NUMERICAL SOLUTION OF MHD FLOW AND HEAT TRANSFER IN POROUS MEDIUM OVER A POROUS SHRINKING SURFACE WITH RADIATION AND VISCOUS DISSIPATION

Shamila Khalid[1], Sajjad Hussain[2], Hassan Waqas[3]
[1,3]Department of Mathematics, Chenab College of Advance Studies, Faisal Abad, Pakistan.
[1,3]Presently: Department of Mathematics, Govt College University Faisal Abad (Layyah Campus), Pakistan.
[2]Punjab Higher Education Department, College Wing, Lahore, Pakistan.
Shamilakhalid26@gmail.com, sajjadgut@gmail.com[2], syedhasanwaqas@hotmail.com[3]
Corresponding Author: shamilakhalid26@gmail.com[1, a]

ABSTRACT: *The magnetohydrodynamic fluid flow through porous medium over a porous shrinking surface with radiation and viscous dissipation is to be investigated. The momentum and energy equations are in the form of nonlinear partial differential type. To obtain a numerical solution of the problem suitable similarity functions are used to convert the mathematical model into an ordinary differential form. The results will be obtained to observe the effects of the physical parameters namely Radiation parameter, Eckert number, Magnetic parameter, Shimidt number, Prandtl number and suction parameter on the flow, temperature and concentration distributions.*

AMS Subject Classification: ?????
Key Words: MHD Flow, Radiation, Porous Shrinking Surface, Vertical surface, MHD, Prandtl number, Porous medium.

1. INTRODUCTION

Thermal and solutal transport by fluid flowing through a porous matrix is a phenomenon of great interest from both the theory and application point of view. These phenomena have several geophysical and technological applications such as drying of porous solids, geothermal reservoirs, heat exchanger, nuclear waste disposal, moisture migration in a fibrous insulation and others. Vafai and Tien [1] have summarized the importance of both boundary and inertia effects in porous media. El-Kabeir *et al* [2] investigated Sort and Dufour effects on heat and mass transfer from continuously moving plate embedded in porous media with temperature dependent viscosity and thermal conductivity. Nakayama and Hossain [3] and Singh and Queen [4] treated an integral method for combined heat and mass transfer by natural convection in porous medium and studied free convection heat and mass transfer along a vertical surface in a porous medium. El-Hakiem *et al* [5] studied the effects of magnetic field and double dispersion on mixed convection heat and mass transfer in non-Darcy porous medium. Convective heat transfer, mathematical and computational modelling of viscous fluids and porous media was investigated by Pop and Ingham [6]. Prasad et al [7] investigated thermal radiation effects on MHD free convection heat and mass transfer from a sphere in a variable porosity regime. El-Hakiem [8] studied radiative effects on non- Darcy natural convection from a heated vertical plate in saturated porous media with mass transfer for non-Newtonian fluid. Combined effects of Joule heating and viscous dissipation on MHD flow past a permeable, stretching surface with free convection and radiative heat transfer was studied by Chen [9]. MHD and mass transfer effect on non-newtonian fluids past a vertical plate embedded in a porous medium studied by El-Hakiem [10]. MHD flow and heat transfer due to shrinking surface is a recent phenomenon Santosh *et al*, [11] considered the unsteady two-dimensional, laminar flow of a viscous, incompressible, electrically conductingfluid towards a shrinking surface in the presence of a uniform transverse magnetic field. Susheela et al, [12] Mathematical analysis is carried out to investigate two-dimensional flow of a viscous incompressible electrically conducting fluid near a stagnation point of a stretching or shrinking surface in a saturated porous medium. S, Jena, [13] deals with a steady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid over a shrinking sheet in the presence of uniform transverse magnetic field with viscous dissipation. Babu1 [14] *et al*, considered the steady 2-dimensional magnetohydrodynamic (MHD) boundary layer flow of heat and mass transfer over a shrinking surface with wall mass suction. Sandeep et al, [9] study we analyzed the influence of thermal radiation and chemical reaction on two dimensional steady magnetohydrodynamic flow of a nanofluid past a permeable stretching/shrinking sheet in the presence of suction/injection. Bhukta [16] analyze the effect in a boundary layer flow of the heat and mass transfer through porous medium of an electrically conducting viscoelastic fluid over a shrinking sheet subject to transverse magnetic field in the presence of heat source. Ahmed *et al* [17] worked on MHD flow and heat transfer through a porous medium over a shrinking surface with suction. Ahmad [18] investigated the incompressible steady, 2- dimensional stagnation point flow, electrically conducting fluid due to a shrinking sheet.

