HYDROMAGNETIC STAGNATION POINT FLOW OF MICROPOLAR FLUIDS DUE TO A POROUS STRETCHING SURFACE WITH RADIATION AND VISCOUS DISSIPATION EFFECTS

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ABSTRACT: *This work intends to examine, the effects of viscous dissipation and radiation on hydro magnetic stagnation point flow of micro polar fluids due to a porous stretching surface. The highly non-linear equations of the problem are transformed in to ordinary differential form by using appropriate similarity transformations. The resulting equations are solved numerically using Rung-Kutta fourth order method with shooting technique. The results have been computed to observe the influence of the physical parameters involved in the study for velocity, temperature and micro rotation. The comparison of the results for Newtonian and micro polar fluids is presented.*

AMS Subject Classification: 76M20.
Key Words: Micropolar fluids, MHD stagnation point flow, Stretching sheet, Vicus dissipation, Prandtl number, Radiation parameter

1. INTRODUCTION

Micropolar fluids introduced by Eringen [1] are the fluid that belongs to a category of fluids that have non symmetrical stress tensor. These fluids could include rigid unsystematically (or spherical) particle suspended in an extremely permeable medium, wherever the deformation of the particle is unheeded. This category of fluids represents, some industrial vital fluids like paints, body fluids, polymers, combination fluids, suspension fluids, animal blood, among the varied non-Newtonian fluids. Eringen [2] after wards developed the theory of thermo micro polar fluids by extending theory of min polar fluids. Moreover, Lukaszewicz [3] provided extensive surveys of literature for micro polar fluids.

Ever since, the micro polar fluids theory has become a popular and interesting research area. Researchers are engaged to explore new useful results using Micropolar fluid theory. Kasiviswanathan and Gandhi [4] obtained a category of achievable solutions of MHD flow of a micro polar liquid restricted between two endless, insulated, similar, non-coaxially pivoting disks. Sajjad and Kamal [5] examined boundary layer stream for miniaturized scale polar electrically conducting liquid on a revolving disk in the vicinity of magnetic field. Kim and Lee [6] found the analytical solution of micropolar fluids. Ashraf et al. [7] obtained numerical simulation for 2- dimensional flow of micropolar fluid between an impermeable and a permeable disk. Lee et al. [8] delineated the results on a wobbly (MHD) flow with chemicals react micro polar fluid over an infinite perpendicular porous plate. Sajjad et al. [9] studied MHD stagnation point flow of micropolar fluids towards a stretching sheet. Ahmad et al [10] investigated convective heat transfer for MHD micropolar fluids flow through porous medium over a stretching surface. Adhikari [11] investigated the magnetohydrodynamic mixed convection stagnation point flow of micropolar fluids due to stretching vertical surface.

The radiation effects may play a major role in engineering, technology and thermal processes.

Some necessary applications of radiative heat transfer are like cooling of nuclear reactors, warmth plasmas, power generation systems, liquid metal fluids and heat transfer control system in polymer processing industry. Abo-Eldahab and Ghonaim [12] described the character of thermal edge cover flow over a stretched sheet in a micro polar fluid in existence of energy. Ahmad Khidir [13] investigated the impacts of thick scattering and Ohmic warming on MHD convective stream because of a permeable rotating disk, considering the variable liquid properties in the vicinity of Hall and thermal radiation. Reddy et al. [14] studied the heat and mass transfer effects on an unsteady MHD flow of a chemically reacting micropolar fluid over an infinite perpendicular porous plate through a porous medium with Hall effects and thermal radiation in the occurrence of radiation absorption. The impact of warm radiation and magnetic field on uneven blended convection stream and heat transfer over a porous extending plate was discussed by Elbashbeshy et al. [15].

In this paper, we obtained the numerical solution for the radiation and viscous dissipation effects on hydromagnetic stagnation point flow of micropolar fluids due to a porous stretching surface, to extend the work of Arthur and Seini [16]. The effects of the physical parameters of the study have been observed and discussed in detail through geographical patteren of the flow and heat transfer.

