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ON OSCILLATIONS OF AN AXIALLY TRANSLATING TENSIONED BEAM UNDER VISCOUS DAMPING

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ABSTRACT. *In this research work, an initial-boundary value problem is considered, which models dynamics in a linear homogeneous axially moving tensioned beam under external viscous damping. Both the ends of beam are supported and general initial conditions are considered. From physical viewpoint, the problem represents a simple mathematical model. And this mathematical model is used to represent damped vertical vibrations of a conveyor belt system. The axial speed of the beam is assumed to be positive and constant. It is also assumed that axial speed is small compared to wave velocity and that the external damping is relatively small as well. A multiple timescales perturbation technique is used to construct the asymptotic approximations of the analytic solutions. It is discussed in details that the introduced damping does in fact affect the solution responses, and that the damping generated in the system does not depend on the mode numbers n.*

Key words. Tensioned beam, Viscous damping, Perturbation, Two timescales, Oscillations

INTRODUCTION

The vibratory systems cover almost all mechanical and structural systems. Elastic systems, which are moving axially, are one of them. These axially moving systems have wide range of applications, for e.g. conveyor belts [1,2], elevator cables [3-5], band-saw blades, aerial cables, magnetic tapes, power-transmission chains, plastic films, textile fibers, paper sheets, crane and mine hoisting cables [6], and pipes transporting liquids and gases [7]. To meet manufacture and design, there are several challenges in technological applications. Therefore, it is important to obtain better insights into the dynamics of these complicated systems. The main goal of applied mathematicians, mechanical and civil engineers and, physicists is to reduce the vibration in these devices because vibrations cause damage to these systems. The study and the analysis of transversal vibrations of axially moving materials has been a challenging subject, which has been studied for many years by researchers and scientists and is still of great interest today. The fundamental work for the axially moving systems was done in Ref. [8], where the moving string and the moving beam with effect of tension for different boundary conditions were investigated. The transversal oscillations of string-like equation have been discussed in details in early works as given in Refs. [9-11]. The horizontal velocity is one of the important characteristics of axially moving systems. In the literature, velocity (constant as well as time dependent) has been studied and analyzed, see for instance, Refs. [12-17]. The responses (free and forced) of a traveling beam were investigated in Ref. [18]. In Ref. [19], the author formulated equations for traveling string with time-changing velocity. In Refs. [20,21] approach of two timescales perturbation with application of the Fourier series and the Laplace transform technique was used to build the solutions for the string-like and beam-like equations. The transversal vibrations of an axially moving externally damped beam have been investigated. The beam is simply-supported at both ends, whereas the general initial conditions (ICs) are considered and the exact approximations of the analytic solutions will be constructed, Refs. [22,23]. The use of external damping can be effective to suppress the oscillation amplitudes and this will be highlighted in detail, and that the damping rate generated in system does not depend on the mode numbers *n*. The use of external viscous damping is new idea in construction of approximations of oscillations for these type of problems. The paper is organized as follows. In section 2 the governing equations of motion are established. In section 3 asymptotic approximations for solutions of IBVP are obtained by making use of the method of two timescales perturbation. In section 4 concluding remarks will be presented in detail.

Governing Equations of Motion

Consider a uniform axially moving beam of mass density ρ, cross-sectional area A, moment of inertial I, flexural rigidity EI, damping coefficient $\bar{\delta}$, and uniform tension T. A stretched beam is simply-supported at $x = 0$ and $x = L$. The functions $f(x)$ and $g(x)$ express the displacement and velocity at $x = 0$, respectively. The beam travels at the uniform constant axial speed $\bar{V}$ between a pair of pulleys that are a distance L apart, as shown in Fig. 1. It is assumed that ρ, A, EI, T, $\bar{V}$, and $\bar{\delta}$ are positive. Gravity and other external forces are not taken into account. The equation, which describes the displacement of beam under external viscous damping, is given by

$$u_{tt} + 2\bar{V}u_{xt} + (\bar{V}^2 - C^2)u_{xx} + \frac{EI}{\rho A}u_{xxxx} + \bar{\delta}(u_t + \bar{V}u_x) = 0. \tag{1}$$

