OBTAINING TIME DOMAIN EXPRESSIONS OF BAND LIMITED SIGNAL PULSES FOR ZERO ISI

Ejaz A. Ansari*

Department of Electrical Engineering, COMSATS Institute of Information Technology

1.5 km Defense Road, off Raiwind Road, 53700, Lahore, Pakistan

***corresponding author's email: {dransari@ciitlahore.edu.pk}**

ABSTRACT: In this paper, we derive time domain expressions of band limited signal pulses used to obtain zero inter symbol interference (***ISI***) for digital communication. Although time domain expressions of raised cosine (***RC***) and square root raised cosine (***SRRC***) pulses have been reported in many Electrical Engineering books on Digital Communications but however, their closed forms have not been computed. We thus, make use of Fourier transform properties along with its tables in a sensible way to derive these expressions in time domain. We consider only four band limited pulses in frequency domain which are rectangular, triangular, raised cosine and square root raised cosine. We start with the Fourier transform pair of unit step function and compute the time domain expression of rectangular band limited pulse using time shift and linearity properties of Fourier transform. Then, we make use of auto correlation / convolution property of Fourier transform to obtain time domain expression of triangular band limited pulse. Finally, we compute the time domain expressions of (***RC***) and (***SRRC***) band limited pulses using linearity, differentiation and even properties of Fourier transform.

***Key Words**:* Closed forms, Raised Cosine, Square Root Raised Cosine, Time domain expressions, Zero ISI

1 INTRODUCTION

Due to infinite absolute bandwidth of multilevel pulses and their improper filtration through a communication system, spreading takes place in time domain which causes the pulses corresponding to each symbol to smear into adjacent time slots and results into inter symbol interference (*ISI*) [*1*]-[*2*]. Restriction on the bandwidth of the pulses is needed so that pulses could have rounded tops instead of flat ones in order to avoid *ISI*. Due to two significant problems present in the overall amplitude transfer characteristic of the signal pulse that causes ISI; *i*) *flat* spectrum over the range of frequencies, $|f| < B$ and zero elsewhere which results into physically *unrealizable* impulse response of the system, *ii*) the *synchronization* of the clock in the decoding sampling circuit needs to be almost perfect which results into *ISI* due to inaccurate synchronization [*3*] – [*5*].

Because of these difficulties, we are forced to include other signal pulse shapes that have a *slightly* different bandwidth. The basic idea is to find pulse shapes that pass through *zero* at adjacent sampling points and yet to have an envelope that decays much faster than $1/t$ so that clock jitter in the sampling times does not cause *appreciable ISI*. Plots of the frequency responses and their corresponding impulse responses of raised cosine (*RC*) and square root raised cosine (*SRRC*) pulses for various roll off factors, β are sketched in section 6 of this paper. It is seen from these graphs that as the absolute bandwidth of the pulse is increased, the filtering and clock timing requirements are relaxed a little bit since the envelope of the impulse response decays as $1/t^3$ which is faster than that of the order $1/t$ for large values of t. The baud rate of these pulses is dependent on the rolloff factor, β and absolute bandwidth of the system which was not the case with either rectangular or triangular shape signal pulse [1 – 5].

In this paper, we thus, compute time domain expressions / closed forms of band limited signal pulses using Fourier transform properties sensibly. We consider four types of signal pulses in the paper which are of rectangular, triangular, raised cosine and square root raised cosine shapes. Time domain expressions of raised cosine and square root raised cosine shape pulses have been reported in many books on digital communications for elimination of inter symbol interference (ISI) without their derivation.