2. MATHEMATICAL ANALYSIS

Consider micropolar fluid flow towards the stagnation point on a porous stretching surface**.** The fluid is electrically conducting. The flow is steady, two-dimensional and incompressible. The magnetic field of strength B_o is perpendicular to the surface. The motion of fluid is always towards the stagnation point over a stretching surface The motion of fluid in always in the positive y direction. The tangential velocity varies proportional to a specified distance x. From the stagnation point, the stream velocity

U_∞ varies proportional to the distance x, so that $U_\infty = bx$ and $U_\infty = ax$, where a and b are constants.The induced magnetic field and pressure gradient are neglected. The temperature of wall is maintained at a constant value T_w. The body couple is absent. The velocity vector: $\underline{V} = V(u,v,w)$ and spin vector: $\underline{\omega} = \omega(\omega_1, \omega_2, \omega_3)$.

Under the above assumptions the equations governing the problems are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$(\mu + k)\left(\frac{\partial^2 u}{\partial y^2}\right) + k\left(\frac{\partial \omega_3}{\partial y}\right) - \sigma B_0^2 u + a^2 x = \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \quad (2)$$

$$\gamma\left(\frac{\partial^2 \omega_3}{\partial y^2}\right) - \kappa\left(\frac{\partial u}{\partial y}\right) - 2\kappa\omega_3 = \rho j\left(u\frac{\partial \omega_3}{\partial x} + v\frac{\partial \omega_3}{\partial y}\right) \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{\alpha}{K}\frac{\partial q_r}{\partial y} + \frac{\rho B_0^2}{\rho c_p}u^2 \quad (4)$$

The boundary conditions are:

$$w_3(x,0) = 0 \;\; u(x,0) = bx, \quad v(x,0) = -v, \quad T(x,0) = T_w \quad (5)$$
$$w_3(x,\infty) = 0 \;\; u(x,\infty) = ax, \quad T(x,\infty) = T_\infty ,$$

Where $\boldsymbol{\upsilon}$ is kinematic viscosity coefficient, α is the thermal diffusivity, σ is the electrical conductivity, K is the thermal conductivity, c_p is the specific heat capacity at constant pressure, q_r is the radiative heat flux and B_o is the magnetic field strength,

μ is dynamic viscosity, κ, γ are additional viscosity coefficients for micropolar fluid. j is micro inertia, ρ is density.

The velocity has two components that are described as u and v in terms of the stream function Ψ (x, y) as given below:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

Using similarity transformations:

$$\psi(x,y) = x\sqrt{\upsilon b}\, f(\eta), \quad \eta = y\sqrt{\frac{b}{\upsilon}}$$

$$u = xbf', \quad v = -\sqrt{\upsilon b}\, f, \quad \omega_3 = bx\sqrt{\frac{b}{\upsilon}}\, g(\eta),$$

$$q_r = -\frac{4\sigma^*}{3K'}\frac{\partial T^4}{\partial y}, \quad T^4 \cong 4T_\infty^3 - 3T_\infty^4, \quad \theta(\eta) = \frac{T - T_\infty}{T_s - T_\infty}$$ where σ^* is Stefan-Boltzmann constant and K' is mean absorption coeffient.

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equations (2), (3) and (4) we get

$$(1 + C_1)f''' + ff'' - f'^2 + C_1 g' - Mf' + \lambda^2 = 0 \quad (6)$$

$$L'' + c_2 f'' - 2c_2 L = C_3(f'L - fL') \quad (7)$$

$$(1 + \frac{3}{4}Ra)\theta'' + \Pr f\theta' + Br(f'' + Mf'^2) = 0 \quad (8)$$

and the boundary conditions are

(9)

where

$$R_a = \frac{4\sigma'}{KK'}T_\infty^3, \quad M = \frac{\sigma B_0^2}{\rho b}, \quad f_w = \frac{-v}{\sqrt{b\upsilon}},$$

$$f'(0) = 1, \; f(0) = f_w, \; \theta(0) = 1, \; L(0) = 0 \qquad B_r = \frac{\mu (bx)^2}{K(T - T_\infty)}.$$
$$f'(\infty) = \lambda, \; \theta(\infty) = 0, \; L(\infty) = 0$$

The dimensional less material constants are

$$C_1 = \frac{\mu}{k}, \; C_2 = \frac{k\upsilon}{rb}, \; C_3 = \frac{\rho j \upsilon}{r}.$$

3. RESULTS AND DISCUSION

The governing highly nonlinear ordinary differential equations (6-8) are solved with the boundary conditions (9). Runge-Kutta 4th order method with shooting technique has been applied to get the numerical result of the problem. Several computations have been made for various values of the parameters involved in the physical and mathematical model of the problem. The effects of these parameters namely magnetic parameter M, velocity ratio number λ, radiation parameter R_a , Prandtl number P_r, Brinkmann number B_r have been observed on the flow kinematics and temperature distribution. The effects of the non-dimensional material constants are also being noticed for velocity and micro rotation function. The results have been presented in graphical form.

