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A MODIFIED ITERATIVE ALGORITHM FOR CLASSIFYING GENERALIZED STRICTLY DIAGONALLY DOMINANT MATRICES

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ABSTRACT: *In several applications, it is extremely important to know whether a matrix is generalized strictly diagonally dominant matrix or not. This article is specially written for the study of generalized strictly diagonally dominant matrix and in this research, an iterative algorithm is developed for identifying an irreducible matrix is generalized strictly diagonally dominant matrix. The performance of proposed algorithm can be verified by theoretical analysis and numerical results.*

Key Words: Generalized strictly diagonally dominant matrix, Iterative algorithm, Irreducible matrix.

INTRODUCTION

Generalized strictly diagonally dominant matrix plays an important role in numerical analysis, matrix theory, control theory and mathematical economics (for detailed information, please refer to (see [2, 4, 8, and 9]. In fact many methods have been obtained for determining whether the given matrix is or is not a generalized strictly diagonally dominant matrix, it is a very important concern in many applications (see [1, 5, 6,and 7]). However, these algorithms have its own drawbacks that increase the concern to further research.

Most of the methods in literature are direct methods, which are suited for only a narrow range and besides, they are complicated and not practical. In comparison with direct methods, iterative methods have more advantages, and they can be implemented through computers. Among studies on iterative methods, we thoroughly review the papers (see [1, 2, 5, 6, 7, and 9]). The basic thought for most iterative methods is to draw conclusions through making the dominant row smaller. It has been assumed that a better result can be achieved by making the dominant row smaller while making the non-dominant row larger. In this paper, through the modification on iterative method in [7] based on the aforementioned idea a new iterative algorithm is proposed for identifying generalized strictly diagonally dominant matrix. The theoretical analysis numerical results prove that our proposed iterative method has become more effective and efficient. This idea can be applied in other algorithms.

To make it more conducive to discussion, some fundamental concepts and common conclusions are given as follows (see [2, 4,and 9]).

Suppose: $A = (a_{ij}) \in \mathbb{C}^{n\times n}$, $N = \{1, 2, \cdots, n\}$.

For arbitrary $i \in N$, where $r_i(A) = \sum_{j\neq i} |a_{ij}|$,

$t_i(A) = \dfrac{r_i(A)}{|a_{ii}|}$ define $N_1(A) = \{i \in N \,||\, a_{ii}| > r_i(A)\}$,

$N_0(A) = \{i \in N \,||\, a_{ii}| = r_i(A)\}$,

$N_2(A) = \{i \in N \,||\, a_{ii}| < r_i(A)\}$, then

$N = N_1(A) \bigcup N_2(A) \bigcup N_0(A)$.

Definition 1. Given $A = (a_{ij}) \in \mathbb{C}^{n\times n}$, if for arbitrary $i \in N$, we have $|a_{ii}| > r_i(A)$ then A is identified as strictly diagonally dominant matrix. If there exist a positive diagonal matrix D such that AD is strictly diagonally dominant, and then A is called generalized diagonally dominant matrix.

Definition 2. Suppose $A = (a_{ij}) \in \mathbb{C}^{n\times n}$, if there exists a permutation matrix P such that

$$PAP^T = \begin{pmatrix} A_{11} & A_{12} \\ O & A_{22} \end{pmatrix}$$

where A11 and A22 are respectively the matrices of $k \times k$ and $(n-k)\times(n-k)$, $1 \le k < n$, then A is reducible. Otherwise A is irreducible.

Definition 3. Suppose $A = (a_{ij}) \in \mathbb{C}^{n\times n}$ is irreducible, if for any arbitrary $i \in N$ such that $|a_{ii}| \ge r_i(A)$, and at least one of them is strictly inequality hold, then A is irreducible diagonally dominant matrix.

The following are some basic properties of generalized strictly diagonally dominant matrix (see [2, 4 and 9]).

Lemma 1.1. Suppose $A = (a_{ij}) \in \mathbb{C}^{n\times n}$is a generalized strictly diagonally dominant matrix, then for any arbitrary $i \in N$, such that $a_{ii} \ne 0$.

Lemma 1.2. Suppose $A = (a_{ij}) \in \mathbb{C}^{n\times n}$is a generalized strictly diagonally dominant matrix, then $N_1(A) \ne \varnothing$.

