DESIGNING OF ADAPTIVE DICTIONARIES BASED ON MEXICAN HAT WAVELET FOR SPARSE REPRESENTATION

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ABSTRACT— *In this paper, we analyzed and compare the over-complete dictionaries based on fixed basis transforms like discrete cosine transform (DCT), Discrete wavelet transform (DWT). The over-complete dictionaries have a very important role to represent the signal or image patch for measuring the sparsity of the signal. In sparse modeling, there is need to design an appropriate dictionary. However, there are many dictionaries used for sparse modeling and were reported in literature. In this paper, we implemented the fixed dictionaries and adaptive dictionaries, i.e., Method of optimal direction (MOD) and KSVD. Both adaptives are used for training the noisy images and computing the error and recovered the number of atoms using adaptive or small patches of images. The dictionaries havea very important role to capture the data of the small image patch and store the redundant representation based on sparsity data. We want to investigate which dictionary is appropriate to store the structure of the image patch, after investigation of pre-specified basis function, we conclude that Wavelet transform based over-complete dictionary perform well and produce accurate results as compared to the other dictionaries. The result showed that our proposed dictionaries based on Mexican hat wavelet performed much better for atom recovery in noisy patches of the image. The dictionary based on discrete wavelet transform basis function with MOD and KSVD produced accurate result as compared to dictionary based on discrete cosine transform basis function.*

Keywords—Sparse Representation; KSVD; DWT; DCT;Over-complete Dictionaries;MOD

1. INTRODUCTION

In Sparse representation can be used in many signal ad image processing applications. Sparse representation of signals in terms of redundant dictionaries is fast evolving research area which produced very high impactful results in the field of image and signal processing [1]. In this approach signals were compressed compactly or approximated in efficient manner in terms of linear combinations of pre specified atom signals. Also the linear coefficients are very sparse i.e., approaching zero [2].

The main ingredient for implementation of above model is to choice of dictionary i.e., analytic approach and other one is learning based approach. In the analytic approach, there is a need to formulate a mathematical model and developed analytic construction efficiently represents this model. This leads towards dictionaries that are highly structured and numerically fast implementation. These analytical dictionaries are fast and also address the structural information of the signals or image patches. The accuracy of the analytical dictionaries may not be up to the standard, but improved using the adaptive algorithms.

These dictionaries are also referred as implicit dictionaries because it only describes the algorithm instead of their explicit matrix. These dictionaries include Wavelets based [2], Curvelets based [3], Contourlets based [4], Complex Wavelets [5] etc. In this paper, we used the analytical dictionaries based on discrete cosine, discrete wavelet transform. These analytical dictionaries further used as a base dictionary and we applied KSVD to train the analytical dictionary to recover the dictionary atoms for sparse image modeling. The Mexican hat wavelet basis function is used in designing the discrete wavelet transform dictionary.

The basic idea is to use the analytical dictionary and trained this dictionary using KSVD algorithm. Also we used MOD algorithm for training purposes. Result comparison among these dictionaries was carried out using small patches of real image with small Gaussian noise factor and results showed that our proposed adaptive dictionaries based on discrete wavelet basis function as compared to discrete cosine basis function based dictionary.

The organization of paper is as follows: The related work is discussed in Section II. The proposed technique is explained in Section III. Section 1V presents the results. The conclusion is presented in Section V.

2. BACKGROUND

Normally two main methods are used to determine a dictionary within a sparse de-composition of signals, i.e., dictionary selection and dictionary learning. Dictionary selection requires selection of a pre-existing dictionaries, e.g., Fourier basis, wavelet basis or modified discrete cosine basis or over-complete dictionary i.e., union of bases. Therefore, it can represent different properties of the signal [8].In Dictionary learning approach; it focuses at constructing the dictionary from the available training data so that the atoms of dictionary can directly capture the particular features of the signals [8]. Dictionary learning methods are mostly based on optimization scheme, in this way the dictionary remains fix and sparse signal decomposition is found. After that the dictionary elements are learned keeping the signal representation is fixed.

