SOLUTION OF QUARTER CAR MODEL BY PATTERN SEARCH METHODS

Nagina, F., M. Saeed, M. F. Tabassum*, J. Ali
Department of Mathematics, University of Management and Technology, Lahore, Pakistan.
* Corresponding Author: Muhammad Farhan Tabassum, farhanuet12@gmail.com +92-321-4280420
(Presented at the 5th International. Multidisciplinary Conference, 29-31 Oct., at, ICBS, Lahore)

ABSTRACT: *This paper involves a relative study of three optimization methods, which are Hooke-Jeeves, Nelder-Mead, and Multi-Directional Search Methods, for design optimizing vehicle suspensions constructed on quarter vehicle model with different types of constrains. In optimization, three design norms are suspension working space, dynamic tire load and vertical vehicle acceleration. To execute design optimization five variables are nominated which are the tire stiffness, damping coefficient, sprung mass, spring stiffness and un sprung mass. It was resulted from the comparative study that the Multidirectional Search Method is more reliable than Hooke-Jeeves Method and Nelder-Mead Method. The optimum results of the quarter car model were obtained by using MATLAB programming environment which demonstrated the effectiveness and applicability of the methods.*

Key Words: Quarter Car Model, Hooke-Jeeves Method, Nelder-Mead Method, Multi-Directional Search Method

1. INTRODUCTION

Trial and error method depends on the design and practice of vehicle suspension until the require results achieved [1].This process is slow and requires a long time. Because of the advancement in theoretical method and computational power, by using optimization approaches focus of vehicle suspension design has been diverted from refined numerical investigation to design synthesis. Many algorithms are used to find optimal suspension properties. Many methods are available, but the choice of a systematic method is an important problem. The above methods could be used in different applications such as control system, identification of system and function optimization, which apply in conventional methods by using different principles concurrently and analytically, whose important features expressed as a population wide search and a continuous balance into exploration and exploitation principal of building block combination [2]. To find a prime solution for searching problems we use Hooks-Jeeves method. Yet, HJ are distinguished by a large number of function evaluations. Function comparison techniques provide a base for Nelder-Mead method and Multidirectional Search method, these techniques are no need of derivative evaluation these are helpful in solving many problems where the objective function is not differentiable.

Vehicle suspension of design optimization need the well trade off solution by knowing optimal design variables which has vertical vehicle body's acceleration, suspension working space and dynamic tire load. Which can be found by variables including acceleration of vehicle, suspension working space and dynamic tire load. Such variables could be included damping coefficient, suspension spring stiffness, inertial parameters and geometry [3]. In the frequency domain the survey of ride quality, the dynamic tire load, suspension working space and acceleration of a vertical vehicle body are acquired [4]. The prime objective of design optimization is to minimize, the acceleration of a vertical vehicle body. At the same time constrained are dynamic tire load and suspension working space. Vehicle will damage in that case if the value of suspension working space is very low, in this way un-sprung mass collision with sprung mass and damage will occur. In case of consistent tire load is less than dynamic tire load, then vehicle tires will rebound the road, this provides the result of motion of vehicle in an unstable manner. A complex vibrating system is a vehicle suspension system which has multiple degree of freedom [5]. To isolate the vehicle body from the road input suspension system is being used. Dynamics related to the vehicle put different needs on the constituent of the suspension system. Ride comfort of passenger requires that sprung mass acceleration should be small while for dynamic presentation there should be proper road holding i-e consistent force into the tires and the road [6].

2. MATERIALS AND METHODS

The motivation for this research was to modify Quarter Car Model. The derivative free methods were used for the optimization of Quarter Car Model. These methods were basically designed for unconstrained optimization problems. In formulating optimization Quarter Car Model the constraints were handled by using exterior penalty functions [7-10].

2.1 Hooke-Jeeves Method

For an N-dimensional problem HJ method [11] required an initial point x_0, a set of N linearly independent search directions v_i, step-length parameters $\delta_i > 0$ and a parameter $\mu > 1$. The method used two types of moves given below:

Exploratory Move: This move was made on the current point by investigation along each direction according to the following formula:

$x_{new} = x_0 \pm \delta_i v_i$ for all i= 1, 2, 3, …, N.

