ACCELERATING FLOW OF OLDROYD-B FLUID OVER THE BOUNDARY WITH NO SLIP ASSUMPTION

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ABSTRACT: *This paper is devoted for the investigation of accelerating flow of Oldroyd-B fluid at a plate under the assumptions of no slip effects. The analytical solutions are traced out for velocity and shear stress profiles by employing mathematical transforms (integral transforms) on the governing partial differential equations of integer order. The generalized solutions have been transcribed into the product of convolution theorem of transforms, multiple integrals and elementary functions. These solutions fulfill all implemented conditions (natural, boundary and initial conditions) as well. Particularized solutions have been reduced form generalized solutions for Newtonian, Second Grade and Maxwell fluids as the limiting cases of present analysis. At last, in order to bring few physical features from modeled problem, graphical illustration is under lined for contrast between models, flow parameters, retardation and relaxation time, material parameters with appropriate rheology.*

Key Words: No slip assumptions, Integral transforms, Oldroyd-B fluids, Graphical rheology.

NOMENCLATURE

ν	Kinematic viscosity
μ	Viscosity of the fluid
ρ	Density of the fluid
Ω_0	Non zero constant
φ	Fourier sine transform parameter
η	Laplace transform parameter
t	Time parameter
λ	Relaxation time
λ_r	Retardation time
u_{OB}	Velocity field for Oldroyd-B fluid
u_M	Velocity field for Maxwell fluid
u_N	Velocity field for Newtonian fluid
u_{SG}	Velocity field for Second Grade fluid
τ_{OB}	Shear stress for Oldroyd-B fluid
τ_M	Shear stress for Maxwell fluid
τ_N	Shear stress for Newtonian fluid
τ_{SG}	Shear stress for Second Grade fluid
$H(t)$	Heaviside function

INTRODUCTION

The Navier-Stoke's equation is an equation which governs on the characteristics of Newtonian fluids but the description of rheology for complex fluids (non-Newtonian fluids) has become inadequate by Navier-Stoke's equation. The complex fluids lie in non-Newtonian fluids for instance, paints, cosmetic products, exotic lubricants, polymer solutions, clay coatings, colloidal and suspension solutions, certain oils and several others. The non-Newtonian fluid flows have significance because of scientific point of view in which nonlinearity of fluid has practical significance in science, engineering and industries. In order to detect the characteristics of non-Newtonian fluid, many models have been presented in literature such as integral type, rate type and differential type. The second grade fluid lies in the category of differential type known as simplest subclass model which delineates the differences of normal stress, on contrary the phenomena. Of thickening and thinning can not be predicted by this model. In continuation, the complexities in different constitutive equations are apparent like, Maxwell model that describes the relaxation phenomenon of fluids and Oldroyd-B model that demarcates the relaxation as well as retardation phenomenon of viscoelastic fluids [1-4]. Sanela and et al. investigated effects of two vertical oscillating plates for unsteady second grade fluid [5]. Zhang and et al. have analyzed the thermal convection for Oldroyd-B fluid for stability of nonlinear and linear in heated porous medium from bottom [6]. Kashif and et al. examined effects of no slip condition for analytical solutions of Maxwell fluid in unidirectional plate. He obtained analytical results for velocity field and shear stress using mathematical transforms satisfying all imposed conditions [7], same authors extended the results for second grade fluid embedded in porous medium using fractional derivative approach [8]. In brevity, we include here few recent references as well [9-14]. However, our aim is to analyze the investigation of accelerated flow of generalized Oldroyd-B fluid on the plate under the assumptions of no slip effects. The analytical solutions are traced out for velocity and shear stress profiles by employing mathematical transforms (integral transforms) on the governing partial differential equations of integer order. The generalized solutions have been transcribed into the product of convolution theorem of transforms, multiple integrals and elementary functions. These solutions fulfill all implemented condition (natural, boundary and initial conditions) as well. Particularized solutions have been reduced form generalized solutions for Maxwell, Second Grade and Newtonian fluids as the limiting cases of present analysis. At last, in order to bring few physical aspects from modeled problem, graphical illustration is under lined for contrast between models, flow parameters, retardation and relaxation time, material parameters and other rheology. The graphs are drawn using Mathcad package (15) with SI units.