Aharon *et al.* [8] proposed the K-SVD algorithm which involves a sparse coding stage based on a pursuit method incorporated by an update step, in which the dictionary matrix is updated single column at the time, while permitting the expansion coefficients to change. More recently, dictionary learning methods for exact sparse representation based on $\ell 1$ minimization and online learning algorithms have been proposed.

3. PROPOSED METHODS

We proposed the fixed dictionaries which are very fast and results in better accuracy. The pre specified dictionaries are based on the discrete cosine transform basis functions; discrete wavelet transforms basis function.

A. Discrete Cosine Transform

The DCT is based on the Discrete Fourier Transform which replaces the complex analysis with real numbers by a symmetric signal extension. The DCT is an orthonormal transform and normally suited for first order Markov stationary signals. DCT coefficients normally represent the frequency contents of the signal same as obtained from Fourier analysis. To deal with non-stationary sources or signals, DCT is typically applied in blocks, e.g., in the case of JPEG image compression algorithm. Selection of overlapping blocks is preferred to analyze the signals while avoiding the artifact. In this case, again an over-complete transform with a redundancy factor of 4 for an overlap of 50% [8]. Therefore, DCT is an appropriate for a sparse representation of signals either for smooth or periodic behaviors.

B. Discrete Wavelet Transform

In 1980's, a new very powerful tool, known as wavelet analysis [10] was proposed for multi resolution analysis of the signals. The theory was developed for both the continuous and discrete time signals. A major breakthrough came from Meyer's work [10], who found that unlike the Gabor transform, the wavelet transform could be designed to be orthogonal while maintaining stability. Mallat established the wavelet decomposition as a multi-resolution expansion and proposed efficient computing algorithms [11].

In our proposed method, we used the mother wavelet called Mexican Hat wavelet [12] as a basis function to implement the discrete wavelet transform and this mother wavelet is represented mathematically in following equations.

$$T(t_1)=\frac{2}{\sqrt{3\sigma}\pi^{1/4}}\left(1-{t_1}^2\big/\sigma^2\right)e^{-\frac{{t_1}^2}{2\sigma^2}} \tag{1}$$

$$T(t_1)=\frac{2}{\sqrt{3}\pi^{1/4}}\left(1-{t_1}^2\right)e^{-\frac{{t_1}^2}{2}} \tag{2}$$

$$T(t_2)=\frac{2}{\sqrt{3\sigma}\pi^{1/4}}\left(1-{t_2}^2\big/\sigma^2\right)e^{-\frac{{t_2}^2}{2\sigma^2}} \tag{3}$$

$$T(t_2)=\frac{2}{\sqrt{3}\pi^{1/4}}\left(1-{t_2}^2\right)e^{-\frac{{t_2}^2}{2}} \tag{4}$$

$$T(t_1,t_2)=T(t_1)*T(t_2) \tag{5}$$

Where the two 1-D functions are multiplied to each other to give the final product as shown in the equation (5).The functions also depend upon the standard deviation values and time values.

The Mexican hat wavelet basis function is used to design the DWT dictionary for sparse representation. The number of steps is shown in the Fig.1.The fixed dictionaries are used the different element sizes in this algorithm and the proposed dictionary based on DWT used Mexican hat wavelet function to implement the dictionary for sparse representation. The over-complete DWT dictionary has scaling and rotation parameters to stores the basis function as shown in Fig.1.The Mexican hat wavelet function return the matrix based on basis values and used this matrix for further as a dictionary for measuring the sparse coefficients.

Wavelet Transform Dictionary based on Mexican Hat wavelet Basis Function

D is the dictionary size, S_1, S_2 are scale factors, θ_1, θ_2 are the Dilation parameters, M_1 number of dictionary atoms, N_1 size of dictionary, M and N are scaling factors.

