

# THE VARIABLE ELASTICITY OF SUBSTITUTION PRODUCTION FUNCTION: A CASE STUDY FOR PAKISTANI BANKING SECTOR

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**ABSTRACT:** In this study the elasticity of substitution (EOS) between capital and labor inputs estimates for Pakistan's banking sector. The estimation is done on the basis of a variable elasticity of substitution production function (VESPF), using annual time series data for the period 1980-2013. This approach trots out configuration of EOS and rendered its implication on sector as it is an important determinant of sustainability of growth and movements in factor income shares. The FMOLS method has been employed to provide optimal estimates of cointegrating regression for estimation of EOS. Findings facilitate in highlighting the importance of EOS as its high value implies that there prevails flexibility to adjust the factors of production in response to changes in factor prices.

**Key Words:** Production Function, Variable Elasticity of Substitution (VES), Unit Root tests, Cointegration, Fully Modified Ordinary Least Square (FMOLS).

## 1. INTRODUCTION

Banking structure can essentially give a bundle to the growth of an economy. In the last twenty years, the environment in which the banks operate has evolved dramatically around the world as trends towards globalization has increased all over the world. So, much attention is being paid to establishing its production relationship. The possibility of efficient capital-labor substitution is crucial for the success of most fiscal, financial and technological policies that are designed to increase employment in developing countries through the adoption of labor-intensive techniques of production. If the possibilities exist for efficient capital labor substitution then they can be substituted for capital without necessarily resulting in a decline in output. This issue crucially depends on whether elasticity of substitution is positive. The variation in the estimates of the capital labor substitution mainly depends on the type of production function used i.e. Constant Elasticity of Substitution (CES) and Variable Elasticity of Substitution (VES). In the past few decades VES production function (VESPF) has drawn the attention of economists as it relaxes restriction on substitutability between inputs and provides space to study and analyze the impact of technical change, measured by K-L ratio in economic narrative, on production process. The CES production function (CESPF) arbitrarily constrains EOS to be a constant and does not allow it to vary with a change in factor input ratio. As demonstrated by Hicks input can vary depending on the factor combination and output. Thus a CESPF with a constant EOS will lead to specification bias. VESP incorporates the impact of the EOS and the K/L ratio thus eliminating specification bias associated with changes in factor ratio. The VESPF does not require that the EOS be the same. The EOS can increase steadily from 0 to  $\infty$ . The contribution of factors in the production process cannot be measured separately as it is always a function of a combination of factors which results in an output. Thus the ratio of factors and changes in it become an important aspect of study.

### 1.1 Objectives

To estimate the elasticity of substitution between labour and

capital in Pakistan's banking industry which is done on the basis of a VES production function.

## 2. LITERATURE REVIEW

There have been a number of studies on the  $\sigma$  since introduction of the concept by Hicks in 1932. Later in 1971, Revankar contributed by giving the idea of variable version of elasticity of substitution. Despite its superiority over constant elasticity of substitution, meagre literature exists on it. Relevant literature is reviewed before conducting the analysis.

Battese and Malik [3] analysed the capital labor substitution for selected manufacturing industries in Pakistan for both large and small scale firms and found that CRS exist for majority of industries. By using efficient method of evaluation of the substitution elasticities, for firm level data found that almost all industries included in the sample have elasticities greater than zero. For small scale firms the elasticities are less than one though for large scale firms the elasticities are not different from one. He concluded that stringent conditions on which substitution is analysed from aggregate data, analysis of data has to be identified at more disaggregated levels.

Adekoya [2] examined the notion of productivity in a farm firm. He measured that how suitably resources are utilized and combined in a farm firm to accomplish outputs. He concluded that productivity of Nigerian farms increased by government's involvement in manpower development. He also argued that there is a need to introduce community development programmes by the Directorate for Foods, Roads and Rural Infrastructure and it requires participation by women and elderly to implement such programmes.

Ekanem and Oyefusi [5] estimated the CD and CES for the manufacturing industry in Nigeria for the period 1980-1997. He has taken into account the process of unused capacity that was characterized by the industry in recent times. He found that among the two production functions the CD production function estimations give better results while taking into considering all the pertinent econometric criteria. He

concluded that the CDPF gives better explanation of aggregate PF in Nigerian manufacturing industry for the period studied.