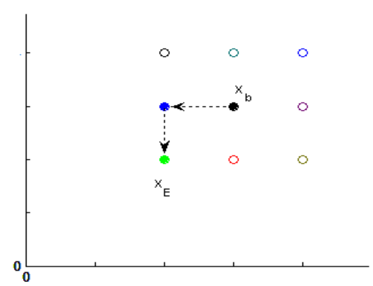

Fig-1: Successful exploratory move

Pattern Move: When an exploratory move was completed and was accomplished successfully, then pattern move was executed, by jumping from present base point along with a direction connecting and a new point was found. Once a pattern move was established it was possible to move as much as allowed. An enlargement parameter η, η ≥ 1, was used for this purpose. The pattern direction was found by the formula applied as $\underline{d} = z_E - z_b$. Therefore the new point, through pattern move, was found as given below

$\underline{y}_b = z_E + \eta\, \underline{d} = z_E + \eta\,(z_E - z_b)$.

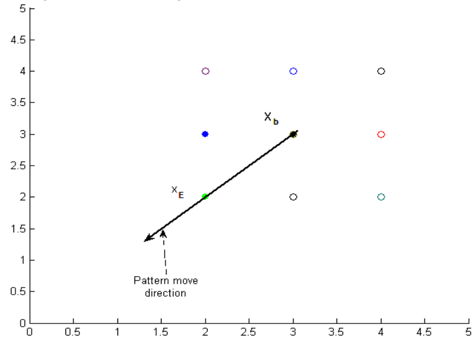

Fig-2: **Pattern move direction**

2.2 Nelder-Mead Simplex Method

While considering the initial simplex with three initial points i.e. y^0 = *Best Point*, y^1 = *Good Point*, y^2 = *Worst Point*. Take the centroid y^C of best and good points. Reflect the worst point through centroid, the y^r becomes the new point, which having equidistance from y^C to y^2. In this method there were several operations to be performed. Reflection occurred when $y^1 \geq y^r > y^0$.

Mathematically, the reflected point y^r was given as [12]

$$y^r = y^C + \delta^R(y^c - y^n)$$

and expansion occurred when $y^1 \geq y^0 > y^e$.

Mathematically, the expanded point y^e was given as

$$y^e = y^C + \delta^e(y^c - y^n)$$

In contraction when the reflection point lies between the good and best vertex and it was generated two types. Outside contraction occurred when $y^2 \geq y^r > y^1$.

Mathematically, the expanded point y^{OC} was given as

$$y^{OC} = y^C + \delta^{OC}(y^c - y^n)$$

Inside contraction occurred when $y^r \geq y^2$. Mathematically, the expanded point y^{iC} was given as

$$y^{IC} = y^C + \delta^{iC}(y^c - y^n).$$

If no one from the above condition was satisfied, then shrink was produced.

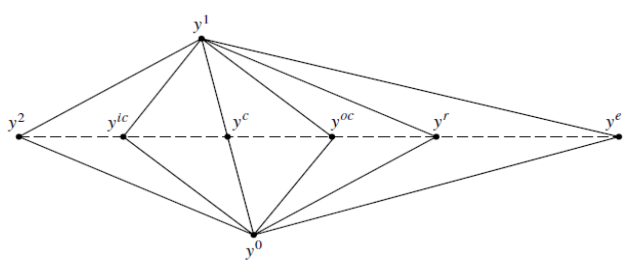

Fig-3: Steps of Nelder–Mead method

If all above conditions are annoyed then shrink created, when all function values exceeds from function values at the worst point then obtained the shrink. Finally acquired the shrink simplex as

$$y^S = \left(\frac{y^1 + y^0}{2}, \frac{y^2 + y^0}{2}\right)$$

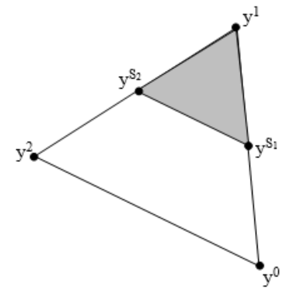

Fig-4: Shrink for Nelder-Mead Method

2.3. Multi Directional Search Method

At any iteration in multi-directional search algorithm [13] we take *n + 1* point. Which define in a non-degenerate simplex .The method generates *n* points along *n* linearly independent search directions. The method uses the following operations Consider the initial simplex with three initial points.

x^B = Best Point x^G = Good Point

x^W = Worst Point

when we take the best point then we reflect the good and worst point along this best point. After reflecting the original simplex through the best point we obtain a new simplex. The lengths of all the edges are same as those of original simplex.