Lemma 1.3. Suppose $A = (a_{ij}) \in \mathbb{C}^{n\times n}$ then A is a generalized strictly diagonally dominant matrix if and only if AD is a generalized strictly diagonally dominant matrix, here D is positive diagonal matrix.

Lemma 1.4. Suppose $A = (a_{ij}) \in \mathbb{C}^{n\times n}$is an irreducible diagonally dominant matrix, then A is a generalized strictly diagonally dominant matrix.

The rest of this paper is organized as follows. Section 1 contains introduction and basic results. Section 2 includes existing iterative method and proposed iterative algorithm with theoretical analysis. Section 3 consists of numerical experiment and comparison. Section 4 contains conclusion remarks.

EXISTING AND PROPOSED ALGORITHMS

In the year 2003, K. Ojiro [7] has proposed an iterative algorithm to identify generalized strictly diagonally dominant matrix. The major steps of the algorithm are discussed as follows.

Algorithm: (see [7])

Input: Matrix $A = (a_{ij}) \in \mathbb{C}^{n\times n}$。

1: If $N_1(A) = \varnothing$ or $a_{ii} = 0$ there exists an $i \in N$, "A is not a generalized strictly diagonally dominant matrix", stop: otherwise

2: if for arbitrary $i \in N$, we have $t_i(A) = 0$, "A is generalized strictly diagonally dominant matrix", stop: otherwise,

3: we make $t_l(A) = \min_{1\le i\le n} t_i(A)$, for $t_i(A) \neq 0$;

4: We make row l in A times $t_l(A)$ to get A' ;

5: normalize row l in A' by a'_{ll} ;

6: For arbitrary $i \in N$, compute $t_i(A')$;

7: If $t_i(A') \le 1$ for arbitrary $i \in N$, and at least one of them is strict, "A is a generalized strictly diagonally dominant matrix", stop,
If $t_i(A') \ge 1$ for arbitrary $i \in N$, then "A is not a generalized strictly diagonally dominant matrix, stop; otherwise

6: Make $A = A'$, return to step 3.

The advantage of the above algorithm is that it has need a fewer steps and O(n) cost for each iteration, good convergence behavior while the previous algorithms required O(n^2) cost for each iteration. Nevertheless, Alanelli*et. al.* highlighted the limitations of above algorithm in (see [1]). Such limitations are: it is not suitable for reducible matrices; the algorithm is not able to end in limited steps when matrix is not generalized strictly diagonally dominant matrix. An example of not generalized strictly diagonally dominant matrix is as follows:

$$A = \begin{pmatrix} 1 & -2 & -1 & 0 \\ -2 & 1 & 0 & -1 \\ 0 & -0.25 & 1 & -0.5 \\ -0.25 & 0 & -0.5 & 1 \end{pmatrix}$$

Through careful study, we discovered that the algorithm is always effective for irreducible strictly diagonally dominant matrix while the aforesaid disadvantages only exist in terms of not generalized strictly diagonally dominant matrix. In fact, the main conclusion in (see [7]) is that suppose A is irreducible, if A is generalized strictly diagonally dominant matrix, then the above algorithm is convergent. Besides, the fifth step in the above algorithm is not necessary. Based on the idea of the original algorithm, a more concise algorithm is given as follows (see Algorithm 1).

Algorithm 1:

Input: irreducible matrix $A = (a_{ij}) \in \mathbb{C}^{n\times n}$

Output: "A is not generalized strictly diagonally dominant matrix" or "A is generalized strictly diagonally dominant matrix".

1: If for some $i \in N$, we have $a_{ii} = 0$, "A is not generalized strictly diagonally dominant matrix", stop; otherwise,

2: set $k = 0$;

3: compute $t_i(A_k)$, for $i \in N$ and $[u, uu] = \arg\min_{1\le i\le n} t_i(A_k)$ $v = \arg\max_{1\le i\le n} t_i(A_k)$;

4: If $u \ge 1$, "A is not generalized strictly diagonally dominant matrix", stop ;
If $v \le 1$, "A is generalized strictly diagonally dominant matrix", stop ;

5: Otherwise compute $A_{k+1} = A_k D_k$, here $D_k = diag(d_1, d_2, \cdots, d_n)$, and
$$d_i = \begin{cases} u, & i = uu \\ 1, & i \neq uu \end{cases};$$

6: Make $k = k+1$. Return to step 3.