Figure 1 shows that when λ < 1 (λ=0.1) the horizontal velocity component f' increases with increase in the values of micro rotation parameter C_1, this figure presents the comparison of Newtonian and micropolar fluids. The velocity of micropolar fluids is greater than that of Newtonian fluids.

But when λ > 1 (λ=2), the velocity component f' is inversely proportional to the micro polar parameter C_1. It decreases with increase in the values of C_1 as show in figure 2. It is also noticed that the velocity for Newtonian fluid is greater than that of micropolar fluids.

Figure 3 demonstrates the pattern of micro rotation for different values of C_1 when λ=0.1. It is noted that the micro rotation increases very small near the stretching surface but decreases away from the surface with increase in the value of C_1. Figure 4 depicts the micro rotation L for various values of C_1 when λ > 1 (λ=2). The micro rotation decreases near stretching surface but increases away from the surface with increase in the values of C_1. It can also be noticed that the micro rotation has opposite sense of rotation when λ < 1 and λ > 1.

Figure 5 presents the velocity $f^{/}$ for different values of suction parameter f_w when $\lambda = 0.1$. The velocity decreases with increasing values of f_w. Figure 6 shows the behavior of micro rotation when $\lambda = 0.1$ and for different values of f_w. The micro rotation increases with increase in the values of f_w.

Figure 7 plots the velocity f' for different values of $\lambda < 1$. The velocity is directly proportional to the value of λ . It increases with increase in the values of λ. The micro rotation has been plotted for different values of $\lambda < 1$ in figure 8. The micro rotation decreases with increasing values of λ. Figure 9 demonstrates the velocity f' for different values of magnetic parameter M. The velocity decreases with increase in the magnetic field strength.

Figure 10 depicts the effect of magnetic force on micro rotation. It is observed that the micro rotation increases near the surface but decreases away from the surface with increase in the magnetic field strength and it changes the sense of rotation at some distance from the surface.

The temperature is inversely proportional to the velocity ratio parameter λ. It reduces with increase in the values of λ and suction parameter f_w as show respectively in figure 11 and figure 12. The temperature function $\theta(\eta)$ is directly proportional to the magnetic parameter M. It increases with increase in the magnetic parameter M as presented in figure 13.

The figure 14 shows that the thermal boundary layer decreases with increase in the values of Prandtl number Pr. The radiation parameter Ra causes increase in the value of temperature function due to addition of radiative heating as shown in figure 15. The thermal boundary layer thickness increases with the increase in the value of Brinkmann number Br. It is due to the fact that viscous dissipation adds up heating to the fluid. The temperature plots are given in figure 16 for various values of Br.