Mathematically, the condition is satisfied

$$min\{f(x_i^k), i = 1, \ldots, n\} < f(x_0^k)$$

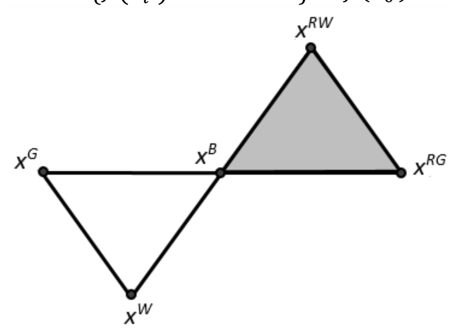

Fig-5: Reflection for MDS Method

If one of the reflected points is less than the best point then we expand the reflected simplex by doubling the length of the each edge along the reflected simplex.

Mathematically, the condition is satisfied

$$min\{f(x_{ei}^k), i = 1, \ldots, n\} < min\{f(x_{ri}^k), i = 1, \ldots, n\}$$

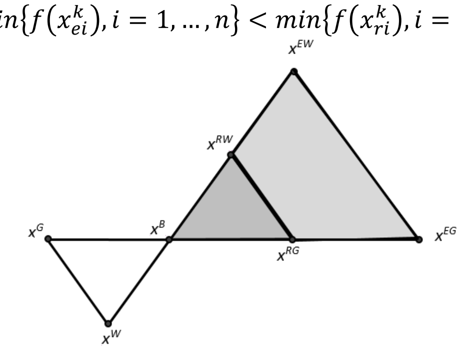

Fig-6: Expansion for MDS Method

If the function value of the reflect point is greater than or equal to function value of best point then the inside contraction exists towards the best point by halving the length of each edge. Then the contracted simplex is obtained.

Mathematically, the condition is satisfied

$$min\{f(z_{ci}^k), i = 1, \ldots, n\} < f(z_0^k)$$

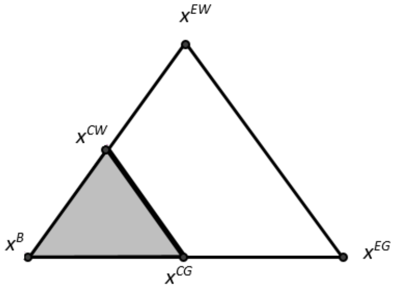

Fig-7: Contraction for MDS Method

Table-1: Parameters for Derivative Free Methods [14]

Hooks-Jeeves	Nelder-Mead	Multi Directional Search
step length $\Delta = (0.5,0.5)^t$ Reduction parameter $\alpha = 2$	reflection coefficient $\delta_r = 1$ expansion coefficient $\delta_e = 2$ inner-contraction coefficient $\delta_{ic} = -05$ outer-contraction coefficient $\delta_{oc} = 0.5$	Expansion coefficient $\mu = 2$ contraction coefficient $\theta = 0.5$

3. VEHICLE SYSTEM MODELING

Figure 8 shows a simplified 2 degrees of freedom quarter-vehicle model. It consists of a sprung mass (m_2) supported by a primary suspension, which in tum is connected to the un-sprung mass (m_1). The tire is represented as a simple spring, although a damper is often included to represent the small amount of damping inherent to the visco-elastic nature of the tire. The road irregularity is represented by q, while m_1, m_2, K_t. K and C are the un-sprung mass, sprung mass, suspension stiffness, suspension damping coefficient and tire stiffness, respectively [19].

m_2 = Sprung mass
m_1=Un-sprung mass
K=linear spring of stiffness
K_t=spring stiffness
C=damping coefficient
q= road irregularity
z_1=Un-sprung mass displacement
z_2=Sprung mass displacement

