SOLUTION OF OPTIMAL PROBLEMS BY USING MODIFIED LAGRANGEAN MULTIPLIER TECHNIQUE

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ABSTRACT: *Optimization techniques represent an analytical tool for the possible solution of a particular problem, for structuring a typical framework into constrained optimization and subsequently solved by the Lagragean multiplier, in product design and process development. These optimization techniques are employed to generate optimal formulation in different industries to locate levels of processing variables in a typical encapsulation. In this paper the research approach was made to resolve the problems by using Lagrangean multiplier method. In this method the objective quadratic function and linear constraint being solved for several variables by eliminating Hessian border matrix. This method is restricted to equal linear constraints, while the Lagragean Multiplier is generally used for obtain maxima and maxima of a linear programing function, whereas in this paper the linear constraints of quadratic function is being achieved by setting the sufficient condition for the stationary point to be maxima or minima. Here we generate a coefficient matrix from the first order partial derivative of Lagrangean equation. i-e coefficient matrix of the type and find det (A) for the extremal points. The purposed modification in Lagrangean method was compared with common method for better understanding through proposed methodology by eliminating the Hessian matrix, as it contains higher order partial derivatives. The results were similar to results obtained by common procedures, it is concluded from obtained results and modification made in Lagrangean Multiplier technique that the proposed method is more reasonable and suitable for linear quadratic functions to solve such problems well in time for better understanding.*

1. INTRODUCTION

The mathematical models for solving problems used in linear and non-linear programming, are not suitable due to their lengthy and complex procedures. Several approaches were made to establish a new method for such distinctive solutions in everyday life. The Lagrangean multiplier (LM) is one of the most widely used techniques for determining the optimum constrains problems, comprising a linear objective function and non-linear inadequate equations. The multiples of a (LM) initially deducted from the objective function. The inequality constraints and additional variables are modified by using LM method by altering and compensating by Shetty, C. M. (2013). The objective functions with a unit change in the constraint values on right-hand-side. The (LM) constraints as multiplier in which the values are taken from an intended function and solved after differentiating w.r.to individual variable (Bazaraa, 2013). The analogous dual variables in linear and non-linear programming problems are resolved. The estimated transform in purposed functions, resulting with a unit change in the quantity on right hand side values of constrained equations. The partial derivate of Langrage functions with respect to each other of three variables were obtained by Luenberger, D. G., & Ye, Y. (2008). Lagragean Multiplier is considered as a valid procedure for the satisfied assumptions of sensitive interpretation as dependent and independent variable in linear and non-linear constraints respectively (Bertsekas, D. P. (2014). The minimal vector of LM is not well defined and quite informative in this respect, in which some of the constraints are duplicates to others. The every set of alternate constraint have non-zero multiplier in minimal LM Vector, whereas in the informative LM vector is also based on either none or all duplicate constraints. In such examples, each of them has two non-zero components with four minimal LM vectors (Sheblé, G. B. 2012).The constrained optimization of Lagrangean Multiplier problems in Lagrangean relaxation of any function is presented as (λ), is also considered as LM problem, which is a dual primal problem, based on constrained optimization. The characterizations of Lagrangean optimality and availability for such convexity also taken in consideration in presence of saddle point, by which a computational potential is also become known for conventional problems. Under certain cases the extended linear-quadratic programming is also expressed (Hemamalini, S., & Simon, S. P. 2010). In past few years some analytical approximations have been also made to solve the non-linear differentials equations. Such as a method to study the variation iteration, considered as the new change in Lagrangean Model. The techniques have certain restrictions and limitations to make corrections in the said functions, but have widespread significance and uses (Abdou and Soliman (2005)). The technique does not need the existence of small parameters in the differential equations, at the same time differentialabiity and derivatives of such a non-lineraties functions w.r.to dependent variable is no more needed. In this technique a series of functions which come together to reach the accurate explanation of the problems. The process corresponded as a technical means to solve several problems. The example for such of the equations are differential equations (He 1999), one and two dimensional coordination of linear and non-linear equations in thermo-elasticity given by the (Sweilam and Khader (2007) and Maxwell equations (Sweilam et al. (2010) respectively. Designing of such optimum values for a set of (n) design variables (x_1, x_2 ,. . . x_n), were also found , which reduces an objective functions of design variable with scalar values. Such that m is an inequality constraint is to be satisfied. In general situations if the objective function is a quadratic function in the design variable and at the same time the constraint equations are also found to be linear in it, then the optimization of the problems have different solution. Now here keep in Consideration a simple constrained minimization problem. In which the basic model the analysis for convergence is made with HI-estimates over the displacement of a finite element, as a variation inequality (D.E. Finkel