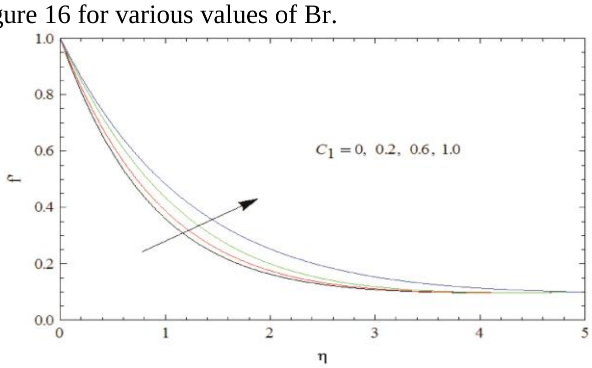

Fig. 1: graph of f' when C_1 = 0, 0.2, 0.6, 1For λ = 0.1

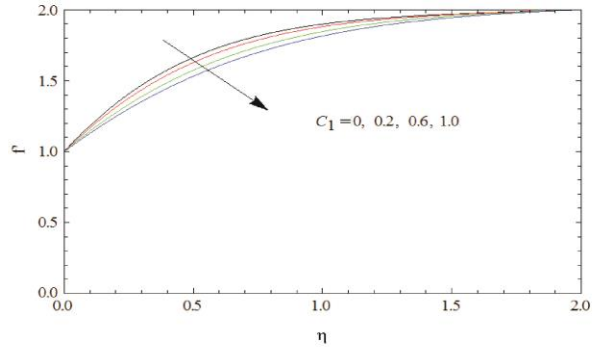

Fig. 2: graph of f' when C_1 = 0, 0.2, 0.6, 1For λ = 2

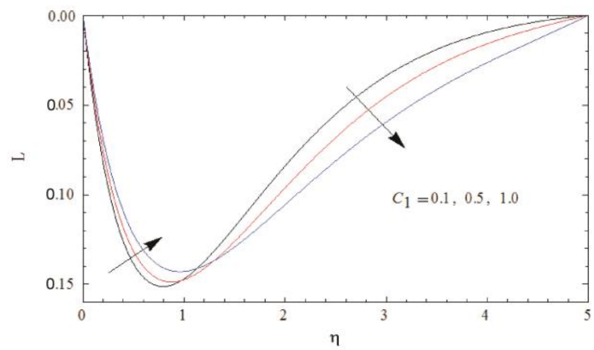

Fig. 3: graph of L when C_1 = 0.1, 0.5, 1 For λ = 0.1.