This work considers MHD flow and heat transfer in porous medium over a porous shrinking surface with radiation and viscous dissipation. The effects of the governing parameters on the velocity, temperature and concentration are presented and discussed in detail.

2. Mathematical Analysis:

Consider a viscous and electrically conducting fluid with steady, incompressible and two-dimensional flow due to a permeable shrinking sheet which coincides with the plane y=0 and the flow is restricted in the region$y$>0. The fluid flows within a porous medium obeying Darcy law. Two balance forces are used along the$x$-axis in opposite direction to keep the sheet boundary moving and keep its origin fix.

The magnetic field of strength B_0 is assumed to be uniform and applied normal to the sheet. Cartesian co-ordinates are used. The magnetic Reynolds number is small. we ignore induced magnetic field. All the properties of fluid are considered constant throughout the motion. The governing equations of motion are given below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}u-\frac{\upsilon}{k_p}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{1}{\rho c_p}\left(\frac{16\alpha}{3\beta}T_\infty^3\right)\frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial x^2} \quad (4)$$

The associated boundary conditions are:

$$u=U(x),\ v=v_w,-k\frac{\partial T}{\partial y}=q_w=E_0x^n,$$

$$-D\frac{\partial C}{\partial y}=m_w=E_1x^i\ at\ \ y=0, \quad (5)$$

$$u\to 0, T\to T_\infty,\ C\to C_\infty\ as\ y\to\infty$$

Where υ is the coefficient of viscosity, ρ is the density of the fluid, k_p is permeability of permeable medium, B is magnetic field, k is thermal conductivity, q_w is the rate of heat transfer, C_p is specific heat at constant pressure, C is the species concentration of the fluid, and m_w is the rate of mass transfer.

It is assumed:

$$U=ax^m,\ B(x)=B_0(x)^{\frac{m-1}{2}} \quad (6)$$

The similarity transformations are:

$$\psi(x,y)=\left(\frac{2\upsilon xU(x)}{1+m}\right)^{\frac{1}{2}}f(\eta),$$

$$\eta=\left[\frac{(1+m)U(x)}{2\upsilon x}\right]^{\frac{1}{2}}y,$$

$$T-T_\infty=\frac{E_0x^n}{k}\sqrt{\frac{\upsilon}{a}}\theta(\eta)$$

$$v_w(x)=-\lambda\sqrt{\frac{\upsilon(m+1)}{2}}x^{\frac{m-1}{2}},$$

$$C-C_\infty=\frac{E_1x^i}{D}\sqrt{\frac{\upsilon}{a}}h(\eta) \quad (7)$$

Where $\lambda>0$ is used for suction at the moving boundary sheet, m is power-law exponent ($m\neq -1$), $n=2m$ is heat flux parameter, i is mass flux parameter, a, B_0 are dimensional constants, E_0 is a positive constant, T_∞ is temperature at wall, C_∞ is the species concentration of the fluid away from the wall, D is the diffusivity coefficient, E_1 is a positive constant.

The Equation (1) is satisfied with stream function ψ,

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$$

By using relation in (7), the equations (2) to (4) respectively become:

$$f'''-f'^2+ff''-(\tfrac{1}{K}+M)f'=0, \quad (8)$$

$$(3R_n+4)\theta''+3R_n(P_rf\theta'-2P_rf'\theta)=-3R_nP_rE_cf''^2, \quad (9)$$

$$h''-f'S_cih+S_cfh'=0, \quad (10)$$

and the associated boundary conditions (5) become:

$$f(\eta)=\lambda=f_w,\ f'(\eta)=1, \theta'(\eta)=1, h'(\eta)=-1\ at\ \eta=0$$

$$f'(\eta)=0, \theta(\eta)=0,\quad h(\eta)=0\ \ as\ \ \eta\to\infty \quad (11)$$

Where,

$K=\frac{k_pa}{\upsilon}$ is permeability parameter and $M=\frac{\sigma(B_0)^2}{\rho a}$ is the magnetic parameter,