This paper is organized as follows*: Section 2 describes the computation of time domain expression of rectangular shape pulse using Fourier transform pair of unit step function, duality, time shift and linearity properties of Fourier transform. Computation of time domain expression of triangular shape pulse is done in section 3. Section 4 and 5 describes the detailed procedure of computing time domain expressions of raised cosine and square root raised cosine shape pulses. We provide the sketch of these pulses and their time domain expressions in section 5. Finally, we present our conclusions in section 6.*

2 COMPUTATION OF TIME DOMAIN EXPRESSION OF RECTANGULAR SHAPE SIGNAL PULSE

Rectangular shaped signal pulse, $r(f)$ as per [6] – [8] is defined as; it is equal to T for $|f| \le 0.5\tau$ and zero otherwise. Shifted form of unit step functions permit us to express this pulse as, $r(f) = T\,x\{u(f + 0.5\tau) – u(f – 0.5\tau)$. We know from Fourier transform tables given in [9] – [12] that $u(t) \leftrightarrow (j2\pi f)^{-1} + 0.5\delta(f)$ and time reversal property of Fourier transform immediately allows us to write $u(-t) \leftrightarrow (-j2\pi f)^{-1} + 0.5\delta(f)$, where we have used the even property of Dirac delta function. Now, duality property of Fourier transform is utilized to express the inverse Fourier transform of $u(f)$ as $(-j2\pi t)^{-1} + 0.5\delta(t) \leftrightarrow u(f)$. If $X(f)$ denote the Fourier transform of $x(t)$, then $x(t)exp(\pm j2\pi f_0 t)$ denote the Fourier transform of $X\left(f \mp f_0\right)$ using frequency translation property. We apply this property along with linearity of Fourier transform and compute time domain expression of $r(f)$ as,

$$(-j2\pi t)^{-1} + 0.5\delta(t) \leftrightarrow u(f)$$
$$\left((-j2\pi t)^{-1} + 0.5\delta(t)\right)e^{\pm j2\pi f_0 t} \leftrightarrow u(f \mp f_0)$$
$$(-j2\pi t)^{-1} \times e^{\pm j2\pi f_0 t} + 0.5 \times \cancel{e^{\pm j2\pi f_0 t}\Big|_{t=0}}^{=1} \times \delta(t) \leftrightarrow u(f \mp f_0)$$
$$(-j2\pi t)^{-1} \times e^{\pm j2\pi f_0 t} + 0.5\delta(t) \leftrightarrow u(f \mp f_0)$$
$$(-j2\pi t)^{-1}\left\{e^{-j2\pi f_0 t} - e^{j2\pi f_0 t}\right\} \leftrightarrow u(f + f_0) - u(f - f_0)$$
$$(j2\pi t)^{-1}\left\{\underbrace{e^{+j2\pi f_0 t} - e^{-j2\pi f_0 t}}_{(2j\sin(2\pi f_0 t))}\right\} \leftrightarrow u(f + f_0) - u(f - f_0);$$

$where\ f_0 = 0.5\tau\ and\ \tau = 2/T\ ,\ thus;$

$$\tau \times \left(\frac{sin(\pi\tau t)}{(\pi\tau t)}\right) = \tau sinc(\tau t) \leftrightarrow u(f + f_0) - u(f - f_0)$$
$$T\tau sinc(\tau t) \leftrightarrow T \times \left(u(f + f_0) - u(f - f_0)\right) = r(f) \qquad (1)$$

Equation (*1*) has been derived in various books of Signals and Systems using either the definition of Fourier Transform or its differentiation property. But, in this case, it has been derived with a different approach.

3 COMPUTATION OF TIME DOMAIN EXPRESSION OF TRIANGULAR SHAPE SIGNAL PULSE

Triangular shaped signal pulse, $\Delta(f)$ as per [6] – [8] is defined as; it is equal to $T\ (1 \pm |f|\ /\ \tau)$ for $|f| \le \tau$ and zero otherwise. We realize that $\Delta(f)$ can easily be generated by performing either autocorrelation or convolution of $r(f)$ with itself. Final result of both operations are equivalent due to even nature of the pulse $r(f)$. Thus, for $f \ge 0$, we may compute the result of autocorrelation of $r(f)$ using its definition as