The BCs and ICs are given by

$$u(0,t) = u_{xx}(0,t) = u(L,t) = u_{xx}(L,t) = 0; t \geq 0, \tag{2}$$
$$u(x,0) = f(x), \text{and } u_t(x,0) = g(x);$$

where $C = \sqrt{\frac{T}{\rho A}}$ is the wave speed. The quantaties:

$u^* = \frac{u}{L},\ x^* = \frac{x}{L}, t^* = \frac{C}{L}t, V^* = \frac{V}{C}, \delta^* = \frac{\bar{\delta}L}{\rho AC}, \mu = \frac{EI}{\rho AC^2L^2}, f^* = \frac{f}{L}, g^* = \frac{g}{C}$ are used in order to put Eqs. (1) and (2). Thus, the Eq (1) into nondimensional form becomes,

$$u_{tt} + 2Vu_{xt} + (V^2 - 1)u_{xx} + \mu u_{xxxx} + \delta(u_t + Vu_x) = 0;\ t \geq 0, 0 < x < 1, \tag{3}$$

with BCs,

$$u(0,t) = u_{xx}(0,t) = u(1,t) = u_{xx}(1,t) = 0;\ t \geq 0, \tag{4}$$

and ICs,

$$u(x,0) = f(x), \text{and}, u_t(x,0) = g(x), 0 < x < 1. \tag{5}$$

The asterisks describing the dimensionless quantities are neglected in Eqs. (3)-(5), and henceforth. In this paper, IBVP (3)-(5) for $u(x,t)$ will be studied and formal approximations will be constructed.

Analytic Approximations

In this section, an approximation of the solution of the IBVP (3)-(5) will be constructed by using a two timescales perturbation method, for details interested reader is referred to Refs. [22,23]. Horizontal velocity $\bar{V}$ of beam is assumed to be small and $O(\varepsilon)$, which is, $V^* = \varepsilon V$, where ε is dimensionless parameter. We assume that $\bar{\delta}L$ is small compared to ρAC and $O(\varepsilon)$, that is, $\delta^* = \varepsilon\delta$. Based on these assumptions, Eqs. (3)-(5) can be written as follows

$$u_{tt} - u_{xx} + \mu u_{xxxx} = -2\varepsilon V u_{xt} - \varepsilon^2 V^2 u_{xx} - \varepsilon\delta u_t - \varepsilon^2 \delta V u_x, \tag{6}$$

the BCs,

$$u(0,t;\varepsilon) = u_{xx}(0,t;\varepsilon) = u(1,t;\varepsilon) = u_{xx}(1,t;\varepsilon) = 0, \tag{7}$$

and the ICs,

$$u(x,0;\varepsilon) = f(x), u_t(x,0;\varepsilon) = h(x). \tag{8}$$

In two timescales method $u(x,t;\varepsilon)$ is supposed to be a function of x, t_1 and t_2. For this reason,

$$u(x,t;\varepsilon) = v(x,t_1,t_2;\varepsilon). \tag{9}$$

By using Eq. (9), transformations given below, are required for time derivatives:

$$u_t = v_{t_1} + \varepsilon v_{t_2}, \quad u_{tt} = v_{t_1t_1} + 2\varepsilon v_{t_1t_2} + \varepsilon^2 v_{t_2t_2}. \tag{10}$$

By substituting Eqs. (9)-(10) into Eqs. (6)-(8), the problem in v up to $O(\varepsilon)$ is given as follows:

$$v_{t_1} - v_{xx} + \mu v_{xxxx} = -2\varepsilon v_{t_1t_1} - 2\varepsilon V v_{xt_1} - \varepsilon\delta v_{t_1} + O(\varepsilon^2), \tag{11}$$
$$v(0,t_1,t_2;\varepsilon) = v_{xx}(0,t_1,t_2;\varepsilon) = v(1,t_1,t_2;\varepsilon) = v_{xx}(0,t_1,t_2;\varepsilon) = 0,$$
$$v(x,0,0;\varepsilon) = f(x), v_{t_1}(x,0,0;\varepsilon) + \varepsilon v_{t_2}(x,0,0;\varepsilon) = h(x).$$