FOR each Scale $S_1 = 1:M$ **do**
  FOR each Scale $S_2 = 1:M$ **do**
    For each translation $\theta_1 = N$ **do**
      For each translation $\theta_2 = N$ **do**
        For each $n_1 = 1:M_1$ **do**
          For each $n_2 = 1:N_1$ **do**
            $temp(n_1,n_2) = \max icanhat\_wavelet()$
            Store $D_1 = temp(n_1,n_2)$
          END FOR
        END FOR
        IF $D_1(:,count) = temp(:)$
          count=count+1
        END IF
      END FOR
    END FOR
  END FOR
END FOR
Store $D = D_1$
END

Fig.1. Implement DWT dictionary based on Mexican hat wavelet basis function.

C. KSVD Algorithm

A generalization of the K-means algorithm for dictionary learning called the K-SVD algorithm has been proposed by Aharon [8]. The sparse approximation step is updated based on the orthogonal matching pursuit algorithm and update each column of the dictionary using singular value decompositing.in this way the residual error can be minimized. The each patch of the signal has different weights and can be represented in the dictionary with multiple atoms. The only drawback of this algorithm is usually not converge and may not produce good accuracy for the large signals or image patches. However, The KSVD algorithm shows good performance for image de-noising applications. The basic steps of the algorithm can be optimized as shown in the following equations.

Given fixed sparse matrix B, The dictionary D can be updated by solving the following problem:

$$\min_{D}\left\{\|Y-DB\|_F^2\right\} \quad subject \quad to \quad \forall i \quad , \quad \|b_i\|_0 \le N_0 \tag{6}$$

In sparse coding stage, there are two assumptions are exist. First is that the D is fixed and the sparse coefficients are updated using the objective function as shown in the equation (6). The second assumption you can fixed the sparse coefficients and dictionary can be updated using any optimization algorithm like KSVD and MOD. The objective function can be solved using the SVD decomposition

$$\left\|Y-DB\right\|_F^2=\left\|Y-\sum_{k=1}^{j}c_k b_T{}^k\right\|_F^2=\left\|\left(Y-\sum_{k\neq j}^{j}c_k b_j{}^k\right)-c_k b_j{}^T\right\|_F^2 \qquad (7)$$

$$\left\|Y-DB\right\|_F^2=\left\|E_k-c_k B_T{}^K\right\|_F^2 \qquad (8)$$

Where $b_j{}^T$ are the rows of sparse matrix B, the c_k denotes the atom of the dictionary D and the E_k stands for the residual matrix. After the SVD decomposition of the matrix E_k, both the atom c_k and $b_j{}^T$ can be updated. This SVD is time consuming process. We introduced the fixed dictionaries called DCT and DWT instead of using random to improve the accuracy and then apply the K-SVD algorithm to further update the sparse coding stage adaptively.

4. EXPERIMENTAL RESULTS

The two pre-specified dictionaries have been proposed and these dictionaries are called discrete cosine transform (DCT), discrete wavelet transforms (DWT) and trained these dictionaries using K-SVD as well as MOD algorithms. Different size of dictionaries was implemented as shown in Fig. (2, 3, 4, 5) using DCT basis function. When the size of the dictionary increases the computationally complexity also increases. The Fig.2 shows that the dictionary with size 8 is able to store the irregular pattern, features, and structures of the signal as compared to other sizes of dictionaries as shown Fig. (3, 4, 5) In case of other all cases, dictionary atoms capture the regular features of the signal or image patch. However, the computationally complexity is increased with the increase of the size. In Fig.2 the elements of over-complete dictionaries are prominent and these bases are used to store the image patch by using linear combinations of dictionary atoms. Similarly can be shown in Fig. (3, 4 and 5).

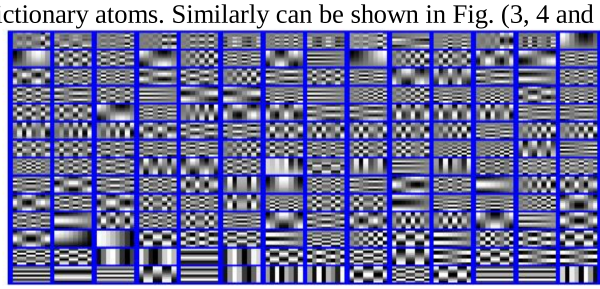

Fig. 2. DCT with element size(8x8).