$for\ 0 \le f \le \tau;$

$$R_x(f) = \int_{-\infty}^{\infty} x(\lambda)x(\lambda - f)d\lambda;\ where\ x(f) = T^{-1}r(f)$$
$$R_x(t) = \int_{\lambda = f - 0.5\tau}^{0.5\tau} (1)\times(1)\,d\lambda = \lambda\Big|_{f-0.5\tau}^{0.5\tau} = \boxed{\tau - f}$$

As Autocorrelation is an even function of time and frequency, we immediately realize that it is equal to $(\tau + f)$ for $-\tau \le f < 0$. In order to scale down its maximum amplitude equal to unity at $f = 0$, we need to multiply the above operation with τ^{-1}. Thus, we develop the relationship between the two band limited pulses as,

$$\Delta(f) = T \times \left(\tau^{-1} \times \left(x(f) * x(-f)\right)\right) = T \times \left(\tau^{-1} \times \left(x(f) * x(f)\right)\right)$$
$$\updownarrow$$
$$T \times \left(\tau^{-1} \times \left(IFT\left\{T^{-1}r(f)\right\}\right)^2\right) = T \times \left(\tau^{-1} \times \left(\tau^2 sinc^2(\tau t)\right)\right) = \boxed{T\tau sinc^2(\tau t)}$$
$$\Delta(f) \leftrightarrow \boxed{T\tau sinc^2(\tau t)};\ where\ \ \tau = 2/T$$
$$\boxed{2sinc^2(2t/T) \leftrightarrow \Delta(f)} \qquad (2)$$

Equation (2) has been derived by utilizing the convolution property of Fourier transform which maps to multiplication property. This equation has also been derived in various books of Signals and Systems using either the definition of Fourier transform or its differentiation property. But, it has been obtained using a different approach.

4 COMPUTATION OF TIME DOMAIN EXPRESSION OF RAISED COSINE (RC) SIGNAL PULSE

A particular pulse spectrum which has desirable spectral properties and has been widely used in practice [1] – [5] is the raised cosine spectrum and its frequency response is described below as:

$$X_{rc}(f) = \begin{cases} 2k & ;\ for\ 0 \le |f| \le a \\ k\left(1 + \cos\left[\pi\tau^{-1}\left(|f| - a\right)\right]\right); & for\ a \le |f| \le b \\ 0 & ;\ otherwise \end{cases}$$

where $k = 0.5T$, $\tau = \beta/T$, $a = (1 - \beta)/2T$, $b = (1 + \beta)/2T$ and $\tau = b - a$ respectively. Now, argument of cosine can also be written as,

$$\pi\tau^{-1}\left(f - a + 0.5T^{-1} - 0.5T^{-1}\right) = \pi\tau^{-1}\left(\left(f - 0.5T^{-1}\right) + \left(0.5T^{-1} - a\right)\right);$$
$$\left(0.5T^{-1} - a\right) = 1/2T - (1 - \beta)/2T = \beta/2T = \boxed{0.5\tau};$$
$$\pi\tau^{-1}\left(\left(f - 0.5T^{-1}\right) + \left(0.5T^{-1} - a\right)\right) = \left(\pi\tau^{-1}\left(f - 0.5T^{-1}\right) + 0.5\pi\right)$$
$$\cos\left(\pi\tau^{-1}\left(f - 0.5T^{-1}\right) + 0.5\pi\right) = -\sin\left(\pi\tau^{-1}\left(f - 0.5T^{-1}\right)\right);$$

Now, raised cosine spectrum, $X_{rc}(f)$ can also be defined as

$$X_{rc}(f) = \begin{cases} 2k & ;\ for\ 0 \le |f| \le a \\ k\left(1 - \sin\left[\pi\tau^{-1}\left(f - 0.5T^{-1}\right)\right]\right); & for\ a \le |f| \le b \\ 0 & ;\ otherwise \end{cases}$$