GOVERNING EQUATIONS

The Oldroyd-B fluid is characterized by the constitutive relations [15-16],

$$\boldsymbol{T} = \boldsymbol{S} - \boldsymbol{I}p\,, \quad \frac{\partial \boldsymbol{S}}{\partial t}\lambda + \boldsymbol{S} = \left[\frac{\delta \boldsymbol{A_1}}{\delta t}\lambda_r + \boldsymbol{A_1}\right]\mu\,, \tag{1}$$

In order to investigate the governing equations, we suppose velocity field of the type

$$\boldsymbol{V} = \boldsymbol{V}(y,t) = u(y,t)\boldsymbol{i}\,, \qquad \boldsymbol{S} = \boldsymbol{S}(y,t), \tag{2}$$

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If fluid is at rest at the moment $t = 0$ then constraint of incompressibility is consistently justified for this flow,

$$\boldsymbol{V} = (y,0) = 0\,, \qquad \boldsymbol{S} = (y,0) = 0\,, \tag{3}$$

solving (1) and (2) reduces to

$$\partial p/_{\partial x}\frac{1}{\rho} + \frac{\partial u(y,t)}{\partial t} - \frac{1}{\rho}\frac{\partial \tau(y,t)}{\partial t} = 0, \tag{4}$$

For nonappearance of pressure gradient, Oldroyd-B fluid has equations are

$$\partial u(y,t)/_{\partial t}\left(1+\lambda\frac{\partial}{\partial t}\right) - \nu\frac{\partial^2 u(y,t)}{\partial^2 t}\left(1+\lambda_r\frac{\partial}{\partial t}\right) = 0 \;, \tag{5}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\tau(y,t) - \mu\,\frac{\partial u(y,t)}{\partial y}\left(1+\lambda_r\frac{\partial}{\partial t}\right) = 0. \tag{6}$$

Suppose an incompressible Oldroyd-B fluid lodging the space overhead an accelerated plate vertical to the y-axis, to arise with fluid is at rest and at the moment $t = 0^+$ the plate is suddenly taken to the variable velocity in plane. Due to shear, the fluid overhead the plate is gradually accelerated. The appropriate problem under initial and boundary conditions are

$$u(y,0) = \frac{\partial u(y,0)}{\partial t} = 0,\;\; \tau(y,0) = 0, \qquad y > 0, \tag{7}$$

$$u(0,t) = \Omega_0 H(t) t^a, \qquad a > 0\,, t \geq 0. \tag{8}$$

Further

$$u(y,t), \quad \frac{\partial u(y,t)}{\partial t} \to 0 \quad as\; y \to \infty \quad and \quad t > 0, \tag{9}$$

equation (9) is satisfied.

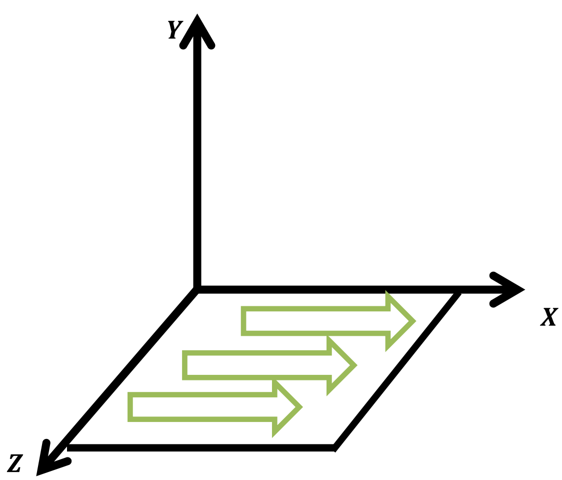

Figure 1. Schematic Graph of $u(0,t) = \Omega_0 H(t) t^a$.

SOLUTION OF THE PROBLEM

Applying Fourier Sine transform on equation (5) and considering equation (7) and (8), we obtain

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\partial u_s/_{\partial t} + \nu\varphi^2\left(1+\lambda_r\frac{\partial}{\partial t}\right)u_s = \Omega_0\nu\varphi\sqrt{\frac{2}{\pi}} \times [H(t)t^a + \lambda_r\delta(t)t^a + \lambda_r a H(t)t^{a-1}], \tag{10}$$

where, Fourier sine transform is defined as

$$u_s(\xi,t) = \int_0^\infty \sqrt{2/\pi}\, u(y,t) Sin(y\varphi) dy, \tag{11}$$

must justify equation (7) as

$$u_s(\varphi,0) = \frac{\partial u_s(\varphi,0)}{\partial t} = 0, \quad \varphi > 0\,, \tag{12}$$

By applying the Laplace transform to equation (10), (7) and (8), we find

$$\overline{u_s} = \frac{\Omega_0\,\nu\,\varphi\, a!\,\sqrt{2/\pi}\,(1+\lambda_r\eta)}{\eta^{a+1}(\lambda\eta^2 + (1+\lambda_r\nu\varphi^2)\eta + \nu\varphi^2)}, \tag{13}$$

equivalently

$$\overline{u_s} = \frac{\Omega_0\sqrt{2/\pi}\;a!}{\varphi}\left[\frac{1}{\eta^{a+1}} - \frac{1+\lambda q}{\eta^a(\lambda\eta^2 + (1+\lambda_r\nu\varphi^2)\eta + \nu\varphi^2)}\right], \tag{14}$$