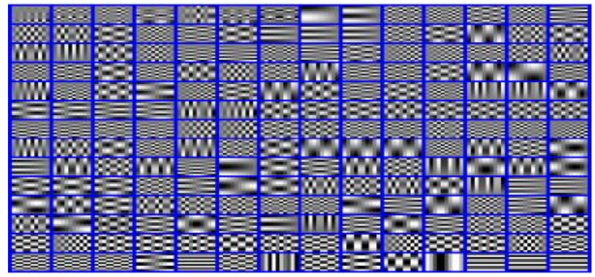

Fig. 3. DCT with element size(10x10).

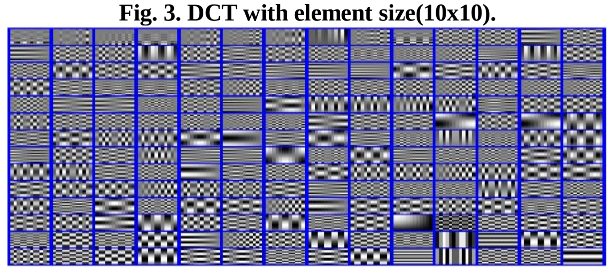

Fig. 4. DCT with element size(12x12).

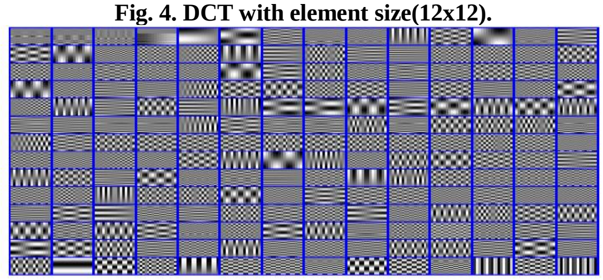

Fig. 5. DCT with element size(16x16).

The proposed dictionary based on discrete wavelet basis function contains atom size of 8x8 as shown in the Fig. 6. It can be seen from there that the image patch features stored is not regular and it can store only singular points of the image patch. When the size of the atoms increases the dictionary atoms stores the more singular points as evident from Fig. 9, where the atom size is 16x16. Also when the size of the dictionary is 10x10 and 12x12 then the storage of the image features is moderate as shown in the Fig.7 and 8.

Fig. 6. DWT with element size(8x8)

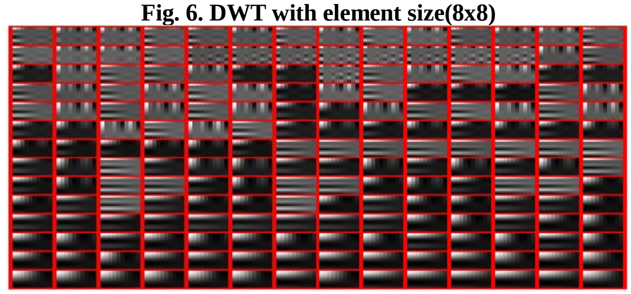

Fig. 7. DWT with element size(10x10)

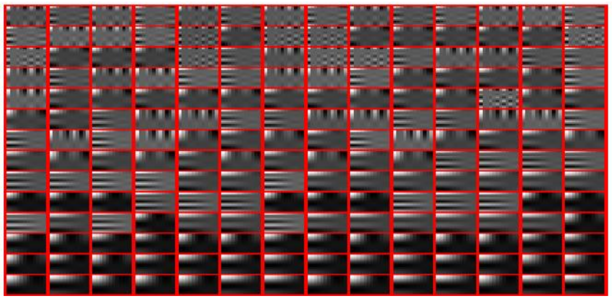

Fig. 8. DWT with element size(12x12)