Ekanem [4] estimated TFP for the Nigerian banking industry for the period 1986-2000. He used the methodology of Growth Accounting Model based on aggregate production functions. He found that the Cobb-Douglas was the most suitable production function that defines the production in Nigerian banking industry. The slopes of the estimated CD function were deployed to refine the Growth Accounting Model. His results showed that aggregate Q at an average grew at 4.29%, while at average TFP grew at rate of 3.33%. He analyzed that TFP provided 78% of the documented growth.

Stella [9] assessed the impact of Information Communication Technology (ICT) on the productivity growth of the Nigerian banking sector. He conceptualized the impact on productivity after deductions for depreciation and labour expenses as capacity to make positive contributions to output. He used translog Production function and the CAMEL framework for estimations. He found a significant increase in bank output such as loans and other assets to changes in costs on information communication technology. He also found that ICT labour costs impacted more on bank output more than capital expenses on ICT gadgets. He gave recommendation to increase investments in information technology in order to increase productivity of banks.

Abiola [1] examined productivity in the banking sector of Nigeria by using two major production functions CD and CES known in the economic literature. The estimation results of ordinary least square (OLS) showed the substitution parameters  $\alpha$  and  $\beta$  have positive values of less than one. The addition of these values of  $\alpha$  and  $\beta$  is greater than one, which shows that as the banking sector inputs becomes double, the output in terms of deposit will be greater than double. The result has supported the theory as substitution parameters in the CESPF were positive. The speed of adjustment was also found to be good for the long run.

Khan et al. [7] investigated the constant version of elasticity of substitution by using time series data of banking sector of Pakistan. They find increasing returns to scale and constant elasticity of substitution was found to be greater than 1. Which reflected a substantial degree of substitution between the inputs. This research, however, did not allow for variability of EOS over time. However, this study dispels the assumption of constancy of EOS and revisits the EOS.

**3. DATA AND METHODOLOGICAL ISSUES**

Data is secondary in nature and is collected from the period, 1980 to 2013 consisting of yearly observations for each variable. Data were obtained from different publication of Money and Banking Statistics provided by the State Bank of Pakistan containing annual information on the balance sheet for all banks operating in Pakistan. To estimate a production function, time series data is needed on output (measured by total deposits, TA). Fixed assets (FA) are used as proxy of capital inputs and these are the values of buildings, land, machinery, tools and office equipment, etc. As labor input Wages and Salaries (SW) are used as proxy for labor instead

of number of employees due to lack of data which include all remunerations, compensation for work done during the year. The version of VES production function is borrowed from [6] and [8] which is:

$$Y = A \cdot e^{\lambda t} \cdot K^\alpha \cdot L^{(1-\alpha)} \cdot e^{\beta k}$$

In current paper the specific VES specification shall be as follows:

$$TD = A \cdot e^{\lambda t} \cdot (FA)^\alpha \cdot (SW)^{(1-\alpha)} \cdot e^{\beta \left(\frac{FA}{SW}\right)}$$

Dividing by SW on both sides.

$$\frac{TD}{SW} = A \cdot e^{\lambda t} \cdot FA^\alpha \cdot (SW)^{-\alpha} \cdot e^{\beta \left(\frac{FA}{SW}\right)}$$

$$\frac{TD}{SW} = A \cdot e^{\lambda t} \cdot \left(\frac{FA}{SW}\right)^\alpha \cdot e^{\beta \left(\frac{FA}{SW}\right)}$$

$$\ln \left| \frac{TD}{SW} \right| = \ln |A| + \lambda \cdot t + \alpha \cdot \ln \left| \frac{FA}{SW} \right| + \beta \cdot \left(\frac{FA}{SW}\right)$$

Econometric specification of the above equation for time series data is as follows:

$$\ln \left| \frac{TD_t}{SW_t} \right| = \ln |A| + \lambda_t \cdot t + \alpha_t \cdot \ln \left| \frac{FA_t}{SW_t} \right| + \beta_t \cdot \left(\frac{FA_t}{SW_t}\right) + \varepsilon_t$$

**4. RESULTS AND INTERPRETATION**

To solve the problem of stationarity a unit root test has been employed. The test of ADF and PP are used to check H0 of a unit root or non-stationary of the series alongside the alternative hypothesis of stationary. These tests have been executed at level as well as 1st difference. The PP and ADF statistics indicate that variables are order integration, I(1).