Inverting (14) by means of the Fourier sine formula, we get

$$\bar{u} = \frac{2\,\Omega_0\,a!}{\pi}\int_0^\infty \frac{\sin(y\varphi)}{\varphi}\left[\frac{1}{\eta^{a+1}} - \frac{1+\lambda q}{\eta^a(\lambda\eta^2 + (1+\lambda_r\nu\varphi^2)\eta + \nu\varphi^2)}\right]d\xi, \tag{15}$$

or equivalently

$$\bar{u} = \frac{2\,\Omega_0\,a!}{\pi}\int_0^\infty \frac{\sin(y\varphi)}{\varphi}\left[\frac{1}{\eta^{a+1}} - \frac{1+\lambda\eta}{\eta^a\lambda(\eta-\eta_1)(\eta-\eta_2)}\right]d\varphi, \tag{16}$$

where

$$\eta_1,\eta_2 = \frac{-\left(\frac{1+\lambda_r\nu\varphi^2}{\lambda}\right) \pm \sqrt{\left(\frac{1+\lambda_r\nu\varphi^2}{\lambda}\right)^2 - \frac{4\nu\varphi^2}{\lambda}}}{2}, \tag{17}$$

are the roots of the algebraic quadratic equation

$$\left\{q^2 + \left(\frac{(1+\lambda_r\nu\varphi^2)}{\lambda}\right)q + \frac{\nu\varphi^2}{\lambda}\right\} = 0\,.$$

Finally, we apply the inverse Laplace transform to (16) using the fact integration

$$\int_0^\infty \frac{\sin(y\varphi)}{\varphi}d\varphi = \frac{\pi}{2}, \qquad y > 0. \tag{18}$$

We find velocity field in convolution form,

$$u = \Omega_0\,H(t)\,t^a - \frac{2\,a\,\Omega_0\,H(t)}{\pi\lambda(\eta_1-\eta_2)}\int_0^\infty \frac{\sin(y\varphi)}{\varphi}\,t^{a-1} * \{(1+\lambda\eta_1)e^{\eta_1 t} - (1+\lambda\eta_2)e^{\eta_2 t}\}\,d\varphi, \tag{19}$$

or equivalently in integral form,

$$u_{OB} = \Omega_0 H(t)t^a - \frac{2\,a\,\Omega_0\,H(t)}{\pi\lambda(\eta_1-\eta_2)}\int_0^\infty\int_0^t \frac{\sin(y\varphi)}{\varphi}(t-q)^{a-1}\{(1+\lambda\eta_1)e^{\eta_1 q} - (1+\lambda\eta_2)e^{\eta_2 q}\}\,d\varphi\,dq. \tag{20}$$

For perusing shear stress, we apply Laplace transform on equation (6), we get

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\bar{\tau} = \mu\left(1+\lambda_r\frac{\partial}{\partial t}\right)\frac{\partial\bar{u}}{\partial y}, \tag{21}$$

precisely solving, we find that

$$\bar{\tau} = \mu\frac{\partial\bar{u}}{\partial y}\left(\frac{1+\lambda_r\eta}{1+\lambda\eta}\right). \tag{22}$$

Finding derivative partially equation (16) with respect to y,

$$\frac{\partial\bar{u}}{\partial y} = -\frac{2\Omega_0 a!}{\pi\lambda}\int_0^\infty \cos(y\varphi)\frac{1+\lambda\eta}{\eta^a(\eta-\eta_1)(\eta-\eta_2)}d\varphi, \tag{23}$$

Employing equation (23) in equation (22), we have

$$\bar{\tau} = -\frac{2\,\Omega_0\,\mu\,a!}{\pi\lambda}\int_0^\infty \cos(y\varphi)\frac{1+\lambda_r\eta}{\eta^a(\eta-\eta_1)(\eta-\eta_2)}\,d\varphi, \quad (24)$$

More equivalently equation (24) can be expressed as

$$\bar{\tau} = -\frac{2\,\Omega_0\,\mu\,a!}{\pi\lambda}\int_0^\infty \cos(y\varphi)\frac{1}{\eta^a}\left\{\frac{1+\lambda_r\eta_1}{(\eta-\eta_1)(\eta_1-\eta_2)} - \frac{1+\lambda_r\eta_2}{(\eta-\eta_1)(\eta_1-\eta_2)}\right\}d\varphi, \quad (25)$$