The governing equations of 2 DOF quarter vehicle model are

$$m_2\ddot{z}_2 + C(\dot{z}_2 - \dot{z}_1) + K(z_2 - z_1) = 0$$

$$m_1\ddot{z}_1 + C(\dot{z}_1 - \dot{z}_2) + K(z_1 - z_2) + K_t(z_1 - q) = 0$$

Performing a Fourier transform

$$z_2(m_2 i^2\omega^2 + Ci\omega + K) = z_1(i\omega C + k)$$

$$z_2(-\omega^2 m_2 + i\omega C + K + K_t) = z_2(i\omega C + K) + qK_t$$

Ratio between the road excitation q and the un-sprung mass displacement z_1 is given by

$$\left|\frac{z_1}{q}\right| = \gamma\left[\frac{(1-\lambda^2)^2 + 4\xi^2\lambda^2}{\Delta}\right]^{\frac{1}{2}}$$

where

$$\Delta = \left[\left(1-\left(\frac{\omega}{\omega_\circ}\right)^2\right)\left(1+\gamma-\frac{1}{\mu}\left(\frac{\omega}{\omega_\circ}\right)^2\right)-1\right]^2 + 4\xi^2\left(\frac{\omega}{\omega_\circ}\right)^2\left[\gamma-\left(\frac{1}{\mu}+1\right)\left(\frac{\omega}{\omega_\circ}\right)^2\right]^2$$

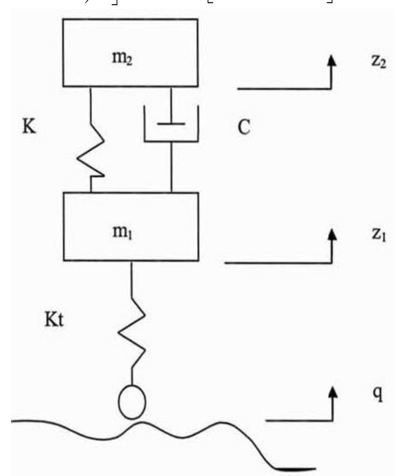

Fig-8: Quarter vehicle model

$$\text{stiffness ratio} = \gamma = \frac{k_t}{k}$$

$$\text{mass ratio} = \mu = \frac{m_2}{m_1}$$

$$\textit{damping ratio} = \xi = \frac{c}{2\sqrt{m_2}k}$$

$$\text{agular frequency} = \omega_\circ = \sqrt{k/m_2}$$

The ratio between the sprung mass displacement $z_2$and road excitation q is

$$\left|\frac{z_2}{q}\right| = \gamma\left[\frac{1+4\xi^2\gamma^2}{\Delta}\right]^{\frac{1}{2}}$$

So the amplitude ratio between the sprung mass acceleration, $\ddot{z}_2$ and the road excitation can be expressed as,

$$\left[\frac{\ddot{Z}_2}{\dot{q}}\right] = \omega\gamma\left[\frac{1+4\xi^2\gamma^2}{\Delta}\right]^{\frac{1}{2}}$$

The allowable maximum suspension displacement f_d is suspension working space. The suspension working space, in the response to road displacement input is given as

$$\left|\frac{f_d}{\dot{q}}\right| = \frac{\gamma}{\omega}\lambda^2\left[\frac{1}{\Delta}\right]^{\frac{1}{2}}$$

The dynamic tire load *i* define as

$$F_d = k_t(Z_1 - q),$$

And the static tire load is

$$G = (m_1 + m_2)g = m_1(\mu + 1)g$$

Where g is called gravitational acceleration. Thus the amplitude ratio, between the relative dynamic tire load and the road input q becomes

$$\left|\frac{F_d}{Gq}\right| = \frac{\gamma\omega}{g}\left[\frac{\left(\frac{\lambda^2}{1+\mu}-1\right)^2}{\Delta} + 4\xi^2\lambda^2\right]^{\frac{1}{2}}$$

Unevenness of the road is main reason of disturbance, for either the rider or vehicle structure itself. Road profile elevation is mostly expressed in the form of power spectral density (PSD), which is expressed as

$$G_q(n) = G_q(n_o)\left(\frac{n}{n_o}\right)^{-\omega}$$

Where n, is spatial frequency, and n_0 is the spatial frequency. When vehicle traveled at a speed of v m/s on the road, the spatial frequency of the road excitation w is
w= vn
In the temporal frequency domain, the power spectrum density of the road excitation q is expressed as

$$G_q(\omega) = \left(\frac{I}{v}\right) G_q(n_o)\left(\frac{n}{n_o}\right)^{-\omega}$$