Theorem 2.1. Suppose A is irreducible, if A is generalized strictly diagonally dominant matrix, then algorithm 1 is convergent.

Proof: Suppose A is non-negative. First, it can be easily proved that as k increases, set $N_0(A_k) \bigcup N_1(A_k)$ increases progressively, while $N_2(A_k)$ decreases progressively, although not in a rigorous way.

Second, suppose the theorem is untenable, then for arbitrary positive integer k, we have $N_2(A_k) \neq \varnothing$. Then there exists a positive integer l such that $\forall m > 0$,

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$N_2(A_l) = N_2(A_{l+m})$ 。

Suppose $N_2(A_l) = \{1, 2, \cdots, k\}$,

$$A_l = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

Here A_{11} is in the form of $k \times k$. Then the first k rows of A_l is not strictly diagonally dominant and for arbitrary $m > l$, the first k rows of A_m is not strictly diagonally dominant. For this reason A is generalized strictly diagonally dominant matrix, it is convinced that there exists strictly diagonally dominant rows in the last $n-k$ rows of A_l, for arbitrary $m > l$. Based on such iterative sequence $\{A_m\}$ as m increases, the elements in matrix will decrease, which is to say that for arbitrary m, we have $A_m \geq A_{m+1}$. However, on the other hand, since we always have $A_m \geq 0$, iterative sequence $\{A_m\}$ is sure to have limits. Suppose

$$\lim_{m\to\infty} A_m = A_\omega = \begin{pmatrix} A_{11} & B \\ A_{21} & C \end{pmatrix},$$

since the first k rows are not strictly diagonally dominant, they are not change in the whole iteration, only $n-k$ rows change.

The following are analysis on limited class of matrix A_ω. First, there does not exist zero rows in the last $n-k$ rows of A_ω. Provided there is a zero row p, $1 \leq p \leq n-k$, and p is from the last rows, then the rows after p in B and C are zero rows. If

$$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix},$$

Here C_{22} is square matrix of rank p. In iteration, elements from the matrix where there is C_{22} decrease progressively and converge to zero matrix. Since A is irreducible, there exists at least one non-zero element in the last p rows of A_{21} and C_{21}. Then the row where there is the non-zero element will be not diagonally dominant, which is contradictory to the hypothesis. Then there does not exist any zero rows in the last n-k rows of A_ω.

Secondly, $\lim_{m\to\infty} \min_{1\leq i\leq n} t_i(A_m) = 1$.

If $\lim_{m\to\infty} \min_{1\leq i\leq n} t_i(A_m) = \alpha < 1$, then in iteration the elements of the last n-k rows in A_ω will decrease progressively which result in zero rows. Therefore, for the last n-k rows of A_ω, we have $t_i(A_\omega) = 1$. Then $N_1(A_\omega) = \varnothing$, which indicates that A_ω is not a strictly diagonally dominant matrix. Since $A_\omega = AD$, here D is positive diagonally matrix. However, according to lemma 1.3, A is not strictly diagonally dominant matrix, which is contradictory to our conclusion. Therefore the original hypothesis is invalid. In limited iteration, we have $N_2(A_k) = \varnothing$, which is to say, algorithm 1 is convergent.

When A is not generalized strictly diagonally dominant matrix, suppose A is irreducible, we make a sequence $\{A_k\}$ as follows:

$A_0 = A$, $A_0 = (a_{ij}^{(0)})$. Do $t_i(A_0) = \dfrac{r_i(A_0)}{|a_{ii}^{(0)}|}$, $\forall i \in N$. Suppose $t_k(A_0) = \max_{1\leq i\leq n} t_i(A_0)$, make $A_1 = A_0 D_0$, here $D_0 = diag(d_1, d_2, \cdots, d_n)$, and

$$d_i = \begin{cases} t_k(A_0), & i = k \\ 1, & i \neq k \end{cases}。$$

After $A_k = (a_{ij}^{(k)})$ is done, do $t_i(A_k) = \dfrac{r_i(A_k)}{|a_{ii}^{(k)}|}$, $\forall i \in N$. Provided $t_p(A_k) = \max_{1\leq i\leq n} t_i(A_k)$, make $A_{k+1} = A_k D_k$, here $D_k = diag(d_1, d_2, \cdots, d_n)$, and

$$d_i = \begin{cases} t_k(A_k), & i = p \\ 1, & i \neq p \end{cases}.$$

On the analogy of this, from the above process we can get sequence $\{A_k\}$ and elements in the sequence become larger progressively. A conclusion about the sequence is drawn as follows, which is especially important for the following algorithm.