We'll differentiate the above spectrum of raised cosine pulse twice in order to express it in the desired form. Thus,

$$X^{/}_{rc}(f) = \begin{cases} 0 & ;\ for\ 0 \le |f| \le a \\ -k\pi\tau^{-1}\cos\left[\pi\tau^{-1}\left(f - 0.5T^{-1}\right)\right] & ;\ for\ a \le |f| \le b\ ; \\ 0 & ;\ otherwise \end{cases}$$
$$and\ \ X^{//}_{rc}(f) = \pi^2\tau^{-2}\left(k\sin\left[\pi\tau^{-1}\left(f - 0.5T^{-1}\right)\right]\right);\ for\ a \le |f| \le b$$

In order to represent the above expression in the desirable form, we must see the simple addition of two rectangular pulses, each centered at the origin and with widths equal to $2a$ and $2b$ respectively.

$$k \times \left(\Pi\left(\frac{f}{2b}\right) + \Pi\left(\frac{f}{2a}\right)\right) = \begin{cases} 2k & ;\ for\ 0 \le |f| \le a \\ k & ;\ for\ a \le |f| \le b \end{cases}$$

With this logic, the double derivative of raised cosine spectrum can thus be written in desirable form as,

$$X^{//}_{rc}(f) = -\pi^2\tau^{-2}\left(X_{rc}(f) - k \times \left(\Pi\left(\frac{f}{2b}\right) + \Pi\left(\frac{f}{2a}\right)\right)\right);$$

The above relation in frequency domain can easily be transformed into time domain using differentiation, linearity properties of Fourier transform and the relation given in eq. (*1*) as

$$(-j2\pi t)^2 x_{rc}(t) = -\pi^2\tau^{-2}\left(x_{rc}(t) - k \times IFT\left\{\left(\Pi\left(\frac{f}{2b}\right) + \Pi\left(\frac{f}{2a}\right)\right)\right\}\right);$$
$$-(2\pi t)^2 x_{rc}(t) = -\pi^2\tau^{-2}\left(x_{rc}(t) - k \times 2 \times \left\{asinc(2at) + bsinc(2bt)\right\}\right);$$
$$\left(\pi^2\tau^{-2} - 4\pi^2 t^2\right)x_{rc}(t) = \pi^2\tau^{-2}k \times \left(\underbrace{2 \times \left\{asinc(2at) + bsinc(2bt)\right\}}_{\{y(t),\ say\}}\right) \ldots\ldots(3)$$

$$asinc(2at)+bsinc(2bt)\leftrightarrow \cancel{a}\times\frac{\sin(2\pi at)}{2\pi\cancel{a}t}+\cancel{b}\times\frac{\sin(2\pi bt)}{2\pi\cancel{b}t};$$

$$=(2\pi t)^{-1}\{\sin(2\pi at)+\sin(2\pi bt)\};$$

$$2\times\{asinc(2at)+bsinc(2bt)\}=y(t)=(\pi t)^{-1}\{\sin(2\pi at)+\sin(2\pi bt)\};$$

$$Now,\ 2\pi at=(2\pi t)((1-\beta)/2T)=(1-\beta)\pi t/T;$$

$$Similarly,\ 2\pi bt=(2\pi t)((1+\beta)/2T)=(1+\beta)\pi t/T;$$

$$\{\sin(2\pi at)+\sin(2\pi bt)\}=2\sin(\pi(a+b)t)\cos(\pi(a-b)t)$$

$$=\boxed{2\sin(\pi t/T)\cos(\pi\tau t)};$$

$$Thus,\ y(t)=\boxed{(\pi t)^{-1}\times(2\sin(\pi t/T)\cos(\pi\tau t))}$$

Substituting the value of y(t) in eq. (*3*) and performing some manipulation and simplification, we can find the desired time domain expression of raised cosine pulse as

$$\cancel{\pi^2\tau^{-2}}\times(1-4\tau^2t^2)x_{rc}(t)=\cancel{\pi^2\tau^{-2}}\times\left(2k(\pi t)^{-1}\times\sin(\pi t/T)\right)\times\cos(\pi\tau t);$$