The function $u(x,t;\varepsilon)$ not only can be approximated by the asymptotic expansion, but also the function $u(x,t;\varepsilon) = v(x,t_1,t_2;\varepsilon)$ can be approximated by the powers of ε in the asymptotic expansion, that is:

$$v(x,t_1,t_2;\varepsilon) = v_0(x,t_1,t_2) + \varepsilon v_1(x,t_1,t_2) + \varepsilon^2 \cdots, \tag{12}$$

and that all the $v_j's$ for $j = 0,1,2,\cdots$, are found in such a way that no secular terms arise. It is also assumed that the unknown functions v_j are $O(1)$. Now, by substitution of Eq. (12}) into Eq. (11), and then equating the powers of ε^0 and ε^1, and neglecting the ε^2 and the higher powers of ε, it follows

$$v_{0_{t_1t_1}} - v_{0_{xx}} + \mu v_{0_{xxxx}} = 0, \tag{13}$$
$$v_0(0,t_1,t_2) = v_{0_{xx}}(0,t_1,t_2) = v_0(1,t_1,t_2) = v_{0_{xx}}(1,t_1,t_2),$$
$$v_0(x,0,0) = f(x), \quad v_{0_{t_1}}(x,0,0) = h(x),$$

and,

$$v_{1_{t_1t_1}} - v_{1_{xx}} + \mu v_{1_{xxxx}} = -2v_{0_{t_1t_2}} - 2V v_{0_{xt_1}} - \delta v_{0_{t_1}}, \tag{14}$$
$$v_1(0,t_1,t_2) = v_{1_{xx}}(0,t_1,t_2) = v_1(1,t_1,t_2) = v_{1_{xx}}(1,t_1,t_2),$$
$$v_1(x,0,0) = 0, \quad v_{1_{t_1}}(x,0,0) = -v_{0_{t_2}}(x,0,0).$$

It can be observed that in $O(1)$-problem the partial differential equation and the BCs are linear and homogeneous, the separation of variables method can be applicable. Following product solutions of the form are assumed:

$$v_0(x,t_1,t_2) = \phi(x) g(t_1,t_2), \tag{15}$$

By substituting Eq. (15) into Eq. (13), it follows

$$\frac{g_{t_1t_1}(t_1,t_2)}{g(t_1,t_2)} = \frac{\phi''}{\phi} - \frac{\mu\phi^{(4)}}{\phi} = -\lambda \tag{16}$$

(7)

A separation constant $-\lambda$ is introduced so that the time-dependent part of the product solution oscillates if $\lambda > 0$. It can easily be shown that the the eigenvalues turn out be real and positive. Eq. (16) yields two ordinary differential equations (ODEs), that is, a time-dependent part

$$g_{t_1t_1}(t_1,t_2) + \lambda g(t_1,t_2) = 0, \tag{17}$$

and a space-dependent part

$$\phi^{iv}(x) - \frac{1}{\mu}\phi''(x) - \frac{\lambda}{\mu}\phi(x) = 0. \tag{18}$$

The four homogeneous BCs in Eq. (13) imply that

$$\phi(0) = \phi''(0) = \phi(1) = \phi''(1) = 0. \tag{19}$$

Thus, Eqs. (18) and (19) form a BVP. Instead of first studying the Eqs. (18)-(19), let us first analyze the time-dependent equation (17). Thus, the general solution of Eq. (17) is a linear combination of sines and cosines in t_1, that is,

$$g(t_1,t_2) = \sigma_1(t_2)\cos(\sqrt{\lambda}t_1) + \sigma_2(t_2)\sin(\sqrt{\lambda}t_1), \tag{20}$$

and it oscillates with frequency $\sqrt{\lambda}$. The values of λ determine the natural frequencies of the oscillations of a vibrating beam. Analyzing the BVP (18)-(19) with $\phi(x) = \mathrm{e}^{\alpha x}$ (α to be determined), following characteristic equation is obtained