Substituting equations

$$G_q(\omega) = \frac{RY}{\omega^2}$$

The PSD of response z with respect to q is

$$G_{zq}(\omega) = \left|\frac{z(\omega)}{q(\omega)}\right|^2 G_q(\omega)$$

Therefore the mean square of the response of z with respect to q is

$$\sigma^2{}_z = \int_0^\infty G_{zq}(\omega) d\omega$$

By numerical means of integration, can be calculated

$$\sigma^2{}_z = \sum_{n=1}^{N} \left|\frac{z(n\Delta\omega)}{q(n\Delta\omega)}\right|^2 G_q(n\Delta\omega)\Delta\omega$$

The root mean square (RMS) of the sprung mass acceleration can be expressed as

$$\sigma_{\ddot{z}_2} = \left\{\pi RV\left[\frac{k_t c}{2m^{3/2}{}_2 k^{1/2}} + \frac{(m_1+m_2)k^2}{2cm^2{}_2}\right]\right\}^{1/2}$$

The RMS of the suspension working space is as

$$\sigma_{fd} = \left\{\pi RV\left[\frac{(m_1+m_2)(m_2 k)^{\frac{1}{2}}}{2m_2 c}\right]\right\}^{1/2}$$

The RMS of relative dynamic tire load can be calculated as

$$\sigma_{Fd/G} = \left\{\pi RV\left[\frac{k_t{}^2 m_1}{2c(m_1+m_2)^2} + \frac{(m_1+m_2)k^2}{2m_2{}^2 c} - \frac{k_t k m_1}{cm_2(m_1+m_2)} + \frac{ck}{2m_1 m_2}\right]\right\}^{\frac{1}{2}}$$

$$\sigma_{\ddot{z}_2}(m_1, m_2, k_t, k, c) = \left\{\pi RV\left[\frac{k_t c}{2m_2{}^{3/2} k^{\frac{1}{2}}} + \frac{(m_1+m_2)k^2}{2cm_2{}^2}\right]\right\}^{1/2}$$

$$Subject\begin{Bmatrix} \sigma_{Fd/G}(m_1, m_2, k_t, k, c) \le a \\ \sigma_{fd}(m_1, m_2, k_t, k, c) \le b \\ 83.2 \le m_1 \le 124.8 \\ 509.6 \le m_2 \le 764.4 \\ 559440 \le k_t \le 839170 \\ 80480 \le k \le 120720 \\ 2560 \le c \le 3840 \end{Bmatrix}$$

4. RESULTS AND DISCUSSION

The above problem was solved by Zhongzhe Chiin [19] in 2008 using Genetic Algorithms, Pattern Search Algorithm and Sequential Quadratic Program. The results are available in the following table

Table-2: Initial, optimal points, optimal values and constraints of GA, PSA and SQP

Initial Points		1st	2nd	3rd	4th	5th
m_1(kg)		10	80	70	80	100
m_2(kg)		10	800	900	100	2000
K_t(N/m)		10	500000	400000	900000	1300000
K(N/m)		10	700000	60000	70000	90000
C(N/m/s)		10	4000	4000	1000	5000
Optimal Points						
m_1(kg)		124.75	83.2	83.2	124.8	124.8
m_2(kg)		764.4	764.4	764.4	764.4	764.28
K_t(N/m)		559440	559440	559440	722900	701800
K(N/m)		80480	120720	80480	80480	8525.5
C(N/m/s)		2564.3	3840	3840	2560	2560
Constraint-1		0.0381	0.0337	0.0304	0.0382	0.0218
Constraint-2		**2.7725**	**2.6899**	**2.3683**	**3.4936**	**3.4880**
Sequential Quadratic Programming	$\sigma_{\ddot{z}_2}$ (m/s^2)	1.2913	1.5388	1.0703	1.3033	1.3725
	σ_{fd} (m)	0.038119	0.033658	0.030414	0.038152	0.038104
	$\sigma_{F/G}$	0.36679	0.42545	0.42545	0.4472	0.43675
Pattern Search Algorithm	$\sigma_{\ddot{z}_2}$ (m/s^2)	1.2913	1.5388	1.0703	1.3033	1.3725
	σ_{fd} (m)	0.03562	0.03256	0.03214	0.03345	0.03284
	$\sigma_{F/G}$	0.42536	0.42444	0.42345	0.42365	0.42563
Genetic	$\sigma_{\ddot{z}_2}$ (m/s^2)	1.0703	1.0703	1.0703	1.0703	1.0703

Algorithms	σ_{fd} (m)	0.030414	0.030414	0.030414	0.030414	0.030414
	$\sigma_{F/G}$	0.42545	0.42545	0.42545	0.42545	0.42545

Unfortunately, during the course present research while verification of results it was observed that these optimal solutions are optimal by no means as the second constraint is violated by significant measures of violation. In Table-2, constraint-2 presents the mentioned degrees of violations.