$$(1-4\tau^2t^2)x_{rc}(t)=\underbrace{2k\times(\sin(}_{T}\pi t/T)/\pi t)\times\cos(\pi\tau t);$$

$$(1-4\tau^2t^2)x_{rc}(t)=(\sin(\pi t/T)/(\pi t/T))\times\cos(\pi\tau t)=\sin c(t/T)\times\cos(\pi\tau t);$$

$$x_{rc}(t)=\left(\frac{\sin c(t/T)}{1-4\tau^2t^2}\right)\times\cos(\pi\tau t)\leftrightarrow X_{rc}(f); \qquad \ldots\ldots\ldots\ldots(4)$$

$$where\ \tau=\beta/T\ and\ 0\le\beta\le 1$$

- ***Special Cases***

Two cases are of interest, one is for $\beta = 0$ and the other is for $\beta = 1$. When $\beta = 0$, then $\tau = \beta/T = 0$ and the pulse in time domain from eq.(4) simply reduces to $x_{rc}(t) = sinc(t/T)$ which corresponds to the spectrum $X_{rc}(f) = 2k$, for $0 \le |f| \le 1/2T$ in frequency domain. And when $\beta = 1$, then $\tau = 1/T$ and the product of sinc and cos functions simply reduces to,

$$Sinc(t/T)\times cos(\pi t/T)=1/(\pi t/T)\times(sin(\pi t/T)cos(\pi t/T))$$

$$=1/(2\pi t/T)\times(2sin(\pi t/T)cos(\pi t/T))$$

$$=sin(\pi(2t/T))/(\pi(2t/T))=sinc(2t/T)$$

$$and\ 1-4\tau^2t^2=1-4(t/T)^2; \qquad for\ |f|\le 1/T$$

$$x_{rc}(t)=\left(sinc(2t/T)/\left(1-4(t/T)^2\right)\right)\leftrightarrow X_{rc}(f)=k\left(1+cos(\pi\tau^{-1}f)\right);$$

5 COMPUTATION OF TIME DOMAIN EXPRESSION OF SQUARE ROOT RAISED COSINE (SRRC) SIGNAL PULSE

The frequency response of this pulse is defined as [4];

$$X_{srrc}(f)=X_{rc}^2(f)=\begin{cases}2k & ;\ for\ 0\le|f|\le a\\ k\left(1+\cos\left[\underbrace{\pi\tau^{-1}(|f|-a)}_{\theta}\right]\right) & ;\ for\ a\le|f|\le b\\ 0 & ;\ otherwise\end{cases}$$

Making use of the identity, $cos(2\theta) = 2cos^2(\theta) - 1$ and after taking the square root of the amplitude of the pulse, we can write it in the following form as

$$X_{srrc}(f)=\begin{cases}\sqrt{2k} & ;\ for\ 0\le|f|\le a\\ \sqrt{2k}\left(\cos\left[\underbrace{0.5\pi\tau^{-1}}_{\tau_1}(|f|-a)\right]\right) & ;\ for\ a\le|f|\le b\\ 0 & ;\ otherwise\end{cases}$$

We'll apply here the definition of inverse Fourier transform to find out the corresponding pulse in time domain. First, we see that root raised cosine pulse spectrum is a real and even function of frequency, *f*. It means that its corresponding pulse in time domain will also be a real and even function of time, *t* when inverse transformed in time domain. Using inverse definition of Fourier transform, we have

$$x_{srrc}(t)=\int_{f=-\infty}^{\infty}X_{srrc}(f)e^{j2\pi tf}df$$

$$X_{srrc}(f)e^{j2\pi tf}=\underbrace{X_{srrc}(f)\cos(2\pi tf)}_{even\ function}+j\underbrace{X_{srrc}(f)\sin(2\pi tf)}_{odd\ function}$$

$$x_{srrc}(t)=\left(\int_{f=-\infty}^{\infty}X_{srrc}(f)\cos(2\pi tf)df\right)+j\left(\cancelto{=0}{\int_{f=-\infty}^{\infty}X_{srrc}(f)\sin(2\pi tf)df}\right);$$