$$\alpha^4 - \frac{\alpha^2}{\mu} - \frac{\lambda}{\mu} = 0. \tag{21}$$

Note that for the cases $\lambda < 0$ and $\lambda = 0$ the solutions $\phi(x) \equiv 0$. Thus, $\lambda < 0$ and $\lambda = 0$ are not the eigenvalues of the problem. Thus nontrivial solutions of Eq. (18) are given by

$$\phi(x) = c_1\cosh(\beta x) + c_2\sinh(\beta x) + c_3\cos(\gamma x) + c_4\sin(\gamma x), \tag{22}$$

where c_1, c_2, c_3 and c_4 are the constants of integration, and where $\beta = \sqrt{\frac{(1+\sqrt{4\lambda\mu+1})}{2\mu}}$ and $\gamma = \sqrt{\frac{-1+\sqrt{4\lambda\mu+1}}{2\mu}}$. Applying the BCs (19), it is observed that the nontrivial solutions are found when $c_1 = c_3 = 0$, and when

$$f_\mu(\lambda) = (\beta^2 + \gamma^2)\sinh(\beta)\sin(\gamma) = 0, \quad (23)$$

Eq. (23) implies $\sqrt{\lambda} = \sqrt{\lambda_n} = n\pi\sqrt{1 + n^2\pi^2\mu}$ for $n = 1,2,3\cdots$. From Eqs. (18), (19) and (23) the n-th eigenfunction $\phi_n(x)$ corresponding to the n-th eigenvalue λ_n can be determined and is given by (up to a multiplicative constant)

$$\phi_n(x) = \sin(\gamma_n x) - \frac{\sin(\gamma_n)}{\sinh(\beta_n)}\sinh(\beta_n x). \quad (24)$$

The general response of $O(1)$-problem is

$$v_0(x,t_1,t_2) = \sum_{n=1}^{\infty}\left(A_{n0}(t_2)\cos\left(\sqrt{\lambda_n}t_1\right) + B_{n0}(t_2)\sin\left(\sqrt{\lambda_n}t_1\right)\right)\phi_n(x), \quad (25)$$

where $A_{n0}(t_2)$ and $B_{n0}(t_2)$ are unknown Fourier coefficients. By eigenfunctions' orthogonality properties and the ICs given in Eq. (13), values of the constants $A_{n0}(0)$ and $B_{n0}(0)$ can easily be obtained. The eigenfunctions $\phi_n(x)$for $n = 1,2,3,\cdots$ satisfy the following orthogonality properties, as given by

$$< \phi_n, \phi_m > = \int_0^1 \phi_n(x)\,\phi_m(x)dx\,, \quad (26)$$

Thus, by using the ICs described in Eq. (13) and the orthogonality properties of the eigenfunctions as given in Eq. (26), it follows that

$$A_{n0}(0) = \frac{\int_0^1 f(x)\phi_n(x)dx}{\int_0^1 \phi_n^2(x)dx}, \quad (27)$$

$$\sqrt{\lambda_n}B_{n0}(0) = \frac{\int_0^1 h(x)\phi_n(x)dx}{\int_0^1 \phi_n^2(x)dx}. \quad (28)$$

Now, to solve the $O(\varepsilon)$-problem, the eigenfunction expansion method is introduced. In this method it is assumed that the solution $v_1(x,t_1,t_2)$ can be expressed as the linear combination of the orthogonal eigenfunctions $\phi_n(x)$. Therefore, it is reasonable to assume the following form for the solution $v_1(x,t_1,t_2)$, such that

$$v_1(x,t_1,t_2) = \sum_{m=1}^{\infty} w_m(t_1,t_2)\phi_m(x), \quad (29)$$

where $w_m(t_1,t_2)$ are unknown generalized Fourier coefficients. Thus, by substituting the Eq. (29) into the $O(\varepsilon)$-equation, it follows that,

$$\sum_{m=1}^{\infty}\left(w_{m_{t_1t_1}}(t_1,t_2) + \lambda_m w_m(t_1,t_2)\right)\phi_m(x) = -2v_{0_{t_1t_2}} - 2Vv_{0_{xt_1}} - \delta v_{0_{t_1}}. \quad (30)$$