Applying inverse Laplace transform on equation (25), we get shear stress

$$\tau = -\frac{2\,\mu\,a\,\Omega_0\,H(t)}{\pi\,\lambda(\eta_1-\eta_2)}\int_0^\infty \cos(y\varphi)\,t^{a-1} * \{(1+\lambda_r\eta_1)e^{\eta_1 t} - (1+\lambda_r\eta_2)e^{\eta_2 t}\}d\varphi \quad (26)$$

or equivalently in integral form,

$$\tau_{OB} = -\frac{2\,\mu\,a\,\Omega_0\,H(t)}{\pi\,\lambda(\eta_1-\eta_2)}\int_0^\infty\int_0^t \cos(y\varphi)\,(t-q)^{a-1} \times \{(1+\lambda_r\eta_1)e^{\eta_1 q} - (1+\lambda_r\eta_2)e^{\eta_2 q})\}\,d\varphi\,dq. \quad (27).$$

LIMITING CASES

Maxwell Fluid

Substituting $\lambda_r \to 0$ into equations (20) and (27)

$$u_M = \Omega_0 H(t)t^a - \frac{2\,a\,\Omega_0\,H(t)}{\pi\lambda(\eta_1-\eta_2)}\int_0^\infty\int_{\mathbf{0}}^{\boldsymbol{t}}\frac{\sin(y\varphi)}{\varphi} (t-q)^{a-1}\{(1+\lambda\eta_1)e^{\eta_1 q} - (1+\lambda\eta_2)e^{\eta_2 q}\}\,d\varphi\,dq. \quad (28)$$

$$\tau_M = -\frac{2\,\mu\,a\,\Omega_0\,H(t)}{\pi\lambda(\eta_1-\eta_2)}\int_0^\infty\int_0^t \cos(y\varphi)\,(t-q)^{a-1} \times (e^{\eta_1 q} - e^{\eta_1 q})\,d\varphi\,dq \quad (29)$$

Second Grade Fluid

Employing the limit $\lambda \to 0$ into equations (20) and (27)

$$u_{SG} = \Omega_0 H(t)t^a - \frac{2\,a\,\Omega_0\,H(t)}{\pi}\int_0^\infty\int_{\mathbf{0}}^{\boldsymbol{t}}\frac{\sin(y\varphi)}{\varphi} \times (t-q)^{a-1}(e^{\eta_1 q} - e^{\eta_2 q})\,d\varphi\,dq. \quad (30)$$

$$\tau_{SG} = -\frac{2\,\mu\,a\,\Omega_0\,H(t)}{\pi}\int_0^\infty\int_0^t \cos(y\varphi)\,(t-q)^{a-1} \times \{(1+\lambda_r\eta_1)e^{\eta_1 q} - (1+\lambda_r\eta_2)e^{\eta_2 q})\}\,d\varphi\,dq\,. \quad (31).$$

Newtonian Fluid

Employing $\lambda \to 0$ and $\lambda_r \to 0$ into equations (20) and (27) and having following relations

$$\lim_{\lambda,\lambda_r\to 0}\eta_1 = -\nu\varphi^2,$$
$$\lim_{\lambda,\lambda_r\to 0}\eta_2 = -\infty\,,$$
$$\lim_{\lambda,\lambda_r\to 0}\lambda(\eta_1-\eta_2) = 1$$

the solutions are reduced

$$u_N = UH(t)t^a - \frac{2\,a\,\Omega_0\,H(t)}{\pi}\int_0^\infty\int_0^t\frac{\sin(y\varphi)}{\varphi} \times (t-q)^{a-1}e^{-\nu\varphi^2 q}\,d\varphi\,dq \quad (32)$$

$$\tau_N = -\frac{2\,a\,\mu\,\Omega_0\,H(t)}{\pi}\int_0^\infty\int_0^t \cos(y\varphi) \times (t-q)^{a-1}\,e^{-\nu\varphi^2 q}\,d\xi\,dq\,. \quad (33)$$

are achieved.

NUMERICAL RESULTS AND CONCLUDING REMARKS

In this section, analytical results have been plotted using various numerical values in order to bring some physical aspects from accelerating plate. The analysis for investigation of accelerated flow of generalized Oldroyd-B fluid on the plate under the assumptions of no slip effects is sought out. The analytical solutions are traced out for velocity and shear stress profiles by employing mathematical transforms (integral transforms) on the governing partial differential equations of integer order. Finally the validations and accuracy of the fluid flow is illustrated by making several graphs using Madcad software (15). However the main results and outcomes are generated below:

- Figure 2 depicts the increasing behavior of velocity as well as shear stress profile when time is varying and all remaining parameters are fixed.
- The relationship between relaxation and retardation phenomenon is under lined in figures 3 and 4. Dynamically oscillations of fluid flows have been examined on both velocity field and shear stress.
- Figure 5 is plotted to shoe effects of viscosity of fluid on plate, in which velocity field has sequestrating behavior of fluid, on contrary shear stress tends to scattering.
- The opposite rheology is observed from figures 6 and 7. This is due to increment of nonlinearity on the fluid motion.
- By seeing figures 8 and 9, motion of Newtonian fluid for the velocity field and corresponding to the shear stress are faster than as compare to and contrast with Oldroyd-B, Maxwell and Second Grade fluid.
- Seeing figures 8 and 9, amongst all mentioned fluids such as Maxwell, Second Grade and Newtonian, sometimes Newtonian fluid moves fastest.