**Table 1: Unit Root Test**

	ADF		PP	
	Level	1 <sup>st</sup> diff.	Level	1 <sup>st</sup> diff.
ln  TD	0.2018 (0.969)	-5.3733 (0.000)	0.1970 (0.968)	-5.3734 (0.000)
ln  FA	0.2112 (0.969)	-8.2605 (0.000)	0.0900 (0.960)	-8.5168 (0.000)
ln  SW	0.7190 (0.991)	-3.4374 (0.017)	0.4876 (0.984)	-3.4374 (0.017)
$\left(\ln \left  \frac{FA}{SW} \right  \right)^2$	-1.1697 (0.675)	-2.1988 (0.030)	-1.2603 (0.636)	-15.4183 (0.000)
$\left(\ln \left  \frac{FA}{SW} \right  \right)$	-0.1067 (0.940)	-8.3550 (0.000)	-0.3698 (0.903)	-8.8751 (0.000)
$\left(\ln \left  \frac{TD}{SW} \right  \right)$	-0.3110 (0.913)	-5.2462 (0.001)	-0.3347 (0.909)	-5.2855 (0.000)

Source: Authors' estimations

The AIC suggests 4 lags of VAR model. The reported trace test statistic for the H0 of no cointegration (H0: r = 0) is 31.5680 which is sound above the critical value of 29.7970 at the 5 percent (5%) significance level. This test as a result, concludes that there is only one cointegrating vector among the TD, FA and SW. The maximum eigenvalue statistic testing the H0 of no cointegration (H0: r = 0) is rejected at the 5% significance level. This result provides an additional evidence in support of the above conclusion that there exists only one cointegrating relationship among the three variables under examination.

**4.1 Estimation Results of the FMOLS**

The prerequisite for Fully modified ordinary least square in favor of approximation of parameters in the long run parameters reveal a cointegrating relation between a set of I (1) variables is contented in support of our data. So we can pertain this method to estimate the vector of cointegration.

Table 4 presents the estimation results from the FMOLS analysis. Other parameters in VES production are found

using the above mentioned VES function given by [6] and [8].  $\lambda_t$ ,  $\alpha_t$  and  $\beta_t$  are these parameters values of which are listed in table. For fixed t the ratio between capital and labour income is a linear function of capital intensity. Here  $\beta$  is the income share of capital.  $\lambda_t$  is the technological estimates of inputs.

**Table 2: The Trace Test**

Hypothesis		Eigen value	Trace test	5% Critical Value	p-value
H <sub>0</sub>	H <sub>1</sub>				
r=0	r=1	0.575	31.568	29.797	0.031
r≤1	r=2	0.196	6.756	15.495	0.606
r≤2	r=3	0.014	0.418	3.841	0.518

Note: Here r is number of cointegrating vectors.

Source: Authors' estimations.

**Table 3: The Maximum Eigenvalue Test**

Hypothesis		Eigen value	Max Eigen statistics	5% Critical Value	p-value
H <sub>0</sub>	H <sub>1</sub>				
r=0	r=1	0.575	24.812	21.132	0.015
r≤1	r=2	0.196	6.338	14.265	0.570
r≤2	r=3	0.014	0.418	3.842	0.518

Source: Authors' estimations

**Table 4: Results of FMOLS for VES**

**Variable Elasticity of Substitution (VES) PF Estimates**

Ln(A)	-4.3693	A	0.0127
$\lambda_t$	0.0798		
$\alpha_t$	0.3061		
$\beta_t$	0.4332		

Source: Authors' estimations

$$\ln \left( \frac{TD_t}{SW_t} \right) = -4.3693 + 0.0798 \cdot t + 0.3061 \left( \ln \left( \frac{FA_t}{SW_t} \right) \right) + 0.4332 \left( \frac{FA_t}{SW_t} \right)$$

$$TD = 0.0127(e^{0.0798t}) \cdot (FA)^{0.3061} \cdot (SW)^{0.6939} \cdot e^{0.4332 \left( \frac{FA}{SW} \right)}$$

For variable elasticity of substitution production function, the  $\sigma$  depends on K and L. Due to which it is called as variable elasticity of substitution (VES). It also implies that variable elasticity of substitution ( $\sigma_{ves}$ ) shall assume a number of values equivalent to the number of observations in time series study. Accordingly, Table 5 lists the  $\sigma_{ves}$  values for each year.

