k) - k\,sat(\frac{s(k)}{\varepsilon})\right] \quad (29)$$

Where T_s is the sampling time, And

$$\nabla^2 y_r(k) = y_r(k+2) - 2y_r(k+1) + y_r(k)$$

SIMULATION RESULTS

The specification of BLDC motor is shown in Table 2.

The effectiveness of the controller (14), (15), (16), Fuzzy Controller, and the observer (22), (25) and (29) is verified by the simulation results.

The BLDC motor is controlled by the sliding mode controller, discretized SMC and DT SMC (14), (15) and (16) respectively, which are obtained by choosing a=2 and k=50. SMC controller is termed as good if sliding surface comes to zero in very short interval of time. With the given parameters SMC reaches the sliding surface in 0.1382s, while discretized SMC takes 0.26s to reach the manifold surface s=0 and DT SMC took 0.29s, while the sampling time was 0.1s. Fig. 5, Fig. 7 and Fig. 9 show the input and output plot for these three controllers that how effectively output tracks the time varying signal. Finally Fig. 6, Fig. 8 and Fig. 10 show the error between the input and output plots of three controllers on a magnified scale. Maximum absolute error in case of SMC is 0.0793, 0.0838 for discretized SMC and 0.0908 for DT SMC. However if we keep all other parameters same and increase the multiple of saturation function i.e. k then the results are changed. Setting k=100, yields the following results. The three controllers, SMC, discretized SMC and DT SMC reach the manifold surface s=0 in 0.0443s, 0.18s and 0.14s respectively. Similarly the maximum absolute error for the three controllers is observed to be 0.0443, 0.0648 and 0.0512 respectively.

Output of the Fuzzy Controller obtained on the basis of membership functions and fuzzy rules explained in the above Fuzzy Controller section is shown in Fig. 11 and tracking error between input and output is shown in Fig. 12. Fig. 11, show the reference signal and the output signal, while Fig. 12 shows the error between input and output plot. For a certain gain, error is found to be of order of 0.005, but this error remains throughout the tracked signal. It is evident from the figure.

Sliding Mode Observer is designed which is highly robust. Since there is no hard non-linearity so the states are tracked to the perfection by the sliding mode observer (22), discretized SMO (25) and DT SMO (29). Fig. 13~14 show the states tracked by SMO. First plot of both figures show the original states. Second plot show the tracked states and finally the third plot show the error between the original and tracked states. Fig. 15~16 and Fig. 17~18 follow the same pattern for Discretized SMO and DT SMO respectively.

In comparing the simulation results of discretized SMC designed based on the discretized SMO and DT SMC design based on the DT SMO, the results are different for differet values of k. For k=50 discretized SMC is better, while at k=100 DT SMC is better. However both give the satisfactory performance at both values of k. Hence it can be concluded that for any values of k, discretized SMC and DT SMC are equally good. Further, the comparison between the sampled SMC (discretized or DT) and Fuzzy Controller, error of the Fuzzy is less than the sampled SMC but the error in the fuzzy remains for all the time between input and output, while the error in sampled SMC is vanished in max 0.3s. Further for comparison consider Fig. 19, Fig. 20 and Fig. 21 these are the outputs of Discretized SMC, DT SMC and Fuzzy Controller respectively with the parameter uncertainty. These plots are for highly uncertain parameters. Fig. 19 and Fig. 20 show that performance of sampled SMC is not affected by the uncertainty of parameters. Hence it is concluded that SMC is highly robust to parameters uncertainties. Hence it gives the best tracking result. Fig. 21 shows the Fuzzy Controller with the perturbation in the parameters. Under small perturbations there is no effect in the output of Fuzzy Controller, while under large parameter uncertainty, the error is introduced of the order of 10^{-5}. Hence Fuzzy Controller is also robust and results in good tracking of time varying signals, but it is less robust compared to SMC. Hence it is concluded that sampled SMC is better technique compared to Fuzzy Controller.

Fig. 13~18 clearly indicate that simulation of both sampled techniques for SMO give the same behavior. So just like sampled SMC, any of the sampled SMO can be used and they are declared as equally good.