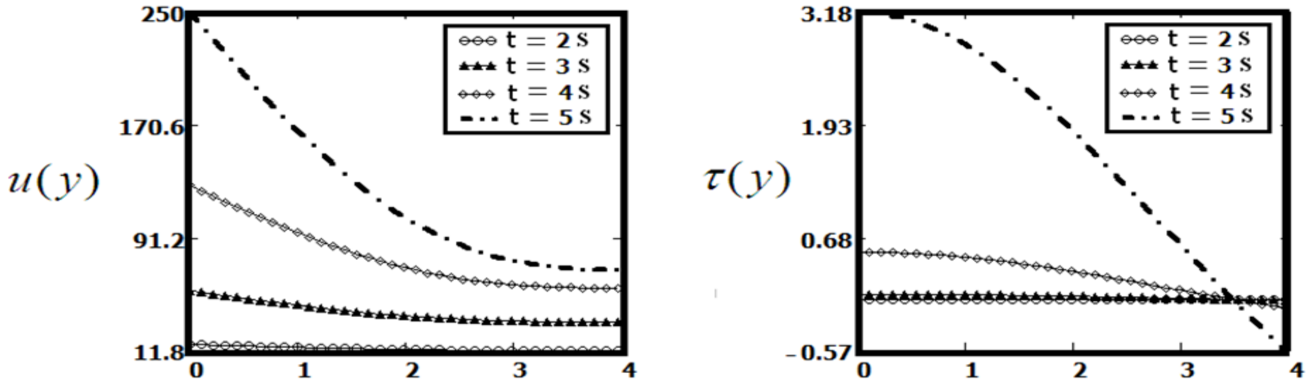

Figure 2: Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for Oldroyd-B fluid from equations (20) and (27) for $\Omega_0 = 2,\ \nu = 0.76,\ \mu = 2.41,\ \lambda = 2,\ \lambda_r = 3\ \ a = 2$ and different values of t.

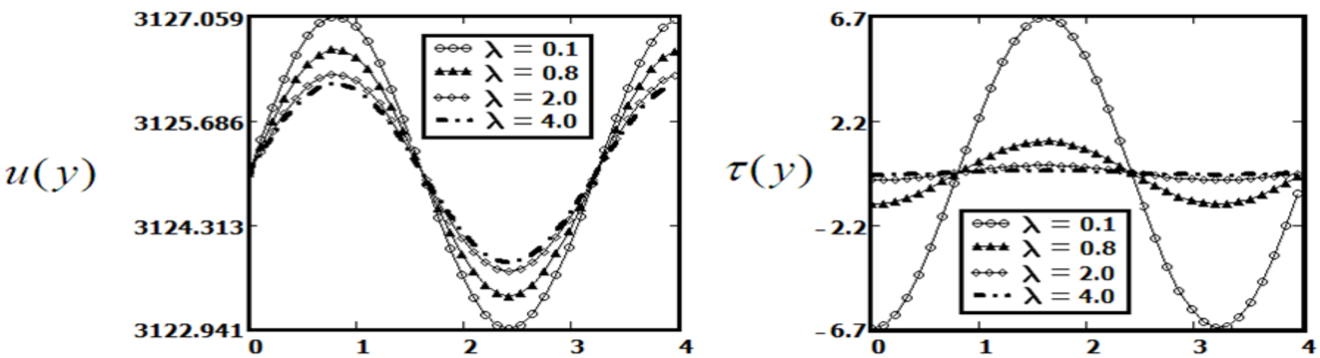

Figure 3. Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for Oldroyd-B fluid from equations (20) and (27), for $\Omega_0 = 2,\ \nu = 0.76,\ \mu = 2.41,\ \lambda_r = 3,\ t = 5\ s,\ a = 2$ and different values of λ.