By substituting Eq. (25) into Eq. (30), it follows that

$$\sum_{m=1}^{\infty}\left(w_{m_{t_1t_1}}(t_1,t_2) + \lambda_m w_m(t_1,t_2)\right)\phi_m(x) = \sum_{m=1}^{\infty}\left\{-\left(2T_{m_{t_1t_2}} + \delta T_{m_{t_1}}\right)\phi_m(x) - 2VT_{m_{t_1}}\phi_m'(x)\right\}, \quad (31)$$

where

$$T_m(t_1,t_2) = A_{n0}(t_2)\cos(\sqrt{\lambda_m}t_1) + B_{n0}(t_2)\sin(\sqrt{\lambda_m}t_1), \quad (32)$$

Multiplying both sides of Eq. (31) by n-th eigenfunction $\phi_n(x)$, then by integrating both sides of the so-obtained equation from $x = 0$ to $x = 1$ with application of the orthogonality properties of the eigenfunctions, it follows that,

$$\begin{aligned} w_{n_{t_1t_1}} + \lambda_n w_n &= -2T_{n_{t_1t_2}} - \delta T_{n_{t_1}} - 2VT_{n_{t_1}}\Theta_{nn} - 2V\sum_{m=1,m\neq n}^{\infty} T_{m_{t_1}}\Theta_{mn} \\ &= \left(2A_{n0}'(t_2) + 2V\Theta_{nn}A_{n0}(t_2) + \delta A_{n0}(t_2)\right)\sqrt{\lambda_n}\sin\left(\sqrt{\lambda_n}t_1\right) \\ &\quad - \left(2B_{n0}'(t_2) + 2V\Theta_{nn}B_{n0}(t_2) + \delta B_{n0}(t_2)\right)\sqrt{\lambda_n}\cos(\sqrt{\lambda_n}t_1) \\ &\quad + \sum_{m=1,m\neq n}^{\infty}\left\{2V\sqrt{\lambda_m}\left(A_{m0}(t_2)\sin\left(\sqrt{\lambda_m}t_1\right) - (t_2)\cos\left(\sqrt{\lambda_m}t_1\right)\right)\right\}\Theta_{mn}, \end{aligned} \quad (33)$$

where,

$$\Theta_{mn} = \int_0^1 \phi_m'(x)\phi_n(x)dx. \quad (34)$$

The solution of Eq. (33) consists the homogeneous solution and the particular integral, that is,

$$w_{n_{P_1}}(t_1,t_2) = -\left(A_{n0}'(t_2) + \left(V\Theta_{nn} + \frac{\delta}{2}\right)A_{n0}(t_2)\right)t_1\cos\left(\sqrt{\lambda_n}t_1\right) - \left(B_{n0}'(t_2) + \left(V\Theta_{nn} + \frac{\delta}{2}\right)B_{n0}(t_2)\right)t_1\sin\left(\sqrt{\lambda_n}t_1\right). \quad (35)$$

It can be seen in Eq. (35) that the solutions are unbounded in t_1. This behavior of the solutions is known as the secular behavior. For secular free behavior, following conditions are used in Eq. (33),

$$2A_{n0}'(t_2) + 2V\Theta_{nn}A_{n0}(t_2) + \delta A_{n0}(t_2) = 0, \quad (36)$$
$$2B_{n0}'(t_2) + 2V\Theta_{nn}B_{n0}(t_2) + \delta B_{n0}(t_2) = 0.$$

The above equations are uncoupled ODEs and they yield,

$$A_{n0}(t_2) = A_{n0}(0)e^{-\left(V\Theta_{nn}+\frac{\delta}{2}\right)t_2}, \quad (37)$$

$$B_{n0}(t_2) = B_{n0}(0)e^{-\left(V\Theta_{nn}+\frac{\delta}{2}\right)t_2},$$

where $A_{n0}(0)$ and $B_{n0}(0)$ are given in Eqs. (27) and (28). Now let us determine the values of Θ_{nn} from Eq. (34). Using integration by parts it follows from Eq. (34) that

$$2\Theta_{nn} = \phi_n^2(x)|_0^1 = \phi_n^2(1) - \phi_n^2(0). \quad (38)$$

It can be observed in Eq. (24) that $\phi_n(0) = 0$ and $\phi_n(1) = 0$, so that $\Theta_{nn} = 0$. Thus, by using Eqs. (37) and (38) into Eq. (25), the zeroth order approximation is given as,

$$\nu_0(x, t_1, t_2) = \sum_{n=1}^{\infty} \mathrm{e}^{-\frac{\delta}{2}t_2}\left(A_{n0}(0)\cos\left(\sqrt{\lambda_n}t_1\right) + B_{n0}(0)\sin\left(\sqrt{\lambda_n}t_1\right)\right)\phi_n(x). \quad (39)$$