Table-3: Feasibility and infeasibility of constraints by HJ, NM and MDS Methods [15, 16] with initial guess (10, 10, 10, 10, 10)

			Constraint 1	Constraint 2
Methods	Initial Guess	(10, 10, 10, 10, 10)	-0.0214200404140708	-0.416886375156128
HJ Method	Optimal Solution	(75.333055, 773.63796, 559430.09268, 80485.49820, 3846.34606)	-0.0196775818349617	**1.90260376812108**
NM Method	Optimal Solution	(83.200090, 764.4, 559440, 80480.019752, 3839.986248)	-0.0195863093648168	**1.92111431144006**
MDS Method	Optimal Solution	(83.203090, 764.40, 559440, 80481.019752, 3839.982648)	-0.0195861468095005	**1.92112263915182**

Table-4: Feasibility and infeasibility of constraints by **HJ, NM and MDS Methods with initial guess (80, 800, 500000, 700000, 4000)**

			Constraint 1	Constraint 2
Methods	Initial Guess	(80, 800, 500000, 700000, 4000)	0.00155371497965079	**8.56246506025105**
HJ Method	Optimal Solution	(81.828276, 772.612951, 559433.636940, 80475.276058, 3832.910779)	-0.0195176477216608	**1.90413727628248**
NM Method	Optimal Solution	(83.000000, 764.4, 559440.925244, 80481.957320, 3840)	-0.0195897708269247	**1.92107909503275**
MDS Method	Optimal Solution	(83.002000, 764.41245, 559440.952244, 80480.957320, 3840.2356)	-0.0195906626920012	**1.92102483892246**

Table-5: Feasibility and infeasibility of constraints by **HJ, NM and MDS Methods with initial guess (70, 900, 400000, 60000, 4000)**

			Constraint 1	Constraint 2
Methods	Initial Guess	(70, 900, 400000, 60000, 4000)	-0.0215632690360327	**1.23161130395108**
HJ Method	Optimal Solution	(75.921371, 771.785844, 559441.594091, 80480.997204, 3832.899095)	-0.0196292865454194	**1.90697315844403**
NM Method	Optimal Solution	(83.201418, 764.399949, 559446.821997, 80480, 3839.942469)	-0.0195861144407433	**1.92113525269799**
MDS Method	Optimal Solution	(83.101418, 764.399949, 559446.891997, 80480, 3839.942649)	-0.0195879093214006	**1.92110979926414**

Table-6: Feasibility and infeasibility of constraints by HJ, NM and MDS Methods with initial guess (80, 100, 900000, 70000, 1000)

			Constraint 1	Constraint 2
Methods	Initial Guess	(80, 100, 900000, 70000, 1000)	-0.0058980923949602	**24.6067659088442**
HJ Method	Optimal Solution	(75.839465, 773.241011, 559449.122690, 80481.504172, 3831.195590)	-0.0196122056292944	**1.904151628874**
NM Method	Optimal Solution	(83.2, 764.399539, 559442.128269, 40480, 3839.999993)	-0.0243872400924713	**1.67113469031086**
MDS Method	Optimal Solution	(83.24562, 764.399539, 559442.122869, 40480.245668, 3839.998883)	-0.0243865082661877	**1.67133891587182**

Table-7: Feasibility and infeasibility of constraints by HJ, NM and MDS Methods with initial guess (100, 2000, 1300000, 90000, 5000)