Theorem 2.2. Suppose A is irreducible and not generalized strictly diagonally dominant matrix and the diagonal elements are non-zero. Provided that comparison matrix $m(A)$ is not singular, then for the aforesaid sequence $\{A_k\}$ there exists a positive integer K such that $N_1(A_k) = \varnothing$ when k > K.

Proof: Notice that as k becomes larger, $N_1(A_k)$ becomes smaller, while $N_0(A_k) \bigcup N_2(A_k)$ becomes larger, though not in a strict way. This is because $\forall i \in N_1(A_k)$, suppose $t_p(A_k) = \max_{1\leq i\leq n} t_i(A_k)$, then $t_p(A_k) > 1$, and $p \neq i$, therefore

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$$t_i(A_{k+1}) = \frac{r_i(A_{k+1})}{|a_{ii}^{(k+1)}|} = \frac{\sum_{j\neq i}|a_{ij}^{(k+1)}|}{|a_{ii}^{(k)}|} = \frac{\sum_{j\neq i,p}|a_{ij}^{(k)}| + t_p(A_k)|a_{ip}^{(k)}|}{|a_{ii}^{(k)}|} \geq \frac{\sum_{j\neq i}|a_{ij}^{(k)}|}{|a_{ii}^{(k)}|} = t_i(A_k)$$

.

Then $t_i(A_{k+1}) \geq 1$ is possible and then $i \notin N_1(A_{k+1})$. For $\forall i \in N_0(A_k) \bigcup N_2(A_k)$, when $i = p$, obviously $i \in N_0(A_{k+1})$, while for $i \neq p$,

$$t_i(A_{k+1}) = \frac{r_i(A_{k+1})}{|a_{ii}^{(k+1)}|} = \frac{\sum_{j\neq i}|a_{ij}^{(k+1)}|}{|a_{ii}^{(k)}|} = \frac{\sum_{j\neq i,p}|a_{ij}^{(k)}| + t_p(A_k)|a_{ip}^{(k)}|}{|a_{ii}^{(k)}|} \geq \frac{\sum_{j\neq i}|a_{ij}^{(k)}|}{|a_{ii}^{(k)}|} = t_i(A_k)$$

,

Therefore we also have $i \in N_0(A_{k+1}) \bigcup N_2(A_{k+1})$.

Secondly, suppose the theorem is untenable, which is to say, for arbitrary positive integer k, we have $N_1(A_k) \neq \varnothing$. According to the above analysis, there exists a positive integer l, such that $\forall m > 0$, $N_1(A_l) = N_1(A_{l+m})$ 。

Provided $N_1(A_l) = \{1, 2, \cdots, k\}$,

$$A_l = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

Here A_{11} is in the form of $k \times k$. Then the first k rows of A_l are strictly diagonally dominant, and according to the above hypothesis, for arbitrary $m > l$, the first k rows of A_m are also strictly diagonally dominant. However, according to the conditions of the theorem, there must exist rows that are not strict diagonally dominant in the last $n-k$ rows of A_l, otherwise A_l would be generalized strictly diagonally dominant matrix. A is generalized strictly diagonally dominant matrix, which is contradictory to the conditions of the theorem. Likewise, for arbitrary $m > l$, there must exist rows that are not strictly diagonally dominant in the last n-k rows of A_m. In this way for A_l, as the increase of l, elements in the corresponding sub block A_{12} will be larger. However, for the fact that the first k rows are strictly diagonally dominant, elements in A_{12} has an upper limit, so there must exist a limit. Provided

$$\lim_{l\to\infty} A_l = A_\omega = \begin{pmatrix} A_{11} & B \\ A_{21} & C \end{pmatrix},$$

now we can have a look at B and C in the result.

Similar to theorem 2.1, it can be proved that the last $n-k$ rows in A_ω are not infinite and that $\lim_{l\to\infty} \max_{1\leq i\leq n} t_i(A_l) = 1$.