Now by substituting $t_2 = \varepsilon t$ into the expression $-\frac{\delta}{2}t_2$, and then by dividing expression by t, it yields that the damping parameter for all oscillations modes can be expressed by,

$$\Gamma_n = -\varepsilon\frac{\delta}{2}. \quad (40)$$

Thus, from Eq. (33) with Eq. (36), it follows that

$$w_{n_{t_1t_1}} + \lambda_n w_n = \sum_{m=1,m\neq n}^{\infty}\left(2V\sqrt{\lambda_m}\left(A_{m0}\sin\left(\sqrt{\lambda_m}t_1\right) - B_{m0}\cos\left(\sqrt{\lambda_m}t_1\right)\right)\right)\Theta_{mn}. \quad (41)$$

The solution of Eq. (41) is given by

$$w_n(t_1, t_2) = A_{n1}(t_2)\cos\left(\sqrt{\lambda_n}t_1\right) + B_{n1}(t_2)\sin\left(\sqrt{\lambda_n}t_1\right) + \sum_{m=1,m\neq n}^{\infty}\frac{2V\sqrt{\lambda_m}\Theta_{mn}}{\lambda_n - \lambda_m}\left(A_{m0}\sin\left(\sqrt{\lambda_m}t_1\right) - B_{m0}\cos\left(\sqrt{\lambda_m}t_1\right)\right), \quad (42)$$

where $A_{n1}(t_2)$ and $B_{n1}(t_2)$ are still unknown functions. Thus, Eq. (29) with Eq. (42) can be expressed as

$$\nu_1(x, t_1, t_2) = \sum_{n=1}^{\infty}\left\{A_{n1}(t_2)\cos\left(\sqrt{\lambda_n}t_1\right) + B_{n1}(t_2)\sin\left(\sqrt{\lambda_n}t_1\right) + \sum_{m=1,m\neq n}^{\infty}\frac{2V\sqrt{\lambda_m}\Theta_{mn}}{\lambda_n - \lambda_m}\left(A_{m0}\sin\left(\sqrt{\lambda_m}t_1\right) - B_{m0}\cos\left(\sqrt{\lambda_m}t_1\right)\right)\right\}\phi_n(x). \quad (43)$$

By using the inner product (26) and the ICs from Eq. (14) into Eq. (43), it follows that

$$A_{n1}(0) = \sum_{m=1,m\neq n}^{\infty}\frac{2V\sqrt{\lambda_m}\Theta_{mn}}{\lambda_n - \lambda_m}B_{m0}(0), \quad (44)$$

$$\sqrt{\lambda_n}B_{n1}(0) = -\sum_{m=1,m\neq n}^{\infty}\frac{2V\lambda_m\Theta_{mn}}{\lambda_n - \lambda_m}A_{m0}(0) - \frac{\delta}{2}A_{n0}(0). \quad (45)$$

It can be observed that the solution (43) still contains infinitely many undermined functions $A_{n1}(t_2)$ and $B_{n1}(t_2)$, for $n = 1,2,3,\cdots$. It is reasonable to take $A_{n1}(t_2) = A_{n1}(0)$ and $B_{n1}(t_2) = B_{n1}(0)$. So far, a formal asymptotic approximation $\nu(x, t_1, t_2) = \nu_0(x, t_1, t_2) + \varepsilon\nu_1(x, t_1, t_2)$ has been constructed for $u(x, t)$. The solutions $\nu_0(x, t_1, t_2)$ and $\nu_1(x, t_1, t_2)$ are continuously differentiable two times with respect to t_1, four times with respect to x, and infinitely many times with respect to t_2.

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