$P_r=\frac{\mu C_p}{k}$ is the Prandtl number and $E_c=\dfrac{a^2}{C_p\left(\frac{E_0}{k}\sqrt{\frac{\upsilon}{a}}\right)}$ is Eckert number, $S_c=\frac{\upsilon}{D}$ is Schmidt number,

$R_n=\frac{\beta k}{4\alpha T^3_\infty}$ is radiation parameter.

3. RESULTS AND DISCUSSIONS

Numerical solution of the non-linear system of equations (8) to (11) has been obtained with help of Mathematica 10.The results in form of graphs have been presented to perceive behaviour of physical quantities of flow and heat transfer.

Fig.1 demonstrates the effect of the magnetic force on velocity f'.It is observed that velocity reduces in magnitude with increase in the values of magnetic parameter M. It is because of the magnetic force causes an opposing force to the fluid flow. When the porosity of the medium increases, the magnitude of the velocity f' also increases. The fig.2, demonstrates the effect of parameter K on velocity f'. The velocity increase with increase in K. The injection also causes an increase in fluid velocity for the shrinking surface and it reduces the boundary layer thickness as presented in the fig.3. Fig.4 shows that velocity component f' increases with increase in suction for stretching surface. The porosity parameter K increases the vertical f as depicted in Fig.5 but Fig .6 shows that magnetic force decreases the vertical velocity component f. It is noted that the magnetic force causes an increase in the velocity f' of the fluid when the surface is shrinking and the boundary layer thickness also decreases. This fact is shown in Fig.7.

The effect of the magnetic field on temperature distribution is presented in Fig.8.The magnetic force causes an increase the temperature function $\theta(\eta)$. The increase in Prandtl number causes a decrease in the value of the temperature distribution as shown in Fig .9. It is because the Prandtl number is reciprocal to the thermal conductivity.Fig.10 and Fig.11 respectively show that increase in Eckert number and radiation parameter both cause the increase in value of function $\theta(\eta)$.

The Schmidt number S_c causes decreases in the value of concentration function as shown in fig.12.
Fig.13 shows that the increase in magnetic field causes increase concentration function $\phi(\eta)$.Fig.14 shows that the increase in the injection parameter causes an increase in the concentration but increase in the suction parameter causes decrease in it.

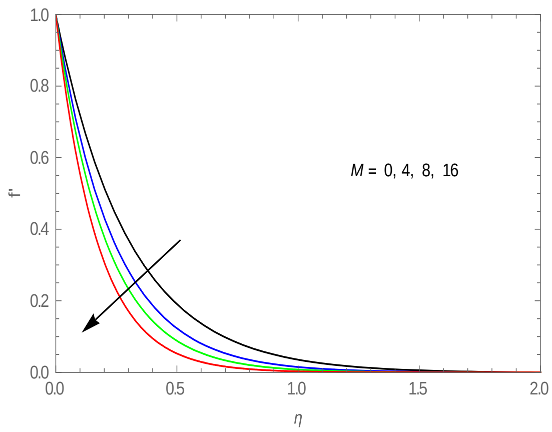

Fig 1:The plot of curve f' under the effect of M.

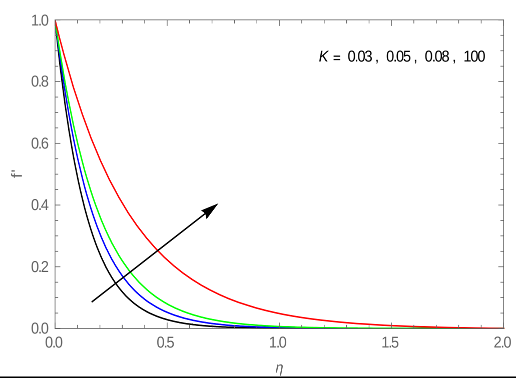

Fig 2:The plotfor curve of f' under the effect of K.