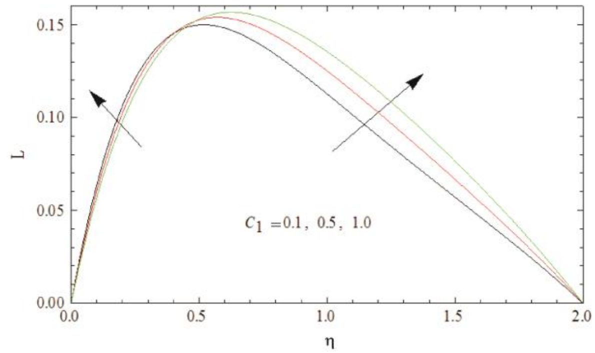

Fig. 4: graph of L when C_1 = 0.1, 0.5, 1For λ = 2

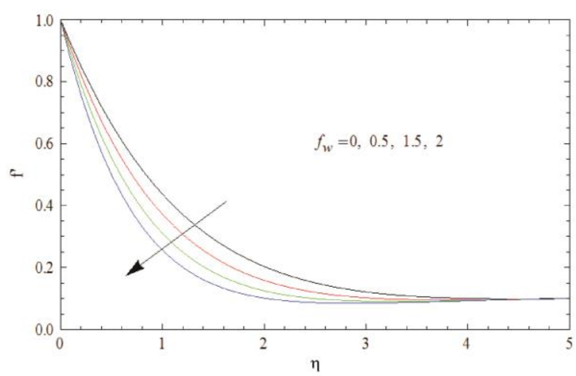

Fig. 5: graph of f' when f_w = 0, 0.5, 1.5, 2, For λ = 0.1

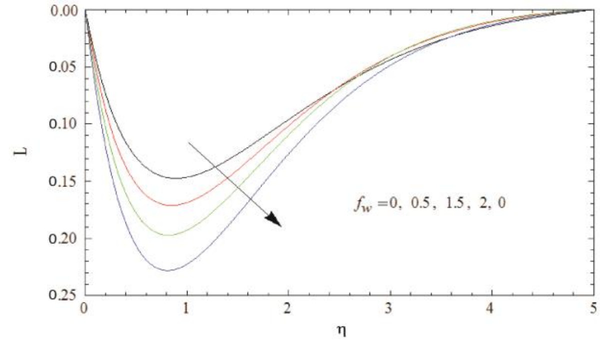

Fig. 6: graph of L when f_w = 0, 0.5, 1.5, 2, For λ = 0.1

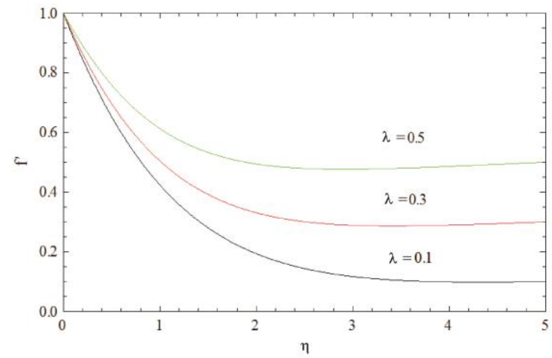

Fig. 7: graph of f' when λ = 0.1, 0.3, 0.5

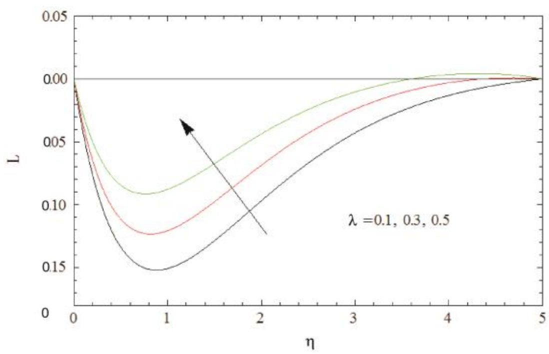

Fig. 8: graph of *L* when λ = 0.1, 0.3, 0.5

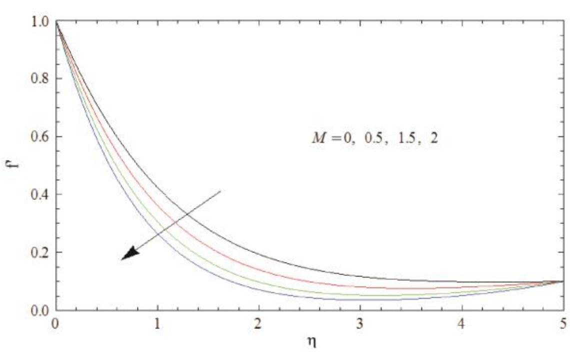

Fig. 9: graph of f' when M = 0, 0.5, 1.5, 2, For λ = 0.1

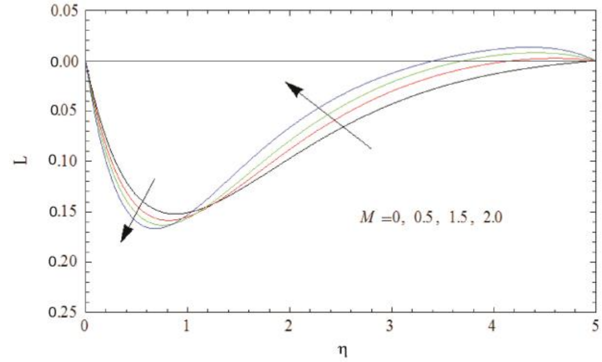

Fig. 10: graph of *L* when *M* = 0, 0.5, 1.5, 2, For λ = 0.1

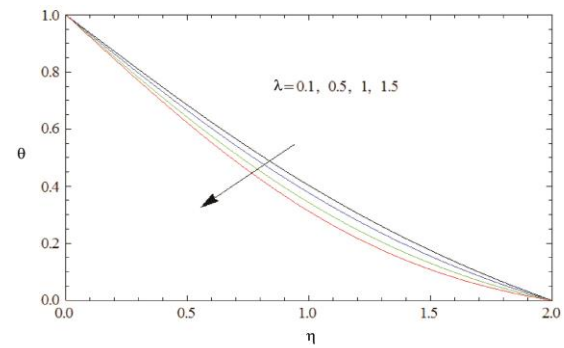

Fig. 11: graph of θ when λ = 0.1, 0.5, 1, 1.5

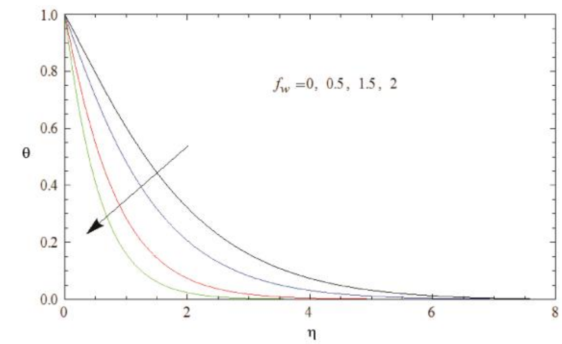

Fig. 12: graph of θ when f_w = 0, 0.5, 1.5, 2

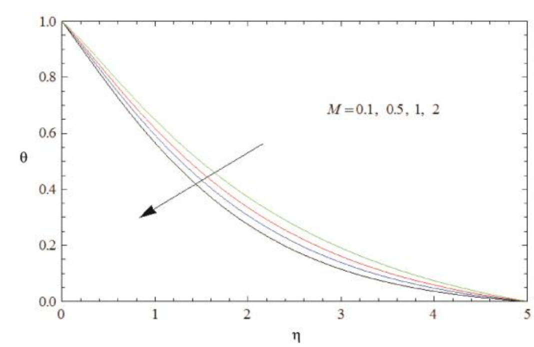

Fig. 13: graph of θ when *M* = 0.1, 0.5, 1,2

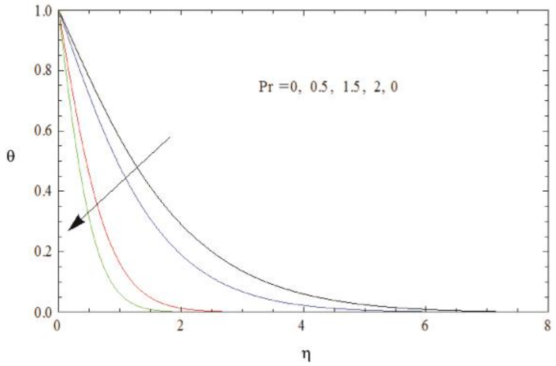

Fig. 14: graph of θ when Pr = 0, 0.5, 1.5, 2

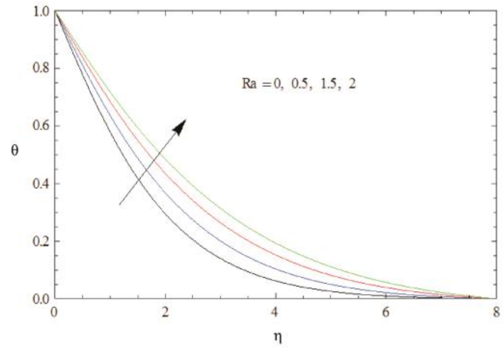

Fig. 15: graph of θ when *Ra* = 0, 0.5, 1.5, 2

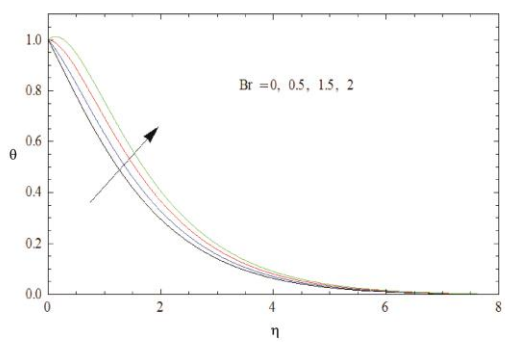

Fig. 16: graph of θ when *Br* = 0, 0.5, 1.5, 2

4. Conclusion

Numerical solution of the radiation and viscous dissipation effects on hydromagnetic stagnation point flow of micropolar fluids due to a porous stretching surface has been obtained in order to observe the effect of physical parameter on velocity, micro rotation and temperature functions. The salient results are summarized as follows.