Table 3: Specification of BLDC Motor

Parameters	Values and units
R	21.2 Ω
Ke	0.1433 V s/rad
D	$1\text{x}10^{-4}$ kg-m s/rad
L	0.052H
Kt	0.1433 kg-m/A
J	$1\text{x}10^{-5}$ kg-m s^2/rad

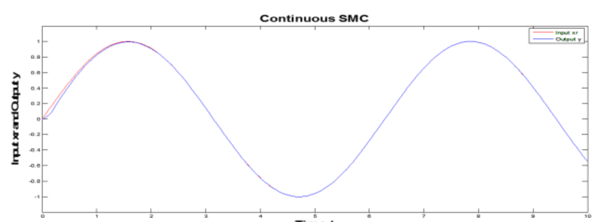

Figure 5: Reference Signal 'r' and output 'y' by Continuous SMC

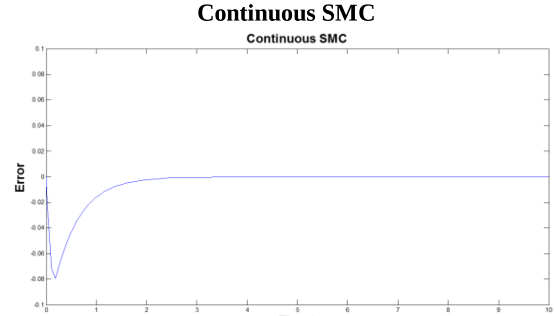

Figure 6: Error between Reference Signal and Output by Continuous SMC

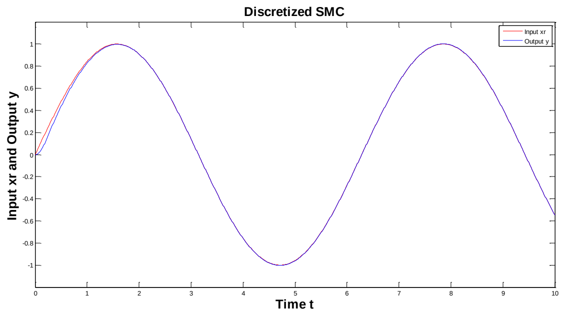

Figure 7: Reference Signal 'r' and Output 'y' by Discretized SMC

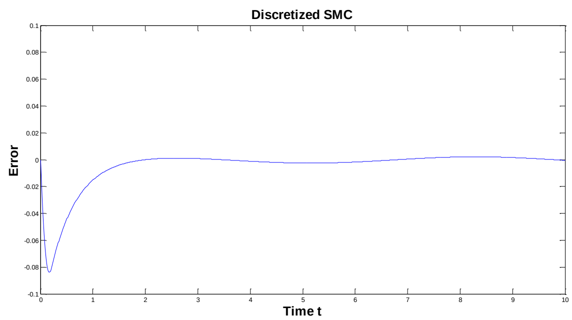

Figure 8: Error between Reference Signal and Output by Discretized SMC

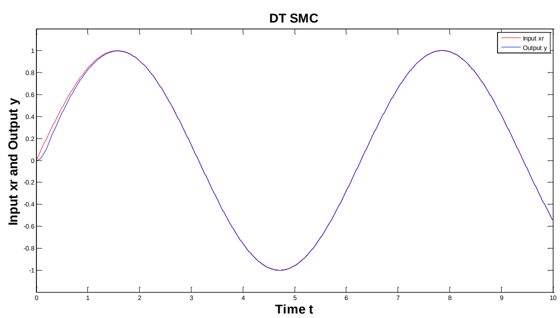

Figure 9: Reference Signal 'r' and Output 'y' by DT SMC

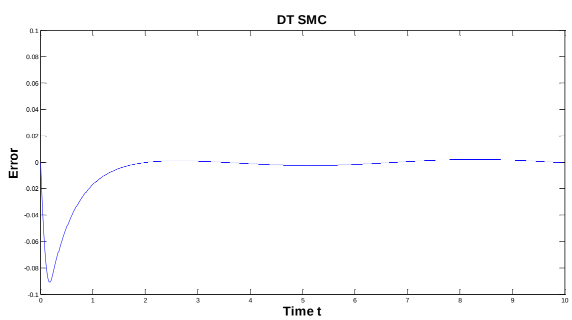

Figure 10: Error between Reference Signal and Output by DT SMC

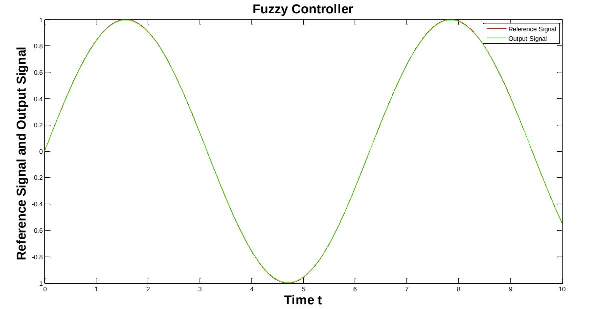

Figure 11: Plot of Reference Signal and Output Signal obtained by Fuzzy Controller