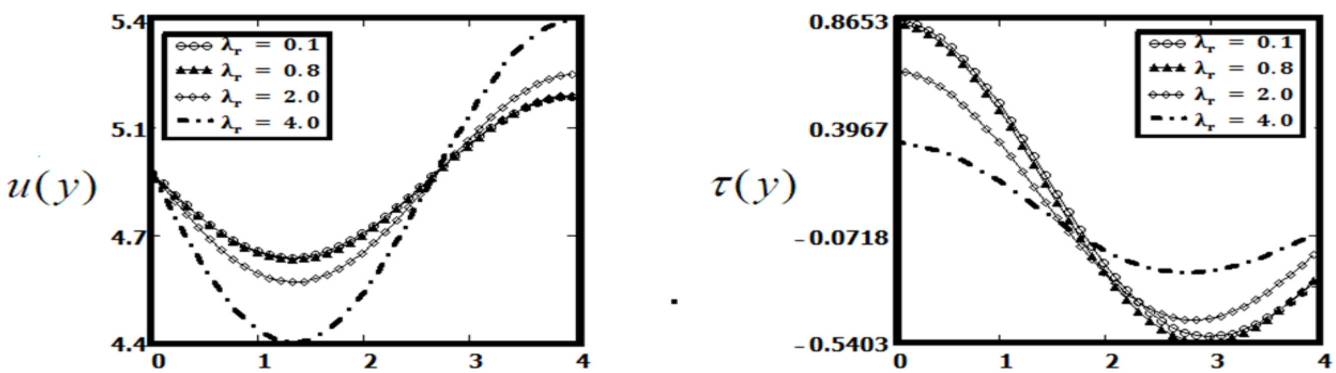

Figure 4. Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for Oldroyd-B fluid from equations (20) and (27), for $\Omega_0 = 2,\ \nu = 0.76,\ \mu = 2.41,\ \lambda = 2, t = 5\ s,\ a = 2$ and different values of λ_r.

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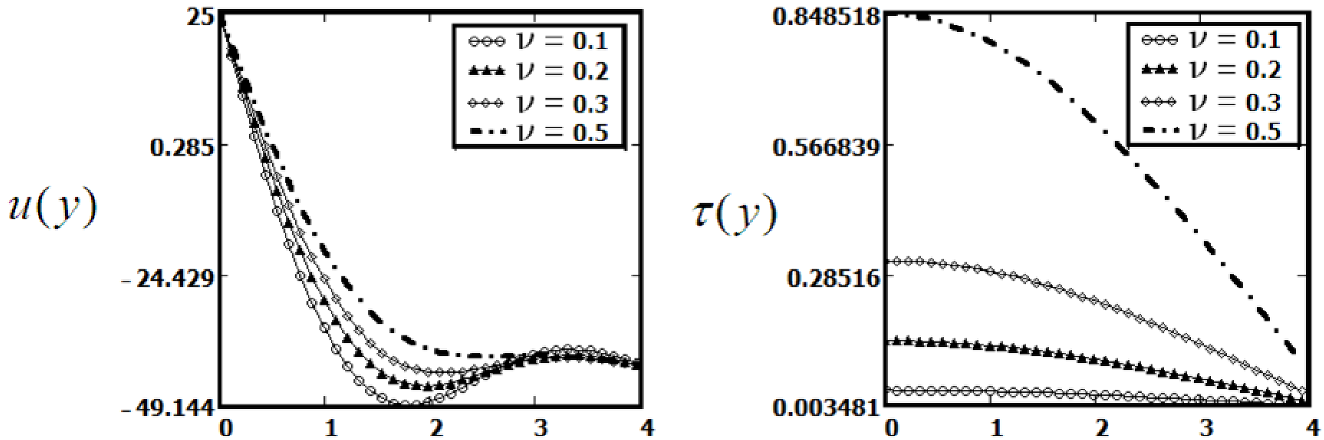

Figure 5. Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for Oldroyd-B fluid from equations (20) and (27), for $\Omega_0 = 2,\ \rho = 2.413,\ t = 5\ s,\ \lambda = 2, \lambda_r = 3, a = 2$ and different values of ν.