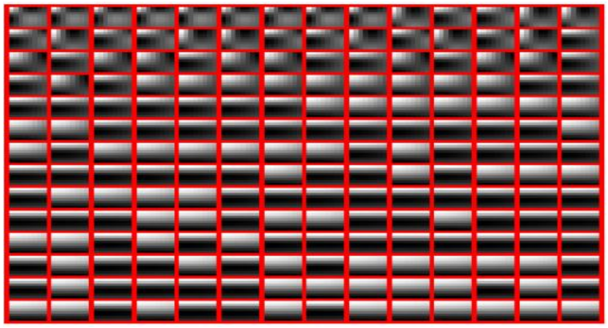

Fig. 9. DWT with element size(16x16)

The root mean square values of different fixed dictionaries have been computed and are shown in the Fig. 12 to Fig. 13. All the base dictionaries has the atom size is 8x8 and these base dictionaries has been updated adaptively with KSVD and MOD algorithms. From Fig. 12 to Fig. 13, it shows that DWT with KSVD and MOD, the root mean square error (RMSE) values are very low as compared to the DCT with KSVD and MOD. Now we discuss the each case individually, In Fig. 12, the RMSE value for DWT+KSVD and DWT+MOD is 0.018 as compared to DCT+KSVD and DCT+MOD having RMSE values 0.02 and 0.027 respectively. Further, it is evident that the convergence capability of DWT with KSVD and MOD algorithm is very fast as compared to DCT with KSVD+MOD. In DWT case, the convergence to stable values occurs in less than five iterations as compared to DCT, which converges to minimum RMS at iteration number 15. This is the significant improvement in terms of convergence for RMSE values to minimum with DWT with KSVD and MOD.

The total number of iterations was 50 to calculate the RMSE values of the dictionaries and RMSE values are decreasing at each iteration because the KSVD trained the dictionaries and ratio of recovered atoms stored in the dictionaries are increases with the increase of the number of iterations. There is no big difference seen in discrete wavelet basis function dictionary as shown in the Fig. 11. When the size of the atoms is increased by 10x10.The RMSE values are different and increase the difference between DCT as well as small increase the difference in DWT. The atom size of 12x12 dictionaries has made no much difference and produced approximately equal RMSE values at the stage of the transitions. However, in case of 16x16, there is significant decrease in RMSE value to 0.009 for DWT with KSVD and MOD in comparison to the DCT with MOD and KSVD i.e., RMSE value is 0.02 as depicted in Fig. 13.

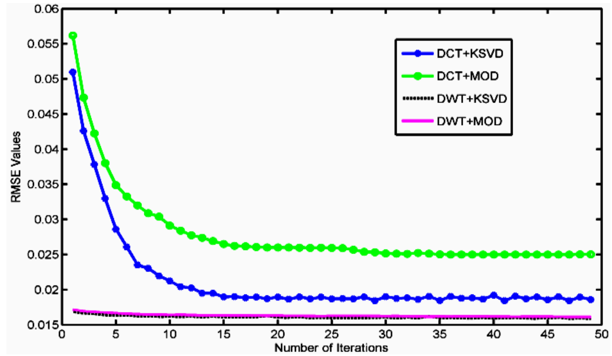

Fig. 10. Root mean Square values of dictionaries with the number of iterations and size is 8.

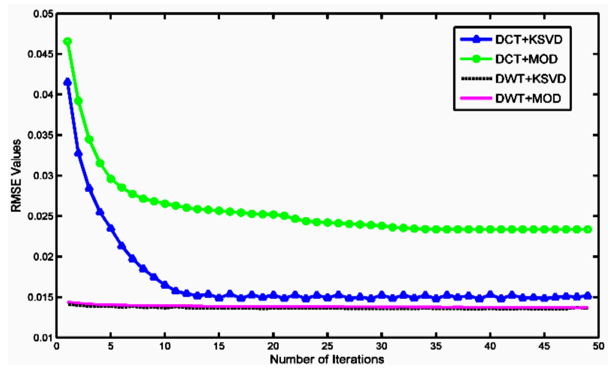

Fig. 11. Root mean Square values of dictionaries with the number of iterations and size is 10.