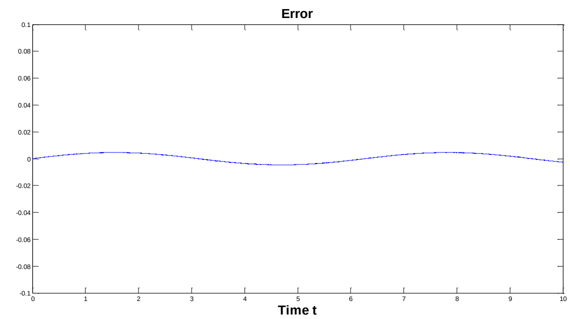

Figure 12: Error between Reference Signal and Output Signal by Fuzzy Controller

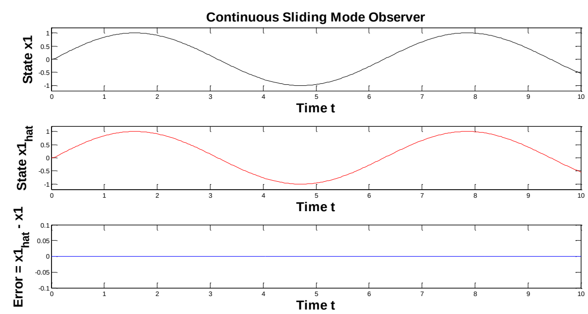

Figure 13: Plots of Original State 'x1', Estimated State '$x1_{hat}$' and difference between them i.e. 'Error=$x1_{hat}$-x1' obtained by using Continuous SMO

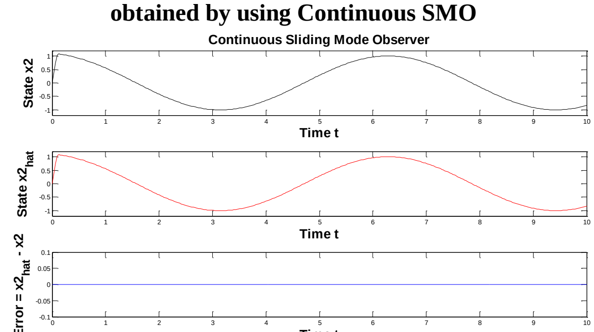

Figure 14: Plots of Original State 'x2', Estimated State '$x2_{hat}$' and difference between them i.e. 'Error=$x2_{hat}$-x2' obtained by using Continuous SMO

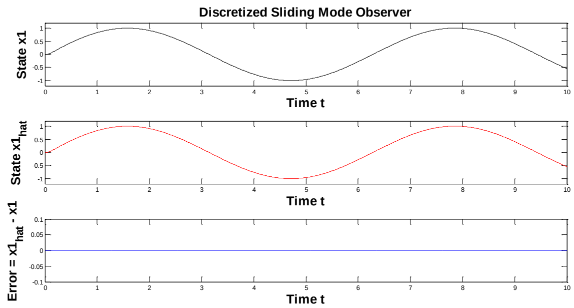

Figure 15: Plots of Original State 'x1', Estimated State '$x1_{hat}$' and difference between them i.e. 'Error=$x1_{hat}$-x1' obtained by using Discretized SMO

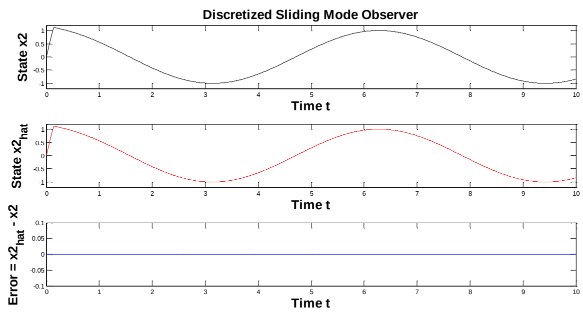

Figure 16: Plots of Original State 'x2', Estimated State '$x2_{hat}$' and difference between them i.e. 'Error=$x2_{hat}$-x2' obtained by using Discretized SMO

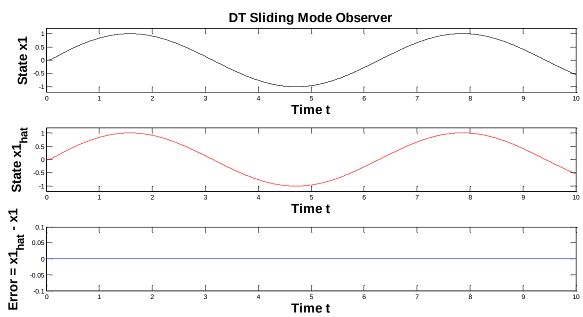

Figure 17: Plots of Original State 'x1', Estimated State '$x1_{hat}$' and difference between them i.e. 'Error=$x1_{hat}$-x1' obtained by using DT SMO