Therefore, no rows in A_ω are not strictly diagonally dominant. If there exist strictly diagonally dominant rows in the first k rows of A_ω, then A_ω is generalized strictly diagonally dominant matrix, which is contradictory to the conditions of the theorem. If there does not exist strictly diagonally dominant rows in the first k rows of A_ω, which is to say that we have $t_i(A_\omega) = 1$, then $m(A_\omega)$ is singular, which is also contradictory to the hypothesis of the theorem. Therefore, for a large enough k, we must have $N_1(A_k) = \varnothing$.

Based on the analysis and above theorem, we can give a modified algorithm 2.

Algorithm 2:

Input: irreducible matrix $A = (a_{ij}) \in \mathbb{C}^{n\times n}$

Output: A is not a generalized strictly diagonally dominant matrix, or A is a generalized strictly diagonally dominant matrix.

1: If $a_{ii} = 0$, for some $i \in N$, "A is not a generalized strictly diagonally dominant matrix", stop; otherwise

2: set $B_0 = A$, $C_0 = A$, $k = 0$;

3: For $i \in N$, compute $t_i(B_k)$, and $p = \arg\min_{1\leq i\leq n} t_i(B_k)$, $[q, qq] = \arg\max_{1\leq i\leq n} t_i(B_k)$;

If $p \geq 1$, "A is not a generalized strictly diagonally dominant matrix", stop;

If $q \leq 1$, "A is a generalized strictly diagonally dominant matrix", stop;

Otherwise, compute $B_{k+1} = B_k D_k$, here $D_k = diag(d_1, d_2, \cdots, d_n)$, and

$$d_i = \begin{cases} q, & i = qq \\ 1, & i \neq qq \end{cases};$$

4: For $i \in N$, compute $t_i(C_k)$, and $[u, uu] = \arg\min_{1\leq i\leq n} t_i(C_k)$, $v = \arg\max_{1\leq i\leq n} t_i(C_k)$;

If $u \geq 1$, "A is not a generalized strictly diagonally dominant matrix", stop;

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If $v \le 1$, "A is a generalized strictly diagonally dominant matrix", stop;

Otherwise, compute $C_{k+1} = C_k D_k$, here $D_k = diag(d_1, d_2, \cdots, d_n)$, and

$$d_i = \begin{cases} u, & i = uu \\ 1, & i \ne uu \end{cases};$$

5: set $k = k+1$, return to step 3.

Note:

(1) In step 3 and step 4 for maximums and minimums, only one of them is needed.

(2) The calculation for each iteration in this algorithm is about twice as that in the algorithm in [7].

(3) The first condition in step 3 and the second condition in step 4 are both necessary. In this way, iterations can sometimes be reduced as it is indicated by the following tables.

(4) Step 3 and step 4 can be done simultaneously as they do not influence each other.

Theorem 2.3. For a given arbitrary irreducible matrix A, algorithm 2 is always convergent.

Proof: If there are zero elements in diagonal of the matrix A, algorithm 2 can be stopped directly. If there are no zero elements in diagonal of matrix A, when matrix A is generalized strictly diagonally dominant, step 4 in algorithm 2 can be stopped in limited steps according to theorem 2.1; when matrix A is not generalized strictly diagonally dominant, step 3 in algorithm 2 can be stopped in limited steps according to theorem 2.2.

Theorem 2.4. For any given irreducible matrix A, if Algorithm 2 converges then it's all conclusions are correct.

Proof: When the algorithm 2 is stopped, there are two outputs:"A is not a generalized strictly diagonally dominant matrix" and "A is a generalized strictly diagonally dominant matrix".

The output "A is not a generalized strictly diagonally dominant matrix" may emerge in step 1, step 3 or step 4. If it is stopped in step 1, then for some $i \in N$, we have $a_{ii} = 0$, according to Lemma 1.1, A is not a generalized strictly diagonally dominant matrix. If it is stopped in step 3, then for arbitrary $i \in N$, we have $t_i(B_k) \ge 1$, then $N_1(B_k) \ne \varnothing$. According to Lemma 1.2, B_k is not a generalized strictly diagonally dominant matrix.