$$x_{srrc}(t)=2\int_{f=0}^{\infty}X_{srrc}(f)\cos(2\pi tf)df$$

$$=2\times\left\{\int_{f=0}^{a}\underbrace{\sqrt{2k}}_{k'}\cos(2\pi ft)df+\int_{f=a}^{b}\sqrt{2k}\cos\left(\underbrace{\tau_1(f-a)}_{\alpha}\right)\cos\left(\underbrace{2\pi ft}_{\gamma}\right)df\right\};$$

$$Now,\ \cos\alpha\cos\gamma=0.5\{\cos(\alpha+\gamma)+\cos(\alpha-\gamma)\};\ \alpha\pm\gamma=(\tau_1\pm 2\pi t)f-a\tau_1;$$

$$=2k'\times\left\{\left.\frac{\sin(2\pi ft)}{2\pi t}\right|_{f=0}^{f=a}+0.5\left.\left(\frac{\sin((\tau_1+2\pi t)f-a\tau_1)}{\underbrace{(\tau_1+2\pi t)}_{\mu}}+\frac{\sin((\tau_1-2\pi t)f-a\tau_1)}{\underbrace{(\tau_1-2\pi t)}_{\eta}}\right)\right|_{f=a}^{f=b}\right\};$$

$$\mu b\ or\ \eta b-a\tau_1=(\tau_1\pm 2\pi t)b-a\tau_1=(b-a)\tau_1\pm 2\pi bt=\underbrace{\tau\tau_1\pm 2\pi bt}_{0.5\pi}=(0.5\pi\pm 2\pi bt);$$

$$\boxed{\sin(\mu b\ or\ \eta b-a\tau_1)=\sin(0.5\pi\pm 2\pi bt)=\cos(2\pi bt)};$$

$$\mu a-a\tau_1=(\tau_1+2\pi t)a-a\tau_1=\ 2\pi at;\eta a-a\tau_1=(\tau_1-2\pi t)a-a\tau_1=-2\pi at;$$

$$\cos(2\pi bt)(\mu^{-1}+\eta^{-1})-\sin(2\pi at)(\mu^{-1}-\eta^{-1});$$

$$2^{nd}\ part\ of\ the\ result\ reduces\ to:$$

$$0.5\times\left(\cos(2\pi bt)(\mu^{-1}+\eta^{-1})-\sin(2\pi at)(\mu^{-1}-\eta^{-1})\right)$$

$$=(\mu\eta)^{-1}\{0.5(\eta+\mu)\cos(2\pi bt)-0.5(\eta-\mu)\sin(2\pi at)\};$$

$$(\eta\pm\mu)=2\tau_1\ or-4\pi t\ and\ (\mu\eta)=(\tau_1^2-4\pi^2t^2)=\tau_1^2\left(1-(2\pi t/\tau_1)^2\right);$$

$$\left(1/2\pi t-\left(0.5\underbrace{(\eta-\mu)}_{-4\pi t}/\mu\eta\right)\right)\sin(2\pi at)=\sin(2\pi at)\left(\left(\tau_1^2-\cancel{4\pi^2t^2}\right)+\cancel{4\pi^2t^2}/\mu\eta(2\pi t)\right)$$

$$=\sin(2\pi at)(\tau_1^2)/(2\pi t)\mu\eta;\ But\ k'=\sqrt{2k}=\sqrt{T}\ and\ \tau_1=0.5\pi/\tau$$

$$x_{srrc}(t)=2k'\times(\tau_1^{-1}\times\cos(2\pi bt)+\sin(2\pi at)/2\pi t)/\left(1-(2\pi t/\tau_1)^2\right);$$

$$x_{srrc}(t)=2\sqrt{T}/\left(1-(4\tau t)^2\right)\times(2\tau/\pi\times\cos(2\pi bt)+\sin(2\pi at)/2\pi t);$$

$$\boxed{x_{srrc}(t)=4\tau\sqrt{T}/\pi\left(1-(4\tau t)^2\right)\times(\cos(2\pi bt)+\sin(2\pi at)/4\tau t)}; \qquad \ldots\ldots\ldots\ldots(5)$$