2006). The foremost revisions were accomplished with a wider class of reliability hypothesis with L2-for the estimations of errors in system. During last few decades, several techniques have been proposed to resolve the ED problems caused by the capability to come together in possible universal optimization. The stochastic systems are inherent algorithm, simulated annealing (Cheng, 2004.), genetic algorithm (Azad, M. 2012). Several conservative justifications of such methods are presently used to explain

$$\frac{\partial}{\partial x}L(x,y,\lambda)=\frac{\partial}{\partial x}f(x,y)+\frac{\partial}{\partial x}\lambda.g(x,y)$$

$$\frac{\partial}{\partial y}L(x,y,\lambda)=\frac{\partial}{\partial y}f(x,y)+\frac{\partial}{\partial y}\lambda.g(x,y)$$

$$\frac{\partial}{\partial \lambda}L(x,y,\lambda)=\frac{\partial}{\partial \lambda}f(x,y)+\frac{\partial}{\partial \lambda}\lambda.g(x,y)$$

the economic development problems in linear (Zadeh, A. K., 2010), nonlinear (Hemam alini, S., 2011) programing, quadratic functions (Sinha, N., 2003) and Lagrangean reduction algorithms (Ravindran, A., 2006). The LM assists in better understanding and potential assessment for the values of objective function in particular constraint problems. The solutions have enormous credibility for economic impact, towards reduced execution as a new technique in cost reduction and maintenance engineering for the improvement of products production as an output (Denardo, E. V. 2012). The systematic approach assist in resolving the issues associated with the substitution method (Pardalos, P. M. 2001).Therefore it needs emphasis and further analysis to shorten these commonly used mathematical models, are still under consideration. Indeed the Lagrangean Multiplier is a better solution for the constrained optimization of problems; at the same time slight complex issues are resolved by this technique in non-linear programming.

2. RESEARCH METHODOLOGY

Indeed an effective and a quite simple mathematical model/framework has been produced for systematic generalization of algorithms by the mean field theory of approximation. The methods of a Lagrangean multiplier that work as an alternative by relaxation single dynamic system. Following procedure was used to apply this method of Lagrangean Multiplier, in order to minimize or maximizing the quadratic functional f(x,y), subjected to linear constraints g(x,y) = K.

2.1 Step 1: First create the Lagrangean quadratic Function

The purpose is creating a Lagrangean functions by optimization and joining with the restraint function as shown here:

$$L(x,y,\lambda)=f(x,y)+\lambda g(x,y)$$

For the best possible explanation。

2.2 Step 2: Finding the first order Partial derivatives

At the second stage, the partial derivatives were found w.r.to individual variables x, y and λ of the given function as given below:

$$L(x,y,\lambda)=f(x,y)+\lambda g(x,y)$$

2.3 Step: 3 setting of each partial derivative equal to zero (necessary condition)

At the third stage the derivatives of each variable were set to zero.

$$\frac{\partial}{\partial x}L(x,y,\lambda)=0$$

$$\frac{\partial}{\partial y}L(x,y,\lambda)=0$$

$$\frac{\partial}{\partial \lambda}L(x,y,\lambda)=0$$

to find

$$L_x=0\,, L_y=0 \; and \; L_\lambda=0$$

Then substituting the λ by using $L_x = 0$, $L_y = 0$

The procedure was carried out to find out the values for x and y. After that reveal for λ = 0, in provisions of λ for the solutions of x and y, as to obtain the value of L which can be used to discover the most favorable values x and y.

2.4 Step:4 Sufficient Condition

The sufficient condition for the stationary point to be maxima or minima are obtained by solving the principal minors of the bordered hessian matrix.