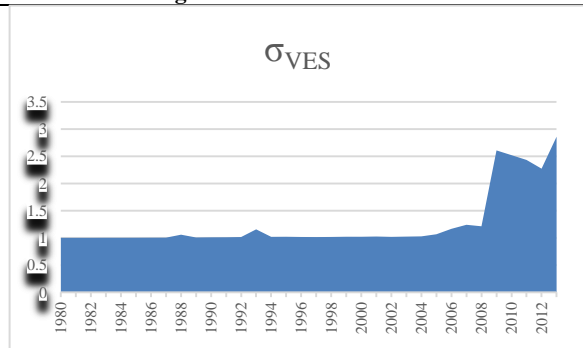
**Table 5: Variable Elasticity of Substitution**

Year	$\frac{FA}{SW} = \frac{\text{Fixed Assets}}{\text{Salaries and Wages}}$	$\sigma_{ves} = 1 - \left[ \frac{\beta \left( \frac{FA}{SW} \right)}{\left\{ \alpha + \beta \left( \frac{FA}{SW} \right) \right\}^2 - \alpha} \right]$
1980	0.0012	1.0025
1981	0.0011	1.0024
1982	0.0013	1.0025
1983	0.0015	1.0032
1984	0.0016	1.0033
1985	0.0017	1.0036
1986	0.0024	1.0048
1987	0.0025	1.0050
1988	0.0303	1.0597
1989	0.0040	1.0081
1990	0.0055	1.0112
1991	0.0059	1.0120
1992	0.0081	1.0164
1993	0.0849	1.1574
1994	0.0100	1.0201
1995	0.0105	1.0212
1996	0.0086	1.0174
1997	0.0080	1.0161
1998	0.0090	1.0181
1999	0.0108	1.0218
2000	0.0107	1.0216

2001	0.0129	1.0259
2002	0.0097	1.0196
2003	0.0121	1.0243
2004	0.0143	1.0286
2005	0.0344	1.0673
2006	0.0909	1.1676
2007	0.1367	1.2416
2008	0.1190	1.2137
2009	1.0528	2.6080
2010	1.0128	2.5206
2011	0.9695	2.4327
2012	0.8823	2.2727
2013	1.1534	2.8599
<b>Average <math>\sigma_{ves}</math></b>		<b>1.2621</b>

Source: Authors' estimations

**Figure 1: VES Over Time**



Source: Authors' drawing

Variable elasticity of substitution ( $\sigma_{ves}$ ) over the years has been greater than 1, showing high substitution-ability between the inputs. An upward trend in its value is also evident as seen in ending years. In other words, in the long run elasticity of substitution is increasing. Overall  $\sigma_{ves}$  over the years 1980-2013 is 1.2621 which is greater than 1. In a way these results corroborate with that in [7] who finds  $\sigma_{ces}$  as 1.129. It adds strength to the reliability of results of  $\sigma_{ves}$ . Both results of  $\sigma$  reveal that degree of substitution-ability between inputs is high. If the possibilities exist for efficient capital-labor substitution, then labor can be substituted for capital without necessarily resulting in a decline in output. The fast growing factor can be substituted for slow growing factor or the factor having higher productivity may be substituted for the factor having low productivity. It also implies that production costs can be cut down by substituting high input price factor with relatively lower input price factor. Banks will then change the combination of inputs in order to produce output that maximize profit or minimize cost because it has forward linkages to profitability and efficiency. Graphical representation of the above values is given below:

**5. CONCLUSION**

To seem into alteration in structure that took place with different developmental processes the EOS has always been an essential parameter. It brings out accessibility of regulation method to the policy makers along with a range of strategies of development. The magnitude of EOS infact influence the choice of possibilities available to the employers. Its value is greater than one over the years

implying high substitutability between inputs. So labour and capital can be substituted for each other without resulting in a decline in output. They are easily substitutable in the long term. Recommendation is given on the need to make policies to assimilate technical knowledge and equipment through the ability of banks in finding their ways to substitute factors of production according to the input price changes. An effort should be made that a rigid factor proportions can become more elastic, if there is any discovery of new production alternatives through the learning efforts of each bank.

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