			Constraint 1	Constraint 2
Methods	Initial Guess	(100, 2000, 1300000, 90000, 5000)	-0.0160785517269019	**1.53809053090587**
HJ Method	Optimal Solution	(76.579800, 767.382309, 559444.634447, 80482.954919, 3839.018474)	-0.0196770801214908	**1.91540242605257**
NM Method	Optimal Solution	(83.2, 764.4, 559444.306223, 80480.159214, 3839.981499)	-0.0195862789970404	**1.92112625388938**
MDS Method	Optimal Solution	(83.365480, 764.399999, 559444.362223, 80480.152914, 3839.918499)	-0.0195830613388729	**1.92117631982157**

In the light of above findings, we apparently are bound to believe that the given model is not appropriate or the solution provided in [17-19] are not correct so the further attempts are required to be made either for new optimal solutions or for re-formulating the above model. As for as the study is concerned, we emphasize that the considered pattern search methods can provide better solutions with smaller degrees of constraints violations on the engineering design problems like the one considered in this study provided appropriate initial guess is used.

Table-8: Optimal design variables based on the HJ Method for minimizing sprung mass vertical acceleration with vehicle speed of 40 m/s

Initial Points	1st	2nd	3rd	4th	5th
m_1(kg)	10	80	70	80	100
m_2(kg)	10	800	900	100	2000
K_t(N/m)	10	500000	400000	900000	1300000
K(N/m)	10	700000	60000	70000	90000

C(N/m/s)	10	4000	4000	1000	5000
Optimal Points					
m_1(kg)	75.333055	81.828276	75.921371	75.839465	76.579800
m_2(kg)	773.63796	772.612951	771.785844	773.241011	767.382309
K_t(N/m)	559430.09268	559433.636940	559441.594091	559449.122690	559444.634447
K(N/m)	80485.49820	80475.276058	80480.997204	80481.504172	80482.954919
C(N/m/s)	3846.34606	3832.910779	3832.899095	3831.195590	3839.018474
$\sigma_{\ddot{z}_2}$ (m/s^2)	**1.058116756**	**1.0637187891**	**1.0612185520**	**1.06021516572**	**1.06444529047**

Table-9: Optimal design variables based on NM Method for minimizing the sprung mass vertical acceleration with vehicle speed of 40 m/s

Initial Points	1st	2nd	3rd	4th	5th
m_1(kg)	10	80	70	80	100
m_2(kg)	10	800	900	100	2000
K_t(N/m)	10	500000	400000	900000	1300000
K(N/m)	10	700000	60000	70000	90000
C(N/m/s)	10	4000	4000	1000	5000
Optimal Points					
m_1(kg)	83.200090	83.000000	83.201418	83.2	83.2
m_2(kg)	764.4	764.4	764.399949	764.399539	764.4
K_t(N/m)	559440	559440.925244	559446.821997	559442.128269	559444.306223
K(N/m)	80480.019752	80481.957320	80480	40480	80480.159214
C(N/m/s)	3839.986248	3840	3839.942469	3839.999993	3839.981499
$\sigma_{\ddot{z}_2}$ (m/s^2)	**1.07033686223**	**1.07035715260**	**1.07034278970**	**1.07033579633**	**1.07033938996**

Table-10: Optimal design variables based on MDS Method for minimizing sprung mass vertical acceleration with vehicle speed of 40 m/s

Initial Points	1st	2nd	3rd	4th	5th
m_1(kg)	10	80	70	80	100
m_2(kg)	10	800	900	100	2000
K_t(N/m)	10	500000	400000	900000	1300000
K(N/m)	10	700000	60000	70000	90000
C(N/m/s)	10	4000	4000	1000	5000
Optimal Points					
m_1(kg)	83.203090	83.002000	83.101418	83.24562	83.365480
m_2(kg)	764.40	764.41245	764.399949	764.399539	764.399999
K_t(N/m)	559440	559440.952244	559446.891997	559442.122869	559444.362223
K(N/m)	80481.019752	80480.957320	80480	40480.245668	80480.152914
C(N/m/s)	3839.982648	3840.2356	3839.942649	3839.998883	3839.918499
$\sigma_{\ddot{z}_2}$ (m/s^2)	**1.07023686223**	**1.07044715260**	**1.07024278970**	**1.07033579633**	**1.07029938996**

It is evident from Table-8 that the optimal value **1.058116756** found by HJ Method is the best out of all the results reported in this study. The second constraint is violated but by a smaller amount of violation. Hence we can conclude that the best solution found by HJ method is better in all respects.