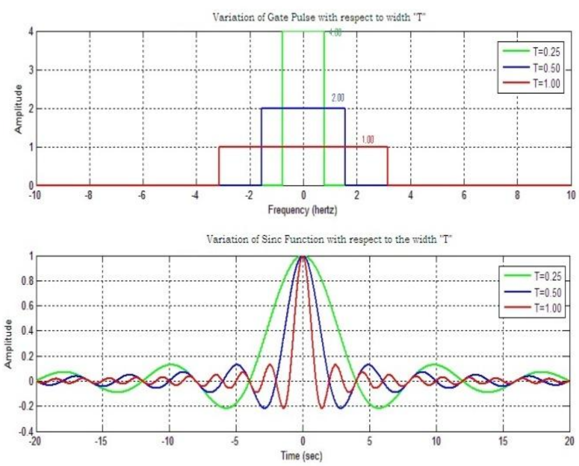
$$where\ \tau=\beta/T;a\ or\ b=(1\mp\beta)/2T,$$

Special Cases

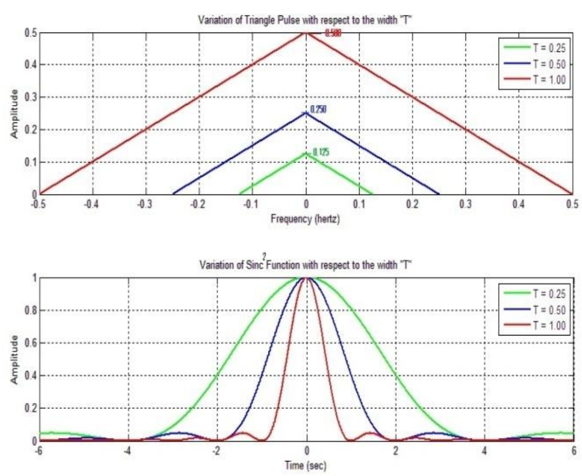
Again here, two cases are of interest, one is for $\beta = 1$ and the other is for $\beta = 0$. When $\beta = 1$, then $\tau = \beta/T = 1/T$, $a = 0$ *and* $b = 1/T$. The 2[nd] term in eq. (5) vanishes and the corresponding pulse in time domain can easily be obtained. Similarly when $\beta = 0$, then $\tau = 0$ and $a = b = 1/2T$ and the pulse in time domain simply reduces to sinc function, *i.e.*, $x_{srrc}(t) = sqrt(T)\ x\ sinc(t/T)$ which corresponds to the spectrum $X_{srrc}(f) = sqrt(2k)$, for $0 \le |f| \le 1/2T$ in frequency domain.

6 PLOTTING OF TIME DOMAIN EXPRESSIONS OF SIGNAL PULSES

In this section, we sketch the time domain expressions of band limited signal pulses described in eqs. (*1*) – (*5*) for different values of τ and excess bandwidth parameter, β. Figure *1* (*a*) and (*b*) shows the variation of rectangular and triangular signal pulses for different values of τ and the variation of their corresponding time domain expressions for these values of τ. The phenomenon of compression / expansion in frequency domain is clearly visible in the sketches of these time domain expressions.