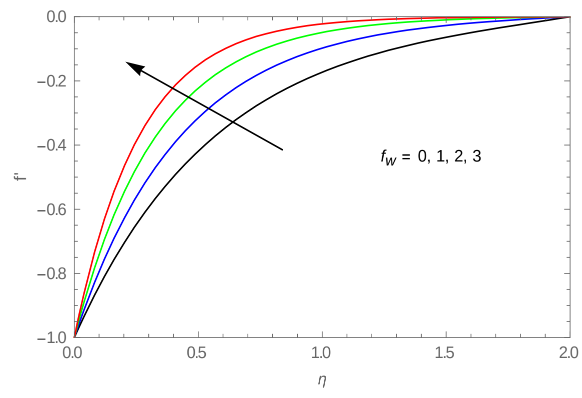

Fig 3: The plot of curve f under the effect of fw.

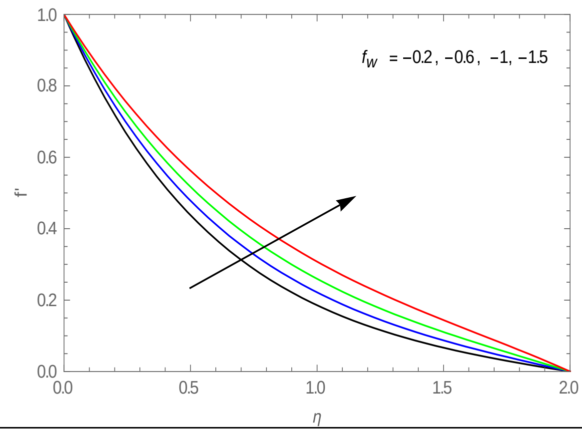

Fig.4: The plot for curve of f' under the effect of fw.

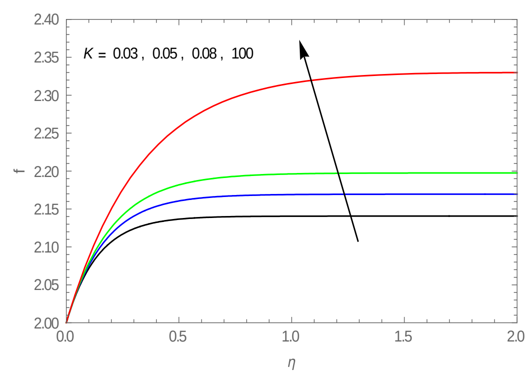

Fig 5:The plot for curve of f under the effect of K.

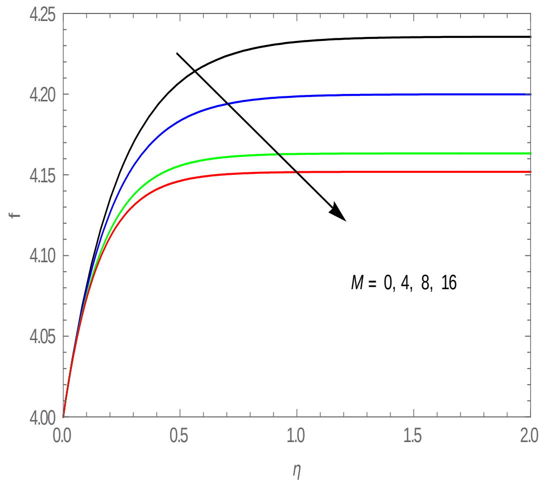

Fig 6:The plot for curve of f' under the effect of M.

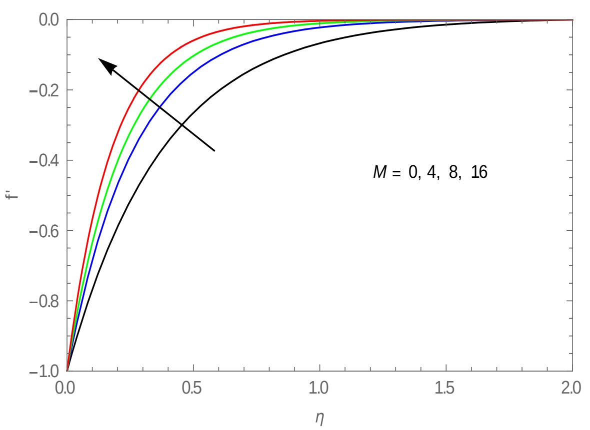

Fig 7: The plot for curve of f under the effect of M.