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

To set the sufficient condition for the stationary point to be maxima or minima, we generate a coefficient matrix from the first order partial derivative of Lagrangean equation. i-e coefficient matrix of the type and find det (A) for the extremal points

The main purpose of the modification in Lagrangean method is to eliminate the Hessian matrix, as it contains higher order partial derivatives.

Example 1

The developed equation for measure profit in the starting of section $P(x, y) = -2x^2+60x-3y^2+72y+100$. In which x indicates the number of handmade chairs whereas y is the number of rockers produced per week respectively on which the imperative benefit is g(x,y). The constraint profit is g(x, y) = x + y = 20, In order to find the optimal solution given this constraint we will follow the method given above. First we will create the Lagrange equation. So as to locate the ideal arrangement given to the limitation, then we will take over the strategy given above. To begin with we made the langrage condition and equation.

$L(x, y) = p(x, y) - \lambda [g(x,y) - k] = (-2x^2 + 60x - 3y^2 + 72y + 100) - \lambda (x+y-20)$

Now find $L_x = 0$, $L_y = 0$ and $L_\lambda = 0$

$L_x = -4x+60 - \lambda = 0$

$L_y = -6y+72 - \lambda = 0$

$L_\lambda = -x-y+20 = 0$

Putting Lx = 0, Ly = 0 to resolve for x and y to get $x = 15-1/4\,\lambda$ and $y = 12-1/6\,\lambda$. Then putting values of x and y into $L_\lambda = 0$ to obtain, $L_\lambda = -x-y+20 = -(15-1/4\,\lambda) - (12-1/6\,\lambda) + 20 = (-27+10/24\,\lambda)+20 = -7 + 5/12\,\lambda = 0$ and $\lambda = 16.8$ so $x = 10.8 \approx 10$ and $y = 9.2 \approx 9$.

The marginal profit was achieved by producing 10 chairs and 9 rockers respectively.

$\det(H^B) = 10 \geq 0$

System of equation in Matrix for proposed work, in this research paper,

$\det = 10 \geq 0$

Example 2

Suppose the function $f(x_1, x_2, x_3) = 1/2\,(x_1^2, x_2^2, x_3^2)$

Subjected to $ø_1(x) = x_1 - x_2$ and $ø_2(x) = x_1+x_2+x_3-1$

Sol: $L(x, \lambda) = f(x) + \lambda_1 ø_1 + \lambda_2 ø_2 = x_2/2 + x_2^2/2 + x_3^2/2 + \lambda_1(x_1 - x_2) + \lambda_2(x_1+x_2+x_3-1)$

$\partial L/\partial x_1 = x_1 + \lambda_1 + \lambda_2$

$\partial L/\partial x_2 = x_2 - \lambda_1 + \lambda_2$

$\partial L/\partial x_3 = x_3 + \lambda_2$

$\partial L/\partial \lambda_1 = x_1 - x_2$

$\partial L/\partial \lambda 2 = x_1+x_2+x_3-1$

By solving (0.333, 0.333, 0.333, 0, -0.333)

$$P = \begin{pmatrix} \partial ø_1/\partial x_1 & \partial ø_1/\partial x_2 & \partial ø_1/\partial x_3 \\ \partial ø_2/\partial x_1 & \partial ø_2/\partial x_2 & \partial ø_2/\partial x_3 \\ \partial^2 L/\partial x_1^2 & \partial^2 L/\partial x_1 \partial x_2 & \partial^2 L/\partial x_1 \partial x_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$Q = \begin{matrix} \partial^2 L/\partial x_1 \partial x_2 & \partial^2 L/\partial x_2^2 & \partial^2 L/\partial x_1 \partial x_3 \\ \partial^2 L/\partial x_1 \partial x_2 & \partial^2 L/\partial x_2 \partial x_3 & \partial^2 L/\partial x_3^2 \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \qquad (H^B)^t = \begin{pmatrix} O & P \\ P^t & Q \end{pmatrix}$$

$$Q = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$H^B = \begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$\det(H^B) = 6 \geq 0$

System of equation in Matrix for proposed work, in this research paper,

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$\det = 6$

3. CONCLUSIONS

The results were similar to results obtained by common procedures, it is concluded from obtained results and modification made in Lagrangean Multiplier technique that the proposed method is more reasonable and suitable for linear quadratic functions to solve such problems well in time for better understanding.

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