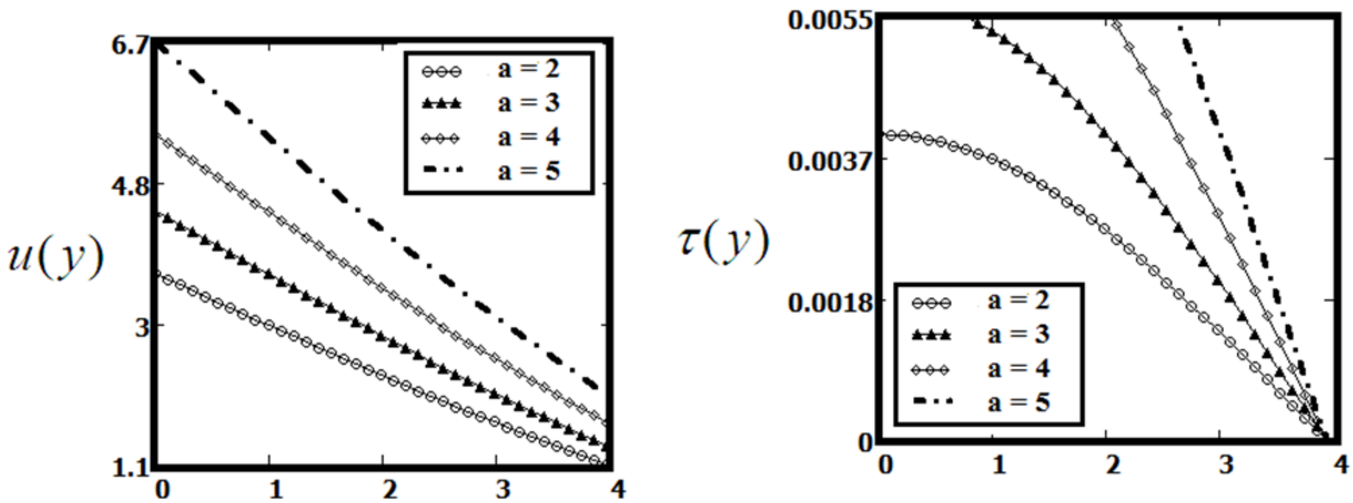

Figure 6. Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for Oldroyd-B fluid from equations (20) and (27), for $\Omega_0 = 2,\ \nu = 0.76,\ \mu = 2.41,\ \lambda = 2, \lambda_r = 3, t = 2$ s and different values of a.

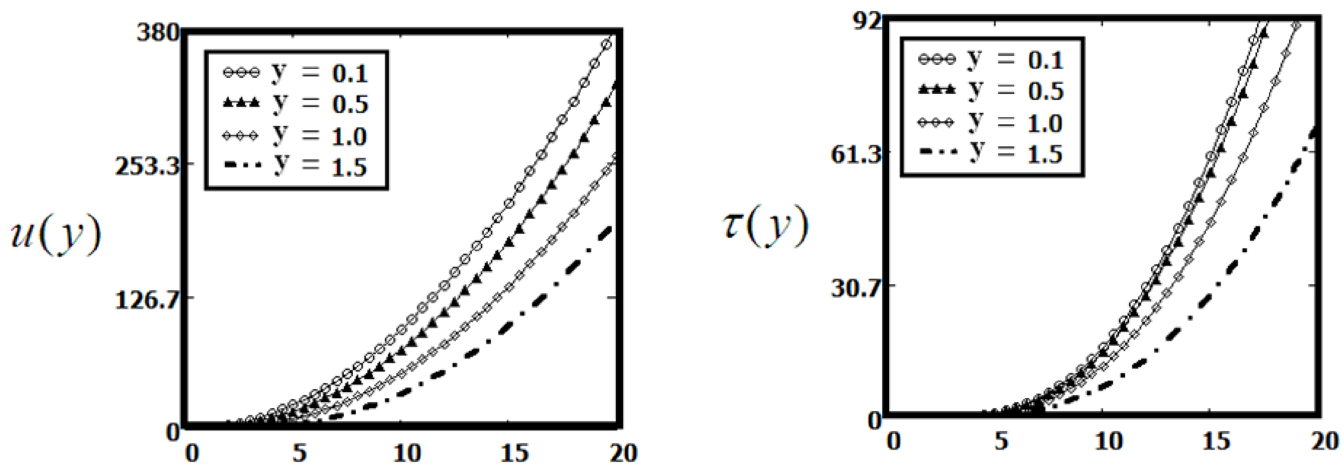

Figure 7. Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for Oldroyd-B fluid from equations (20) and (27) for $\Omega_0 = 2,\ \nu = 0.76,\ \mu = 2.41,\ \lambda = 2, \lambda_r = 3, a = 2$ and different values of y.