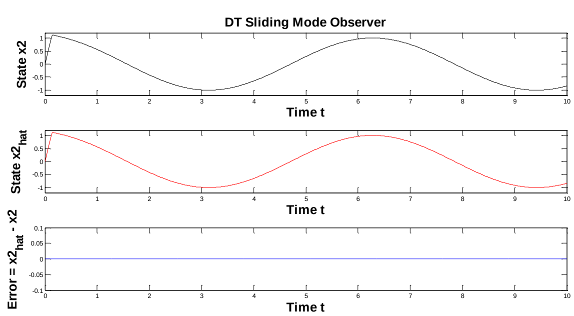

Figure 18: Plots of Original State 'x2', Estimated State '$x2_{hat}$' and difference between them i.e. 'Error=$x2_{hat}$-x2' obtained by using DT SMO

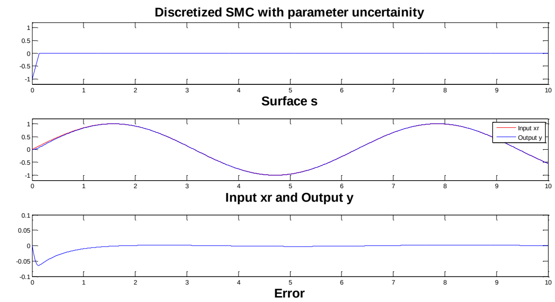

Figure 19: Plot of Sliding Surface, Reference Signal and Output Signal obtained by Discretized SMC and the Tracking Error in the prescence of Parameter Uncertainity

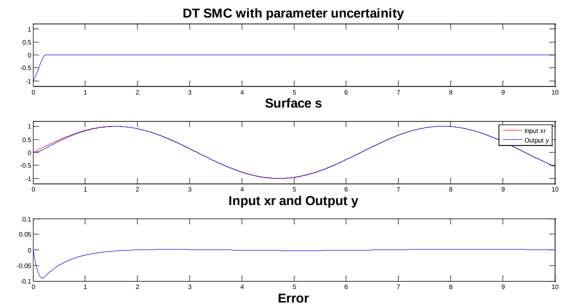

Figure 20: Plot of Sliding Surface, Reference Signal and Output Signal obtained by DT SMC and the Tracking Error in the prescence of Parameter Uncertainity

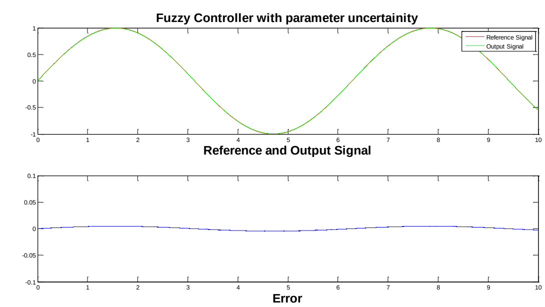

Figure 21: Plot of Reference Signal and Output Signal obtained by Fuzzy Controller and the Tracking Error in the prescence of Parameter Uncertainity

CONCLUSION

In this paper, sampled data control of BLDC motor is formulated using different controllers. For arbitrary low sampling time performance of sampled data techniques equals the performance of continuous time techniques. The performance of different controller is viewed in order to declare which controller is best for the sampled data control of BLDC Motor. Two famous sampled data techniques were simulated first is discretization of continuous time law and second is discrete time (DT) design of control based on approximate discrete time model of system. Both techniques of sampled data control were simulated for SMC. Performance of both techniques is comparable to their continuous time counterpart when system parameters were perturbed. Both techniques give almost same performance. Fuzzy Controller was designed and simulated and then simulated again with the parameter uncertainties. Performance of Fuzzy controller was affected to some extent by parameter uncertainty. Hence for sampled data control of BLDC motor, sampled SMC is the best technique and then Fuzzy Controller also gives acceptable results.

For feedback sliding mode observers were employed continuous, discretized and DT sliding mode observer were formulated and implemented SMO was used with above mentioned continuous time control laws and similarly discretized and DT observer were used with discretized and DTD control law in closed loop.

The effectiveness of the designed controllers and observers is shown by the simulation and experimental results. Moreover, the responses of the system using sampled SMC and Fuzzy Controller are presented to compare their performance.

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