(a) **Sktech of rectangular pulse & its closed form for different** ***T***

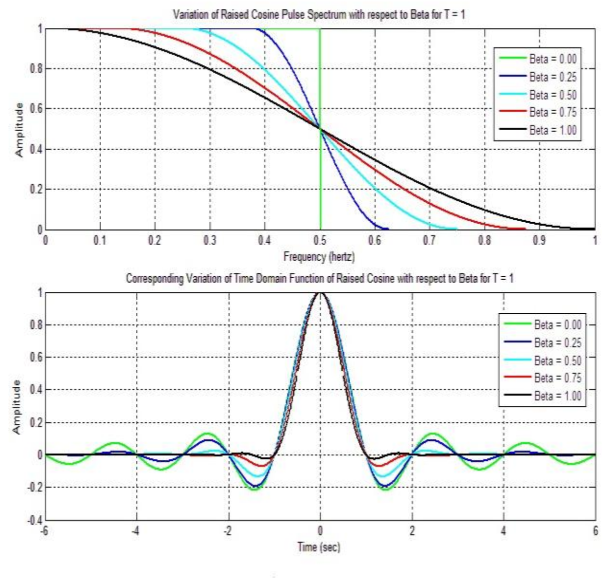


(b) Sktech of Triangular pulse & its closed form for different ***T***

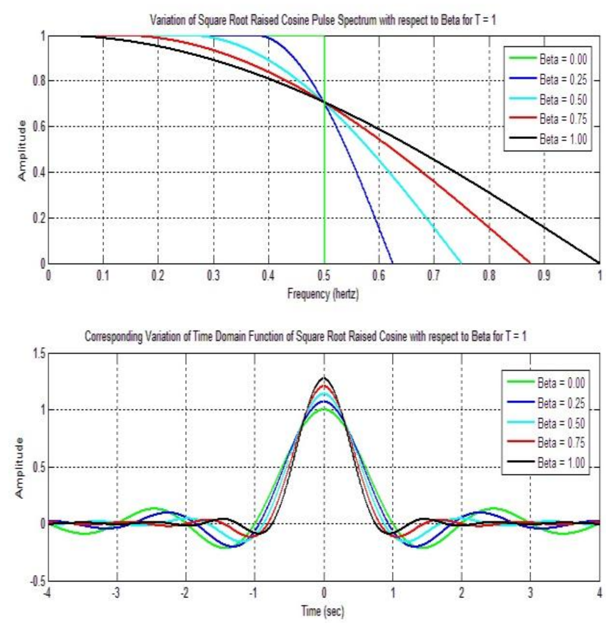
Figure 1, a) and b)**: Variation of Rectangular and Triangular pulses for different values of τ in frequency and time domains**.

Moreover, the rate of decay of closed forms is t and t^2 and both functions in time domain are non-causal which require a time delay of significant length to be able to become physically realizable.

Figure *2* (*a*) and (*b*) shows the variation of raised cosine and its square root signal pulses for different values of excess bandwidth parameter, β and the variation of their corresponding time domain expressions for these values of β. The variation of raised cosine and its square root pulses in frequency domain is clearly visible in the sketches of these time domain expressions.



(a) Sketch of Raised cosine pulse & its closed form for different β



(b) Sketch of Raised cosine pulse & its closed form for different β

Figure 2, a) and b)**: Variation of Rectangular and Triangular pulses for different values of τ in frequency and time domains.**

Figure (*2*) *a*) and *b*) shows clearly the variation of closed forms with respect to the excess bandwidth parameter, β. In comparison to figure *1*, the rate of decay of these functions in time domain is ($1/t^3$) instead of ($1/t$) or ($1/t^2$). However, like figure *1*, a long time delay is required in order to make both functions realizable. Moreover, the bandwidth of the functions in figure *2* depends upon the parameter, β which was not the case in figure *1*.

7 CONCULUSIONS

In this paper, we derived closed forms or time domain expressions of four band limited signal pulses such as rectangular, triangular, raised cosine and square root raised cosine. Although closed forms for rectangular and triangular pulses have been computed in many books on Signals and Systems, however, we used a different approach here in obtaining the closed form expressions of these two pulses. The closed forms of raised cosine and square root raised cosine pulses which are extensively used in digital communications for zero *ISI* have been reported in many books on Digital Communications without their derivation. We used Fourier transform properties along with its tables in a sensible manner to compute the closed form expressions of these important pulses. Finally, we showed the variation of these pulses and their corresponding time domain expressions with respect to the parameters T and β in the form of sketches.

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