Therefore, according to Lemma 1.3, A is not a generalized strictly diagonally dominant matrix. If it is stopped in step 4, then for arbitrary $i \in N$, we have $t_i(C_k) \ge 1$, then $N_1(C_k) \ne \varnothing$. According to Lemma 1.2, C_k is not a generalized strictly diagonally dominant matrix. Therefore, A is not a generalized strictly diagonally dominant matrix.

The output "A is a generalized strictly diagonally dominant matrix" may emerge in step 3 and step 4. If it is stopped in step 3, then for arbitrary $i \in N$, we have $t_i(B_k) \le 1$. The condition $t_i(B_k) \ge 1$ has been analyzed in the paragraph above. Here we have $t_i(B_k) \le 1$, and at least one inequality is strict. Therefore, according to Lemma 1.4, B_k is a generalized strictly diagonally dominant matrix. Then A is a generalized strictly diagonally dominant matrix. If it is stopped in step 4, then for arbitrary $i \in N$, we have $t_i(C_k) \le 1$. The condition $t_i(C_k) \ge 1$ has been analyzed in the above paragraph. Here we have $t_i(C_k) \le 1$, and at least one inequality is strict. Therefore, according to Lemma 1.4, C_k is a generalized strictly diagonally dominant matrix, and then A is a generalized strictly diagonally dominant matrix.

Input: irreducible matrix	A	B	C	D	E	F
Generalized strictly diagonally dominant matrix or not	Yes	No	Yes	No	No	Yes
Number of iteration required in algorithm 1	2	1	18	76	53	22
Number of iteration required in algorithm 2	1	1	18	10	2	22

NUMERICAL EXPERIMENT

In this part, we examine the effectiveness of algorithm 2 through numerical experiments and make a comparison between algorithm 1 and algorithm 2. Matrices in the following examples are from paper [1].

Example 1. Based on the following matrices:

$$A = \begin{pmatrix} 1 & 0 & -0.5 \\ -0.5 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 0 & -0.5 \\ -2 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & -0.8 & -0.1 \\ -0.5 & 1 & -0.3951 \\ -0.8 & -0.6 & 1 \end{pmatrix},$$

$$D=\begin{pmatrix} 1 & -0.8 & -0.1 \\ -0.5 & 1 & -0.3952 \\ -0.8 & -0.6 & 1 \end{pmatrix},$$

$$E=\begin{pmatrix} 1 & -2 & -1 & 0 \\ -2 & 1 & 0 & -1 \\ 0 & -0.25 & 1 & -0.5 \\ -0.25 & 0 & -0.5 & 1 \end{pmatrix},$$

$$F=\begin{pmatrix} 1 & -0.2 & -0.1 & -0.2 & -0.1 \\ -0.4 & 1 & -0.2 & -0.1 & -0.1 \\ -0.9 & -0.2 & 1 & -0.1 & -0.1 \\ -0.3 & -0.7 & -0.3 & 1 & -0.1 \\ -1 & -0.3 & -0.2 & -0.4 & 1 \end{pmatrix}。$$

Our results are shown in the above Table：

According to the results shown in the Table, for generalized strictly diagonally dominant matrices, the two algorithms are almost the same in terms of the required number of iterations; while for non- generalized strictly diagonally dominant matrices, the required number of iteration in algorithm 2 are obviously much fewer than algorithm 1. This proves that our algorithm is more efficient and effective. Besides, theoretically in this paper pointed out that algorithm 1 is not effective for matrix E, in fact algorithm 1 can be effective in real calculation, but a large number of iterations are required.

Example 2. Based on the following matrix

$$A=\begin{pmatrix} 100 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 3 \end{pmatrix},$$

It can be easily concluded that A is a generalized strictly diagonally dominant matrix. Three iterations are required in algorithm 1, while only one iteration is required in algorithm 2. From this example we can see that sometimes for generalized strictly diagonally dominant matrices have fewer iteration are required in algorithm 2 than that in algorithm 1. Therefore, algorithm 2 is the best.

CONCLUSION AND REMARKS

In this paper, we proposed an iterative algorithm to identify generalized strictly diagonally dominant matrix. The efficiency of proposed algorithm is better than existing algorithm that can been proved through theoretical analysis and numerical experiments. A drawback of proposed algorithm is that it is only effective for irreducible matrices whereas it is not suited for reducible matrices. In future, our focus will be how to adapt proposed algorithm in reducible matrices.

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