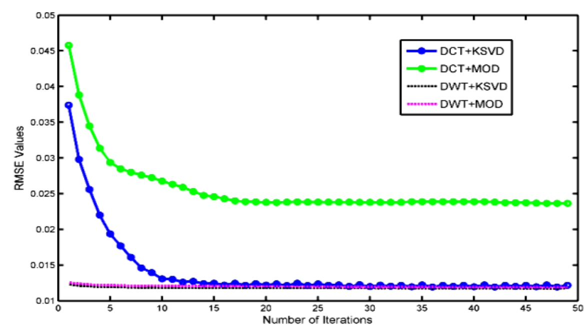

Fig. 12. Root mean Square values of dictionaries with the number of iterations and size is 12.

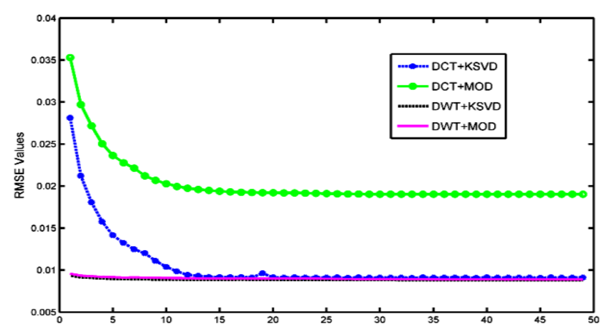

Fig. 13. Root mean Square values of dictionaries with the number of iterations and size is 16.

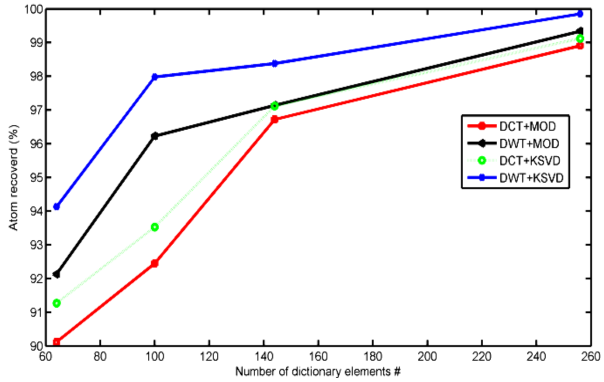

Fig. 14. Atom Recovery based on different dictionary elements using fixed and adaptive dictionaries.

The atom recovered using different dictionaries with several numbers of sizes is shown in the Fig.14.The proposed dictionaries with two algorithms are used to recovered the sparse atoms and also analyzed these dictionaries using different number of elements. The dictionary based on DWT and optimized using KSVD algorithm produces outperform results as compared to other dictionaries and also shows that the atom recovery increases with the increase of the number of dictionary elements during sparse modeling as shown in Fig.14. The DWT using KSVD with element size 256 almost recovered 995 atoms as shown blue lines in the graph. The dictionary builds with DWT basis function using Mexican hat wavelet basis function is more accurate as compared to the discrete cosine transform. The DWT based dictionary captures the singular points, edges and good image structures. The restored atoms achieved through DWT are more as compared to DCT based dictionary.

5. CONCLUSION

In this paper we proposed the over-complete fixed dictionaries (DCT, DWT) for sparse representation and these dictionaries are trained using the well-known KSVD algorithm and MOD algorithms. Results showed that our proposed over-complete DWT dictionary based on Mexican hat wavelet basis function produced accurate results as compared to the over-complete DCT dictionary. The root means square values for DWT based dictionary with KSVD and MOD algorithms are significantly lower as compared to the DCT based dictionary with KSVD and MOD algorithms.

In future work, the dictionary can be designed based on Ridgelet transform for measuring the coefficients of sparse representation and will compare the performance among these dictionaries.

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