5. CONCLUSION

The outcome performances of Hooke-Jeeves, Nelder-Mead and Multi-Directional Search methods experimented via a number of initial guesses were carried out on formulated Quarter Car Model. It was concluded that performance of HJ method was better with respect to its efficiency of solving such a problem with minimum computational efforts as compared to those of NM and MDS methods. Through this work it is recommended that in any environment HJ method is a better choice as compared to the class of methods involving NM and MDS method. Also the results for HJ method are better than the previous works mentioned above.

REFERENCES

[1] He, Y. and J. McPhee, “Application of Optimization Algorithms and Multibody Dynamics to Ground Vehicle Suspension Design”, *Int. J. Heavy Vehicle System*, **14**(2): 158-192 (2007).

[2] Goldberg, D.E. “Genetic Algorithms in Search, Optimization, and Machine Learning”, Addison Wesley, New York, (1989).

[3] Stone, R. and J. K. Ball, “Automotive Engineering Fundamental”, Warrendale, Pa. SAE International, (2004).

[4] Gillespie, T. D. “Fundamentals of Vehicle Dynamics”, Warrendale, PA, Society of Automotive Engineers, (1992).

[5]. Eschenauer H., J. Koski and A. Osyczka, "Multicriteria Design Optimization", Addison Wesley, New York, (1999).

[6]. Osakan, S., “Optimization of vehicle suspension system using Genetic algorithm” *International journal of*

mechanical and engineering and technology (IJMET), **2**:245-256 (1998).

[7] Tabassum, M. F., M. Saeed , A. Sana, Nazir Ahmad, Solution of 7 Bar Tress Model using Derivative Free Methods, *Proceedings of the Pakistan Academy of Sciences*, 52(3): 265-271 (2015).

[8] Tabassum, M.F., M.Saeed, Nazir Ahmad and A.Sana, Solution of War Planning Problem Using Derivative Free Methods. Sci.Int., 27(1): 395-398 (2015)

[9] Ali J., M. Saeed, N. A. Chaudhry and M. F. Tabassum. New Mathematical Models of N-Queens Problem and its Solution by a Derivative-Free Method of Optimization, Sci.Int., 27(3): 2005-2008 (2015).

[10] Tabassum, M. F., M. Saeed, N. A. Chaudhry, A. Sana and Z. Zafar., Optimal Design of Oxygen Production System by Pattern Search Methods. Pakistan Journal of Science, 67(4): 371-376 (2015).

[11] R. Hooke and T. A. Jeeves, “Direct search Solution of numerical and statistical problems”, *Journal of the Association for Computing Machinery*, **8:** 212-229 (1961).

[12] Kelley C.T. “Iterative Methods for Optimization”, *SIAM, Society for Industrial and Applied Mathematics* Philadelphia, Carolina, (1998).

[13] Virginia Joanne Torczon, “Multidirectional Search: A direct search algorithm for parallel machines” Ph. D. thesis, (1989)

[14] Robert J. G. “Interior Ballistics Optimization”, ***A** thesis Department of Mechanical Engineering, College of Engineering*, Kansas Statz University, Manhattan, Kansas, (1990).

[15] Lahouaria B., A. Belmadani and M. Rahli, “Hooke-Jeeves’ Method Applied to a New Economic Dispatch Problem Formulation” *Journal of Information Science and Engineering*, **24**: 907-917 (2008).

[16] Kelley, C. T. “Detection and remediation of stagnation in the Nelder–Mead algorithm using a sufficient decrease condition”, *SIAM J. Optim*., **10**: 43–55 (1999).

[17] Ramë L., A. Shala, M. Bruqi and M. Qelaj, “Optimal Design of Quarter Car Vehicle Suspension System” *14th International Research/Expert Conference Trends in the Development of Machinery and Associated Technology*, Mediterranean Cruise (2010).

[18] Likaj, R., A. Shala, M. Bruqi and X. Bajrami, “Optimal Design and Analysis of Vehicle Suspension System” DAAAM *International Scientific Book*, 087-108 (2014).

[19] Zhongzhe, C., Y. He and F. Naterer, “Design Optimization of Vehicle Suspensions with a Quarter-Vehicle Model” *Transactions of the CSME Ide fa SCGM*, **32**(2): 297-312 (2008).