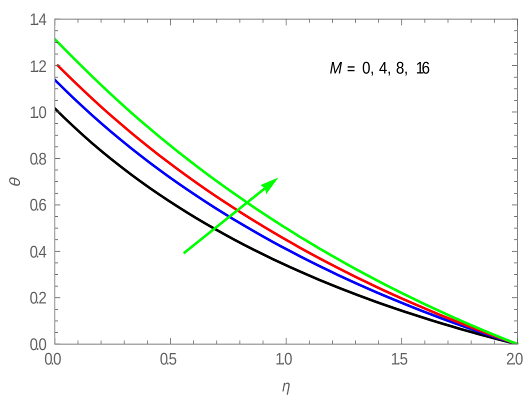

Fig 8:The plot for curve θ under the effect of M.

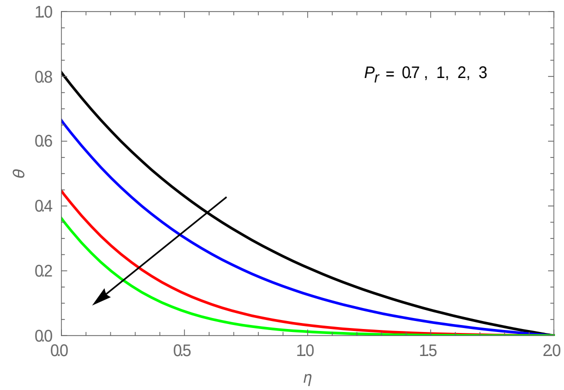

Fig.9 :The plot for curve of θ under the effect of Pr.

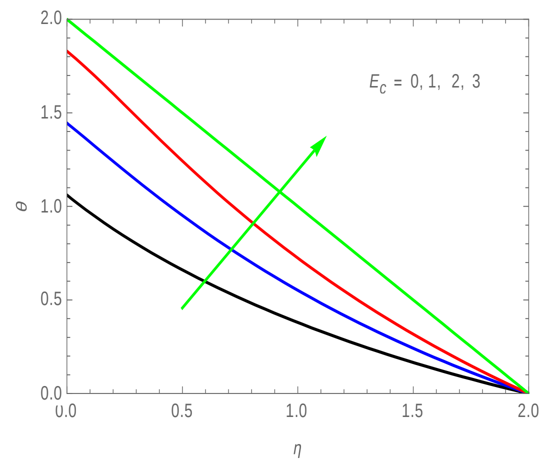

Fig 10:The plot for curve of θ under the effect of M.

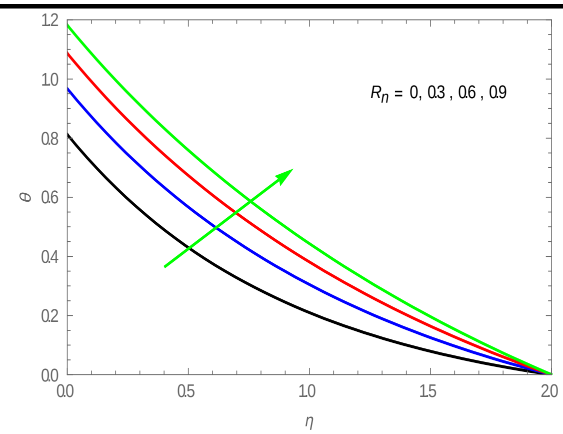

Fig.11: The plot for curves of θ under the effect of R_n.