4168 ISSN 1013-5316;CODEN: SINTE 8 Sci.Int.(Lahore),28(4),4163-4169,2016

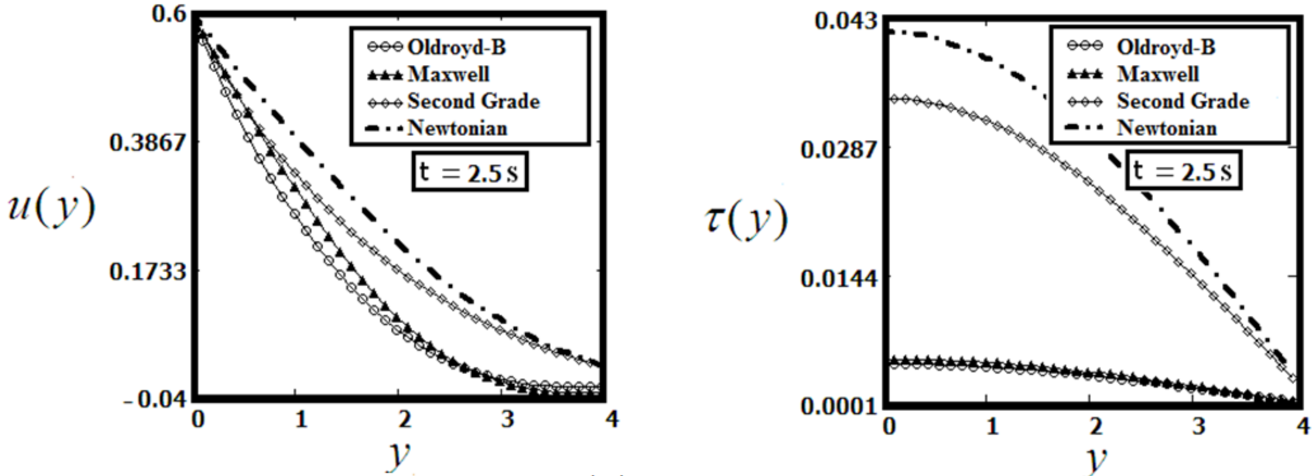

Figure 8. Profiles of the velocity fields $u(y,t)$ and the shear stresses $\tau(y,t)$ for Oldroyd-B, Maxwell, Second grade, Newtonian fluids from equations (20), (28) , (30) , (32) , (27), (29) , (31) and (33) for $\Omega_0 = 2,\ \nu = 0.76,\ \mu = 2.41,\ t = 5\ s,\ \lambda = 2, \lambda_r = 3, a = 2$ and different values of $t = 2.5\ S$.

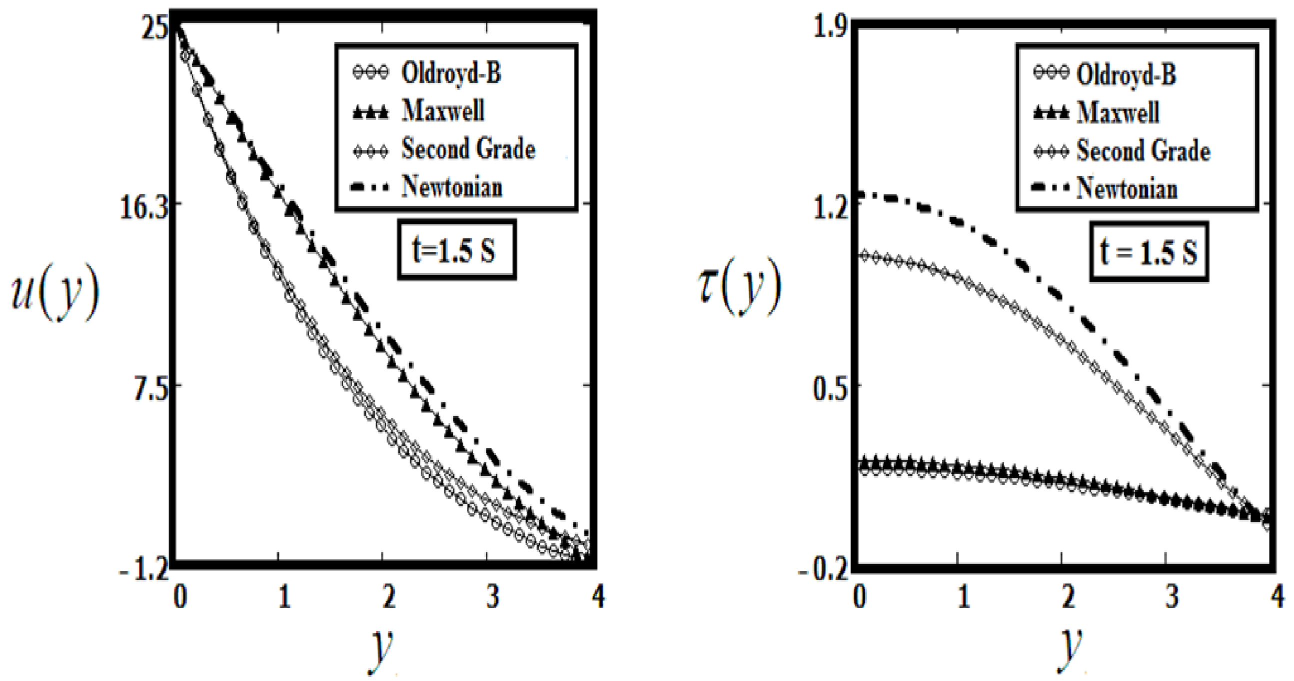

Figure 9. Profiles of the velocity fields $u(y,t)$ and the shear stresses $\tau(y,t)$ for Oldroyd-B, Maxwell, Second grade, Newtonian fluids from equations (20), (28) , (30) , (32) , (27), (29) , (31) and (33) for $\Omega_0 = 2,\ \nu = 0.76,\ \mu = 2.41,\ t = 5\ s,\ \lambda = 2, \lambda_r = 3, a = 2$ and different values of $t = 2.5\ S$.

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