- It is to mention that when vortex viscosity *k* and the micro rotation vector ω are zero, problem of Micropolar fluids corresponding to the Newtonian fluids flow.
- It is noticed that when $\lambda < 1$ the velocity of Micropolar fluid is greater than that of Newtonian fluid but when $\lambda > 1$ the velocity of Newtonian fluid is greater than that of Micropolar fluid.
- The velocity $f^{/}$ increases with increasing value of λ. The velocity decreases with increase in the values of $f_{w.}$ The velocity increases with increase in the values of C_1 when $\lambda < 1$ but velocity decreases with increase in the values of C_1 when $\lambda < 1$. The velocity decreases with increase in the values of *M*.
- When $\lambda < 1$ the micro rotation increases near the surface but decreases away from the surface with increasing values of $C_{1.}$
- When $\lambda < 1$ the micro rotation decreases near the surface but increases away from the surface with increasing values of $C_{1.}$
- The micro rotation has opposite sense of rotation for $\lambda < 1$ and $\lambda > 1$.
- The micro rotation decreases with increasing values of λ.
- The micro rotation increases near the surfaces but decreases away from the surface with increase in the values of *M*.
- The temperature decreases with increasing values of λ and $f_{w.}$
- The temperature increases with increasing values of *M*.

The thermal boundary layer decreases with increase in the values of *Pr* and it increases with increase in the values of *Br*.

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