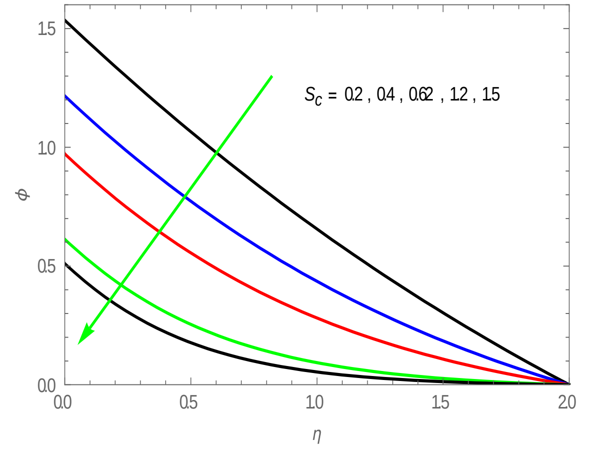

Fig 12:The plot for curve of ϕ under the effect of Sc.

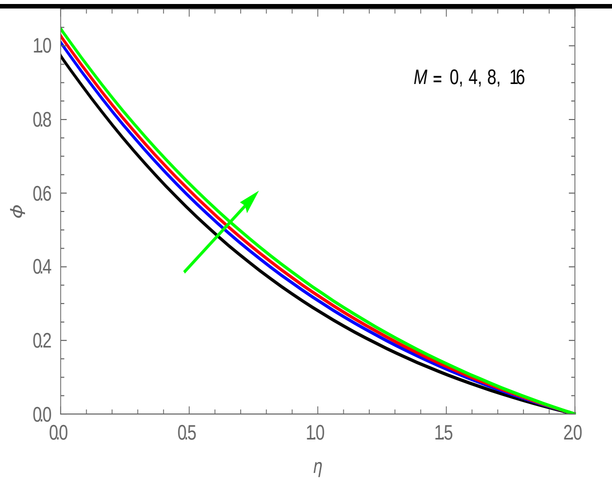

Fig.13: The plot for curves of ϕ under the effect of *M*.

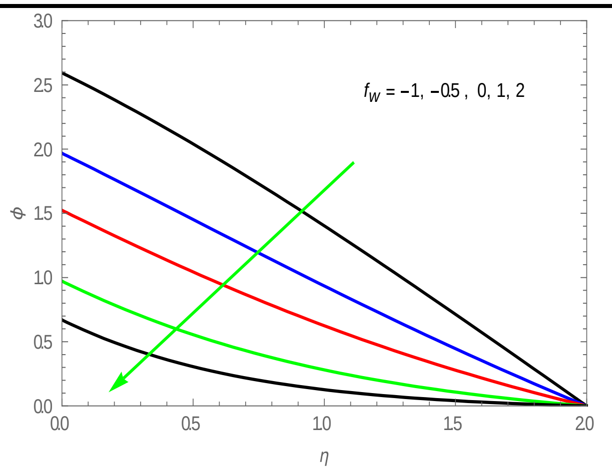

Fig.14: The plot for curves of ϕ under the effect of f_w.

CONCLUSION:

In this work, we considered numerical solution of MHD flow and heat transfer in porous medium over a porous shirking surface with radiation and viscous dissipation. The results of the study have been elaborated with the various sufficient ranges of the parameters effecting the flow and heat transfer in this problem. The main outcomes of the work are summarized below.

- When it is stretching sheet, the velocity f' reduces in magnitude with increase in value of magnetic parameter M. It is because of the magnetic force causes an opposing force to the fluid flow. But the magnetic force causes an increase in the velocity of the fluid when the surface is shrinking and the boundary layer thickness decreases.
- When the porosity of the medium increases, the magnitude of the velocity f' increases.
- The velocity component f' decreases with increase in suction but increases with increase in injection for stretching case and the suction causes an increase in fluid velocity for the shrinking surface and it reduced the boundary layer thickness.
- Magnetic force decreases the vertical velocity component f' . But the increase in porosity of the medium increases this velocity.
- The magnetic force causes as increase the temperature function $\theta(\eta)$. But increase in suction causes decrease in it.
- The increase in Prandtl number also causes a decrease in the value of the temperature distribution.
- The increase in Eckert number and radiation parameter both cause the increase in value of function $\theta(\eta)$.
- The increase in magnetic field causes increase concentration function but Schmidt number causes decreases in it.
- The increase in the injection parameter causes an increase in the concentration but increase in the suction parameter causes decrease in concentration.

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